Slide 1

Global Teacher Empowerment Network GTEN
Saturday 13 August 2022   16:00 – 18:00 London Time
VOLUME, SURFACE AREA AND CAPACITY

Toni Beardon   Caroline Ainslie   Tejumade Ogundipe

1

Slide 2

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

MATHS TOYS

Global Teacher Empowerment Network (GTEN)
PROGRAMME: VOLUME, SURFACE AREA AND CAPACITY

Learning Spiral

Put your name, country and the age group you teach on the chat

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:
Volume
Capacity
Surface area
Metric units gm, cc, ml
Factors
Properties of 3D objects
Volume and surface area of cone
Designing packaging

UPPER SECONDARY

LOWER SECONDARY

UPPER PRIMARY

LOWER PRIMARY

EARLY YEARS

STARTER ACTIVITY

15. **Upper Secondary**: Packing problems, volume of a cone investigation
14. **Upper Secondary**: Number patterns & formulae for 3D sequences
13. Reading scales and measuring doses of medicine
12. **Lower Secondary**: metric system, meaning of centi and milli
11. **Lower Secondary**: scale factors for enlargement in 3D
10. **Upper Primary**: DIFFERENT VOLUMES, SAME SURFACE AREA of cuboids
9. **Upper Primary**: SAME VOLUME, DIFFERENT SURFACE AREAS of cuboids
8. **Upper Primary**: surface area and volume of cuboids with calculations
7. **Lower Secondary**: objects with SAME VOLUME, DIFFERENT SURFACE AREAS
6. **Primary**: revision of area and volume by COUNTING squares and cubes
5. **Lower Primary**: investigating capacity, key questions
4. Diagnostic Assessment
3. **Lower Primary**: language, full, empty, more, less, capacity
2. **Early Years**: water play, learning about capacity
1. **Packets from the kitchen. Nets. Grams, Cubic Centimetres & Millilitres**

2

Slide 3

DO TALK RECORD
the basis of a good lesson
During this session you will need to wear 2 hats.

Teacher

Learner

DO the activities pretending that you only know what your learners know.

TALK to other teachers by commenting on the chat about how your learners might discover new ideas doing this activity.

RECORD in whatever way that is meaningful to you as a teacher and also in a way that will be meaningful to your learners.

Talk about the area being the number of unit squares

Talk about the volume being the number of unit cubes.

3

Slide 4

PACKETS FROM THE KITCHEN
Collect empty packets

What shapes?

How are they made? (nets)

How much do they hold?

HOW TO TURN A BOX INSIDE OUT

4

VOLUME MASS AND CAPACITY

What is the connection between these measures?
Answer on Chat

CUBIC CENTIMETRES (volume)

GRAMS (mass)

LITRES (capacity)

1 cubic centimetre = 1 gram = 1 millilitre

5

LIQUID MEASURES – TALKING ABOUT CAPACITY – EARLY YEARS

Free water play is good early learning experience for babies and small children.
Empty food containers are excellent toys for this play, don't throw them away.
Enrich the play-learning by play with sand and water, either at the beach or in a play area.

Most of the activity should be free play with water and cups of different sizes.
Ask the children to arrange the cups in order of size from the one that
holds the smallest quantity of liquid to the one that holds the most.
How many cupsful of the smallest cup fill each of the others?

Ask the children to test, by pouring from one cup to another,
to see if they have put the cups in order correctly.
They could compare the capacity of these cups with their own favourite drinking cup or mug.

6

LIQUID MEASURES – TALKING ABOUT CAPACITY – LOWER PRIMARY

Do this session outdoors or by the kitchen sink or in the bath. Collect empty cups and containers that you would otherwise throw away. Dry sand/rice can be used instead of water.

The learners should work on this activity in pairs or small groups with a variety of cups and plastic bottles to explore the capacities practically and talk about them.

Talk about pouring water from one container to another, introduce the words
CAPACITY, FULL, EMPTY, MORE THAN, LESS THAN.

Ask learners to put the containers in order.
Before testing this out and discussing the findings, give learners plenty of time to discuss how the capacities compare, and to ask and find answers to their own questions.

7

CAPACITY WORDS
Show this question to your group of learners and say:
**"Put up 1 finger if you think the answer is A,
2 fingers for B, 3 fingers for C and 4 for D".**

Bottle 3 has _____ juice than bottle 1, but _____ than bottle 2.

1   2   3

A — less, more
B — less, less
C — full, empty
D — empty, full

Get EVERY learner in the class to respond with 1, 2, 3 or 4 finger answer.

Help learners to develop the language
'more (or greater) than', 'equal to' and less than', 'full', 'empty'.

For older learners introduce the symbols   > and <.

8

Slide 9

LIQUID MEASURES – CAPACITY – LOWER PRIMARY

Ask learners to help you to find the answers to the questions.

Encourage them to ask other questions and together find the answers.

Key questions

1. Why do you think that cup is the largest?
2. (Or why is it the smallest?)
3. Tell me about your ideas.
4. Why do you think that?
5. Why do you think that's the smallest container?
6. How many of cup 3 (or cup 8) will fill cup 1?
7. How did you decide on that estimate?
8. Tell me what you are thinking.
9. How will you test out your ideas?
10. Why is it important to be able to measure capacity?
11. What do we use measuring spoons for?

Slide 10

AREA & VOLUME - PRIMARY

Use squared paper and cubes.

Ask your learners:

How many squares?

How many cubes?
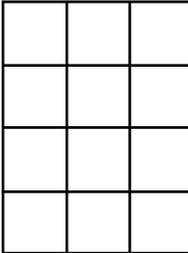
Talk about the area being the number of unit squares and the volume being the number of unit cubes.

What is the area of the rectangle?   **12 square units**

What is the volume of the cuboid?   **24 cubic units**

Put answers on Chat

Slide 11

SURFACE AREA & VOLUME
UPPER PRIMARY OR LOWER SECONDARY

Do these models have the same volume?   Answer 👍 for YES 👎 for NO   **4 cubic units**

Do they have the same surface area?   **18 square units**
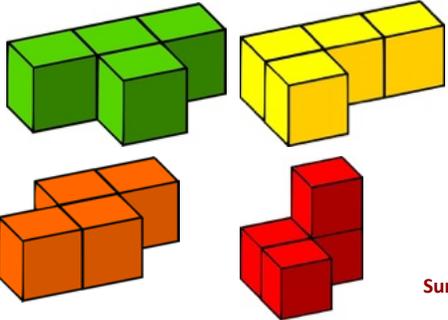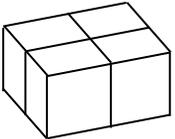
Can you make another model like this with the same volume and a different surface area?

Volume 4 cubic units
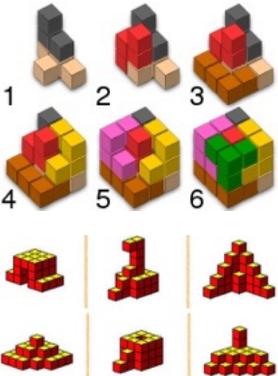**Surface Area 16 square units**

Slide 12

SURFACE AREA & VOLUME * SOMA CUBE PUZZLE
UPPER PRIMARY OR LOWER SECONDARY

What is the same and what is different about the pieces?

Which pieces have the same volume?

Which pieces have the same surface area?

SOLUTION

You can make your own puzzle from cubes. Like the tangram, there are many soma challenges.

Slide 13

SURFACE AREA & VOLUME - UPPER PRIMARY

Volume 24 cubic units

Answers

How many unit cubes? **24**

Find other cuboids with the same volume?

How many are there? **5**

Area 12 square units

Area 8 square units

Area 6 square units

1 × 2 × 12	2 × 2 × 6
1 × 3 × 8	2 × 3 × 4
1 × 4 × 6	

What is the surface area of the 2 × 3 × 4 cuboid in the picture?

Areas of faces
2(6 + 8 + 12)
2 × 26 = 52

13

Slide 14

SAME VOLUME & DIFFERENT SURFACE AREAS - UPPER PRIMARY

If you have 2 shapes with the same volume do they have the same surface area?
Answer 👍 for YES 👎 for NO

6 6 2 2 4 4 6

Volume   = 48 cubic units
Surface area = 2(8 + 12 + 24)
            = 88 square units

3 2 8 8 8 3 2

Volume   = 48 cubic units
Surface area = 2(6 + 16 + 24)
            = 92 square units

These cuboids have the SAME VOLUME and DIFFERENT SURFACE AREAS

What about the same question the other way round?

14

Slide 15

SAME SURFACE AREA & DIFFERENT VOLUMES
UPPER PRIMARY OR LOWER SECONDARY

Answers

- How many unit cubes? **24**
- How many cuboids have the same volume? **5**

These cuboids have the same surface area and different volumes	1 × 2 × 12	2 × 2 × 6
	1 × 3 × 8	2 × 3 × 4
	1 × 4 × 6	

Volume = 2 × 3 × 4
       = 24
Areas of faces
2(6 + 8 + 12)
2 × 26 = 52

How many cuboids have the same surface area 52 square units?

Work out the volume and surface area of this cuboid.
Put answers on Chat
2 cuboids have surface area 52

1 2 8 8 8 1 2

Volume = 1 × 2 × 8
       = 16
Areas of faces
2(2 + 8 + 16)
2 × 26 = 52

15

Slide 16

VOLUME – LOWER SECONDARY

If you double the edge lengths what happens to the volume?

Make these objects with cubes or use this diagram to illustrate **2 x edge length => 8 x volume**

3D object of twice the height is 8 TIMES THE VOLUME

LINEAR SCALE FACTOR   2
AREA SCALE FACTOR   4
VOLUME SCALE FACTOR 8

Ideas about scale factors help us to estimate volume and capacity.
Talk about how the word **CAPACITY** refers to the
VOLUME of liquid that will fill a container

16

4

Slide 17

CAPACITY – LOWER SECONDARY

Use your measuring jug to show the learners how to find the capacity of cups in millilitres.

Introduce the idea of a **LITRE** and the meaning of the word '**milli**' meaning '**one thousandth**'.

Discuss the facts that:
1 cubic centimetre is 1 millilitre
and there are **1000 millilitres in one litre**

Ask learners what is measured in litres and what is measured in millilitres in everyday life.

Slide 18

CAPACITY – SECONDARY

Under each cylinder, on the **top line**, write down the number **of millilitres** of liquid in the cylinder. On the **second line** write the number **of litres**, for example the first one is 500 millilitres which is 0.5 litres.
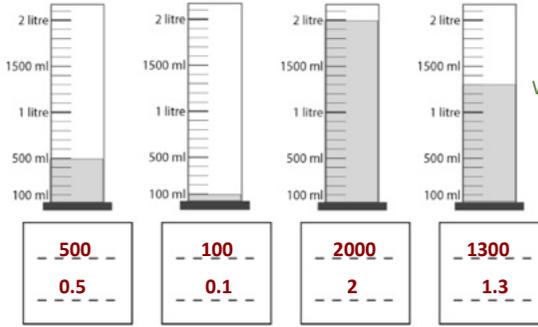
Write answers on chat

500	100	2000	1300
0.5	0.1	2	1.3

Slide 19

CAPACITY – SECONDARY

How to Measure Liquid Medicines

Use the dropper, syringe, medicine cup, or dosing spoon that comes with the medicine.
•Best practice is to measure in milliliters (ml) as read on the dosing device.
•In the past medicine was often measured in teaspoons (tsp), or tablespoons (tbsp).
•Be sure you are using the correct measure for the dose.

Medicine cups
•Be sure to use the cup that comes with the medicine.
•Don't fill it up. Look carefully at the lines and letters on the
Use the numbers and fill the cup to the right line.

Droppers or syringes
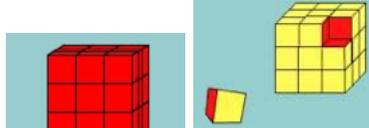Use the numbers to fill it to the right line.

Dosing spoons:
Be sure you are using the correct measure for the dose.
These spoons are for 2.5 ml and 5 ml.
See American Academy of Pediatrics: https://healthychildren.org

Slide 20

PROBLEM SOLVING - SURFACE AREA – PAINTED CUBE - SECONDARY

Imagine a large cube made up from 27 small red cubes each measuring 1cm by 1 cm by 1 cm.
What is its volume?
Imagine dipping the large cube into a pot of yellow paint so the whole outer surface is covered. **What is its surface area?** **54 sq cm**

One coat gloss Yellow

Now imagine breaking the cube up into 27 small cubes.

How many of the small cubes will have yellow paint on their faces? **26**

Will they all look the same? **No**

How many yellow faces do the corner cubes have? **3**

How many faces on the other cubes are yellow? **Edge 2,** **Centre of Face 1**

Now imagine doing the same with other big cubes made up from small red cubes.
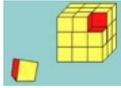
Do all the small cubes have yellow paint on them?

What interesting discoveries can you make?

What results do you get for $4 \times 4 \times 4$, $5 \times 5 \times 5$ ... etc **See next slide**

PAINTED CUBE https://aiminghigh.aimssec.ac.za/painted-cube/

SURFACE AREA – PAINTED CUBE

Can you spot the number patterns and the formulas for the sequences?

1. Size of large cube	2. No. of small cubes with 6 red faces- inside the original cube	3. No. of small cubes with 5 red faces- on the faces of the original cube but not at the edges	4. No. of small cubes with 4 red faces- on the edges of the original cube	5. No. of small cubes with 3 red faces- at the corners of the original cube	6. Total No. of small cubes. Volume of large cube in cm³ Total of columns 2, 3, 4 and 5	Surface area of large cube in cm²
3 x 3 x 3	1	6	12	8	27	54
4 x 4 x 4	8	24	24	8	64	96
5 x 5 x 5	27	54	36	8	125	150
6 x 6 x 6	64	96	48	8	216	216
10 x 10 x 10	512	384	96	8	1000	600
23 x 23 x 23	9261	2646	252	8	12167	3174
n by n by n	$(n-2)^3$	$6(n-2)^2$	$12(n-2)$	8	n^3	$6n^2$

21

PROBLEM SOLVING – PACKING – TOPS – UPPER SECONDARY
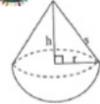
For your company, you have to design spinning tops made from two pieces glued together, a hemisphere and a cone.

You have to plan how they will be manufactured, packaged and sold.
If the radius r of the circular surface where the pieces are glued together is 3 cm, and the slant height s of the cone is 5 cm, work out the total height.

If each top is packed in a box measuring 6.5 cm by 6.5 cm by 7.5 cm how many small boxes will fit into a packing case with internal measurements 55 cm by 55 cm by 55 cm?

Answer on chat

It is possible to pack 7 layers of 64 boxes in each layer in the packing case, 448 small boxes containing tops.

22

INVESTIGATION – VOLUME OF CONE – UPPER SECONDARY

The cone shown below was made from the sector of the circle using scrap plastic.

Make your own cone and cylinder the same height from paper, plastic or scrap card using the templates. Fill the cone with lentils, rice or something similar and investigate the relationship between the volume of the cone and the volume of the cylinder.

Volume of a cone is **ONE THIRD of the volume of the cylinder**

FORMULAE

Volume of a cone is $\frac{1}{3}\pi r^2 h$
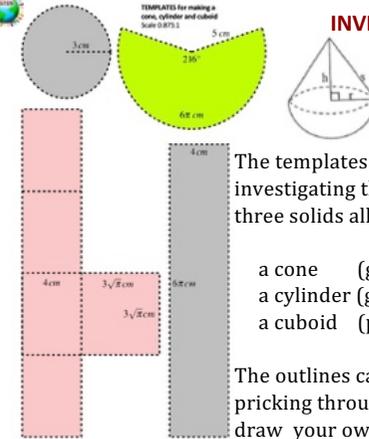
Volume of a cylinder is $\pi r^2 h$

Volume of a sphere is $\frac{4}{3}\pi r^3$

This isn't a proof of the formula; you need to learn integral calculus to prove the formula.
You have all the information to work out the volume and surface area of the top.

23

INVESTIGATION – PACKING - TOPS

TEMPLATES for making a cone, cylinder and cuboid
Scale 0.875:1

Notice the 3-4-5 triangle

The templates can be used to make the resources needed for investigating the volume of a cone. Use the templates to make three solids all with the same base area and height:

a cone (green template),
a cylinder (grey circle and the grey rectangle),
a cuboid (pink template).

The outlines can be marked on scrap card or scrap plastic by pricking through the template, or by tracing, or you could draw your own outlines using compasses and a protractor.

24

GTEN WORKSHOP VOLUME, CAPACITY & SURFACE AREA * SUMMARY

FREE AIMSSEC RESOURCES FOR ACTIVITY BASED LEARNING TO BUILD UNDERSTANDING OF MATHEMATICS FROM AGE 5 TO 18+

See the AIMING HIGH website for worksheets, solutions and suggestions for teaching for all activities in the workshop.

25

AIMINGHIGH TEACHER NETWORK

On the AIMING HIGH website there are freely downloadable worksheets, Notes for Teachers with solutions, Key Questions and Diagnostic Quizzes for formative assessment, also Inclusion and Home Learning Guides with activities for learners of all ages and attainment levels.

CUPS AND CAPACITY https://aiminghigh.aimssec.ac.za/cups-and-capacity/

BOTTLES AND CAPACITY https://aiminghigh.aimssec.ac.za/bottles-and-capacity/

PAINTED CUBE https://aiminghigh.aimssec.ac.za/painted-cube/

TOPS https://aiminghigh.aimssec.ac.za/tops/

SOMA CUBE https://aiminghigh.aimssec.ac.za/soma-cube/

26

AIMS | African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

Global Teacher Empowerment Network (GTEN)
AIMSSEC GTEN YouTube Channel
https://www.youtube.com/c/MathsToys/videos

For teachers in primary and secondary schools, colleges and universities
Each workshop has a recurring Mathematical Theme
with learning activities for ages 5 to 18+
Originally broadcast to groups in Spain, The Caribbean, India,
Nigeria, Kenya, Tanzania, Uganda and South Africa

AIMSSEC FACEBOOK https://www.facebook.com/aimssecsa/

27

AIMS | African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

Toni Beardon LAB11@cam.ac.uk
Caroline Ainslie caroline@bubblymaths.co.uk
Tejumade Oundipe tejumade@aimssec.ac.za
Enquire about signing up for an AIMSSEC course
as a self-funding student admin@aimssec.ac.za

Thanks for coming to this workshop.

Use the AIMSSEC ideas on AIMING HIGH and add comments.

Share what you have learned with other teachers.

Try to help all your learners to have a

'YES I CAN'

attitude to mathematics.

28