

Global Teacher Empowerment Network GTEN PLAYFUL LEARNING

Saturday 3 December 2022 16:00 – 18:00 GMT

Toni Beardon Caroline Ainslie Lenny Williams

Put your name, country and the age group you teach on the chat

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AIMS
African Institute for
Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE

Global Teacher Empowerment Network (GTEN)

PROGRAMME: PLAYFUL LEARNING

Put your name, country and the age group you teach on the chat

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:

- Permutations
- Properties of 3D shapes
- Systematic working
- Mean, median and mode
- Interpreting data
- Transformations

GUEST PRESENTATION

Geometric constructions

Angles

Pythagoras Theorem

Learning Spiral

UPPER SECONDARY 6. Solving FOOT + BALL = GAME puzzle systematically to find all solutions

LOWER SECONDARY 5. Exploring winning strategies in a simple blocking game

UPPER PRIMARY 4. Systematically analyzing all possible positions of game pieces

LOWER PRIMARY 3. Interpreting data about goals scored in a series of matches.

EARLY YEARS 2. Properties of icosahedron and truncated icosahedron

1. Counting teams and matches in the World Cup

2

THE VALUE OF PLAYFUL LEARNING

Finland ranks very highly in international comparisons of student achievement in mathematics.
 Play features powerfully in Finnish education.
 Children in Finland start school at 7.
 Hours in school are fewer than in many other countries.
 Homework is only for the oldest learners.
 Learners are encouraged to play after school hours.
 Teachers and local authorities have greater autonomy than in many other high-performing countries.

Professor Kristina Kumpulainen, Department of Teacher Education, University of Helsinki

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PLAY POWER

WHY DOES PLAY MATTER?

1. Research shows that there are clear relationships between academic achievement, experiences of play and the development of social self-regulatory skills.
2. When you don't understand a piece of mathematics the best way forward is to play with the ideas and create your own examples.
3. Playful learning motivates learners to concentrate better and persevere if at first they don't succeed.
4. Play helps people to build the skills needed in life.
5. Playful learning helps learners to develop good communication, teamworking skills and sensitivity to other people which lead to healthy social interaction.
6. Children growing up now will need to adapt to change, to take new roles and to practise lifelong learning and to understand constantly changing new ideas, technologies and skills needed at work and at home.
7. People will need to engage in creative problem solving and to exercise resilience and flexibility in new conditions.

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LESSON ACTIVITIES DURING THE FOOTBALL WORLD CUP QATAR 2022





Where is the maths?



How many teams?

In the Groups Rounds each team plays all the other teams in their group once.

Two teams from each group qualify for the knockout competition.

How many games in the Groups Rounds?

How many teams play in the knockouts?

How many games are played in the tournament?

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LESSON ACTIVITIES DURING THE FOOTBALL WORLD CUP QATAR 2022





In the Groups Round each team plays the other 3 teams in their group of 4.
 How many games? $(4 \times 3) / 2 = 6$ games in each group, so $6 \times 8 = 48$ games.
 Two teams from each group qualify for the knockout competition.
 How many teams play in the knockouts? **2 teams from each group** so $2 \times 8 = 16$ teams.

How many knockout games?

METHOD 1 There are 16 teams and 15 must be knocked out to determine the winner. There is also a game to determine 3rd place. **Total ? games.**

METHOD 2 Add up the matches in each round.

Round of 16	8 games
Round of 8	4
Round of 4	2
Game to determine 3 rd place	1
Final	1
Total	16 games

How many games are played altogether?

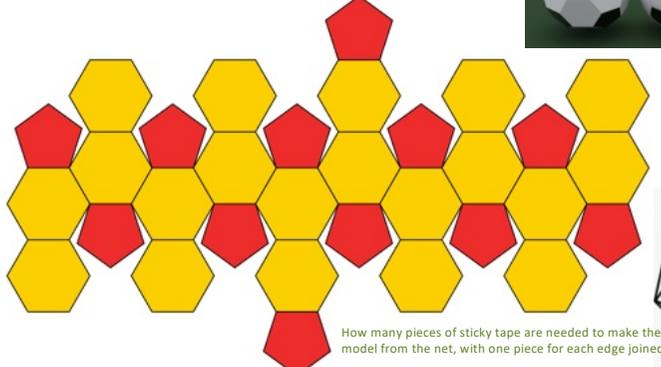
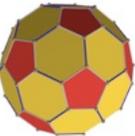
Answer $48 + 16 = 64$ games

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BUCKYBALL INFLATED TRUNCATED ICOSAHEDRON

How many faces, edges and vertices?
https://en.wikipedia.org/wiki/Truncated_icosahedron



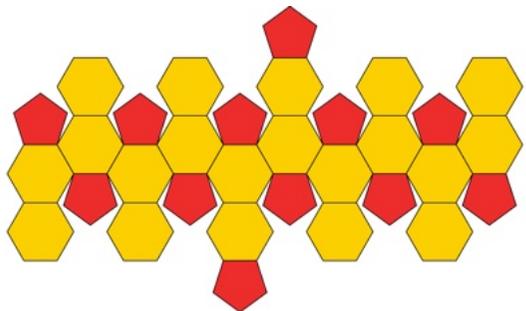





How many pieces of sticky tape are needed to make the model from the net, with one piece for each edge joined?

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BUCKYBALL INFLATED TRUNCATED ICOSAHEDRON

How many faces, edges and vertices?
https://en.wikipedia.org/wiki/Truncated_icosahedron





32 faces
 20 hexagons and
 12 pentagons

90 edges
 $(20 \times 6 + 12 \times 5) / 2 = 90$

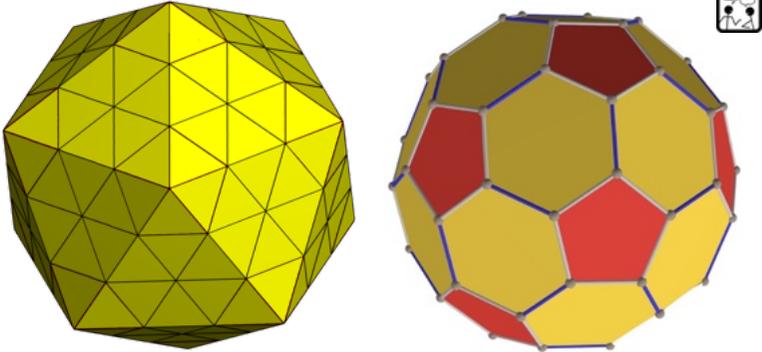
60 vertices
 $(20 \times 6 + 12 \times 5) / 3 = 60$

Number of faces + number of vertices - number of edges = ?

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ICOSAHEDRON
20 faces, 30 edges and 12 vertices

How is a buckyball produced by cutting off pieces of an icosahedron?



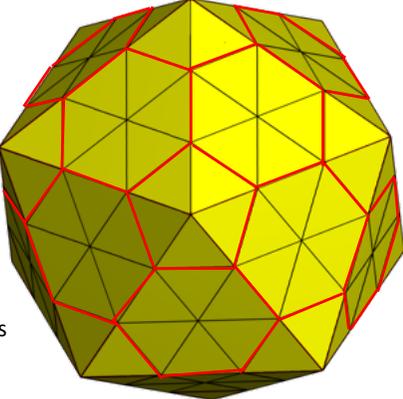
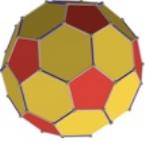
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GTEN Workshop Saturday 3 December TRUNCATED ICOSAHDRON

A pentagonal based pyramid is cut off each of the 12 vertices of an icosahedron.

12 pentagonal faces are formed for the buckyball shape.

Each pentagonal face is surrounded by 5 hexagonal faces which lie in the 20 faces of the original icosahedron.

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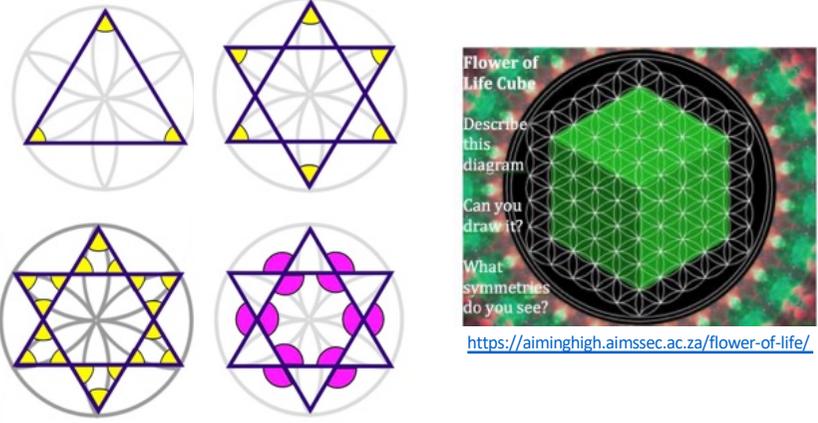
Flower of Life Cube

Describe this diagram

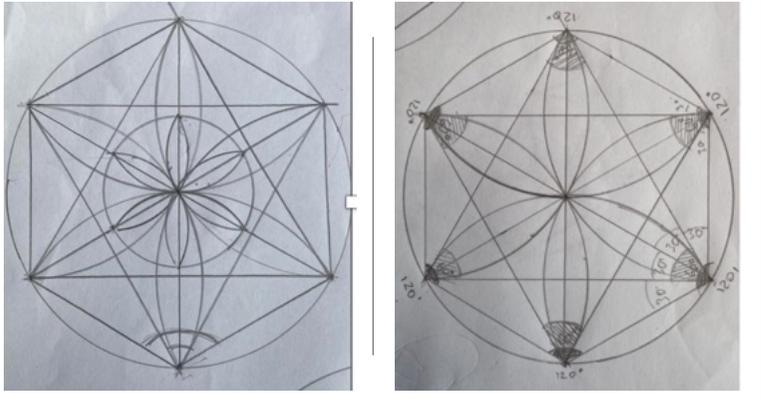
Can you draw it?

What symmetries do you see?

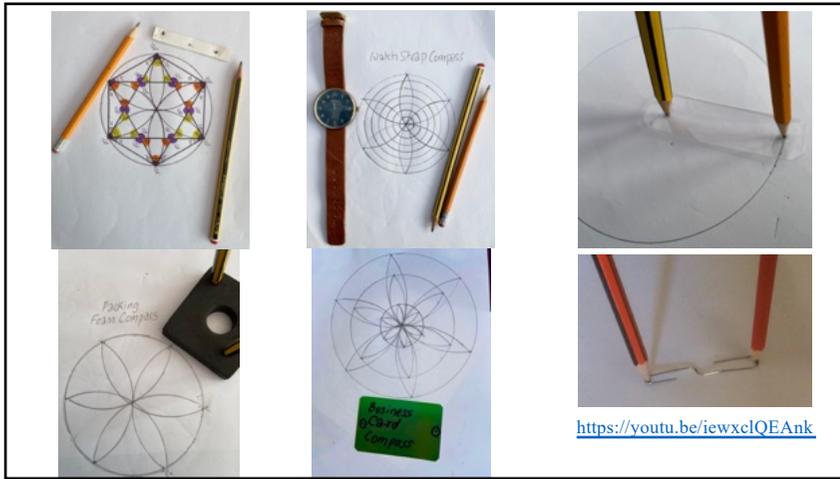
<https://aiminghigh.aimssec.ac.za/flower-of-life/>



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LESSON ACTIVITIES DURING THE FOOTBALL WORLD CUP QATAR 2022

DECODING THE MESSAGE

1	9	9	4
6	5	3	3
8	5	2	7

F	O	O	T
+B	A	L	L
G	A	M	E

2004
BALL +
GAME

1007
2AL8 +
GAME

199T
653L +
852E

Here is your key or code book:
 A = 5, B = 6, E = 7, F = 1, G = 8,
 L = 3, M = 2, O = 9, T = 4.

Now find solutions to the FOOT + BALL + GAME puzzle by replacing the letters with numbers in these examples.

What is the hidden message?

<https://aiminghigh.aimssec.ac.za/football-challenge/>

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2004
BALL +
GAME

1007
2AL8 +
GAME

Replace T, L and E here:
199T
653L +
852E

FOOT + BALL CHALLENGE

Replace the letters with numbers to make the sum correct. Each letter represents a different digit. There are 224 solutions. How many can you find?

EXAMPLE
 F=2, O=9,
 T=7, B=3,
 A=4, L=1,
 G=6,
 M=0, E=8

2997
3411 +
6408

FOOT + BALL = GAME
 2004 3977 5981
 2004 3677 5681

FOOT + BALL = GAME
 1007 2488 3495
 1007 2688 3695
 1007 3288 4295
 1007 3688 4695

FOOT + BALL = GAME
 1994 6533 8527

Once one solution has been found you may be able to find other solutions by changing some of the digits.

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A Systematic method for finding solutions

The letter O can only take values 0 or 9.

Why?

F	O	O	T
+B	A	L	L
G	A	M	E

EITHER 1 is carried over from the tens column to the hundreds column so $1 + O + A = 10 + A$. In this case the letter O stands for 9. **(Case 1)**

OR there is no carry over from the tens column to the hundreds column so $O + A = A$. In this case the letter O stands for zero. **(Case 2)**

Full explanations and solutions are given in the Inclusion and Home Learning Guide for FOOTBALL CHALLENGE

<https://aiminghigh.aimssec.ac.za/football-challenge/>

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LESSON SUGGESTIONS FOR A SYSTEMATIC METHOD FOR FINDING SOLUTIONS

**FOOT
+BAL L
GAME** The letter O can only take values 0 or 9 so take each of the cases in turn.

1. Ask learners what they can deduce about L and M from the tens column when the letter O stands for 9. (Answer $M = L - 1$)
2. In this case there cannot be any carry-over from the units column so what does this tell you about $T + L$? (Answer $T + L \leq 8$ because $O = 9$)
3. List all the T, L pairs such that $T + L \leq 8$.
4. Then share the work of finding the solutions in these cases giving different T, L pairs to different learners.
5. Discuss best way to record the solutions on a chart in a systematic way.

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CASE 1: The letter O stands for nine and so $M = L - 1$.

This means there is no carry-over from units to tens because if there were then $L = M$ which cannot happen. Also $E \neq O = 9$ so $T + L = E \leq 8$.

There are only 11 possible T, L pairs that satisfy these conditions.

Can you find them?

CASE 2: The letter O stands for 0. Because $M \neq L$ there is a carry-over from units to tens so $1 + L = M$ so $L \leq 8$.

Next, $T + L = 10 + E \geq 11$ (because $E \neq 0$) so that $T \geq 3$.

There are only 19 possible T, L pairs that satisfy these conditions.

Can you find them?

For solutions see <https://aiminghigh.aimssec.ac.za/football-challenge/>

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RESULTS FOR 15 MATCHES PLAYED BY ALPHA UNITED AND BETA ROVERS

1. The mode of the number of goals scored by Beta Rovers is one less than the mean number of goals they scored

The mean number of goals scored by Beta Rovers is equal to the median number of goals they scored

Which chart gives the same information as the Tally Chart?

2. A pie chart to show the number of goals scored in fifteen football matches

THE DATA SHOWS HOW MANY GOALS THE TEAMS SCORED IN THE MATCHES.

3. The mode number of goals scored by Alpha United is one more than the mean number of goals they scored

The mean number of goals scored by Alpha United is equal to the median number of goals they scored

Which chart gives the same information as the Pie Chart?

4. A pictogram to show the number of goals scored in fifteen football matches

5. Frequency

6.

Number of goals	Tally
0	
1	
2	
3	
4	

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LESSON ACTIVITIES DURING THE FOOTBALL WORLD CUP QATAR 2022

Results for one team Find the mean, median and mode to determine which team it is.

Number of goals	Tally
0	
1	
2	
3	
4	

Mean $0 \times 3 = 0$ $1 \times 3 = 3$ $2 \times 2 = 4$ $3 \times 5 = 15$ $4 \times 2 = 8$
 $0 + 3 + 4 + 15 + 8 = 30$
 $30 / 15 = 2$

Mode 3

Median Goals: 0 0 1 1 1 2 2 3 3 3 3 4 4 4 4
2

Results for Alpha United Represented in the tally chart and the pictogram

Number of goals	Frequency - number of matches	Total number of goals	Angle in pie chart
0	3	0	72
1	3	3	72
2	2	4	48
3	5	15	120
4	2	8	48
		30	360

Mode 3 goals Mean 2 Median 2

These are results for Alpha United because the mode is 1 higher than the mean and the median is the same as the mean.

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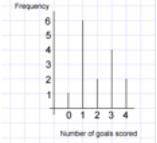
LESSON ACTIVITIES DURING THE FOOTBALL WORLD CUP QATAR 2022

Results for the other team. Find the mean, median and mode

Mean $0 \times 1 = 0$ $1 \times 6 = 6$ $2 \times 2 = 4$ $3 \times 4 = 12$ $4 \times 2 = 8$
 $0 + 6 + 4 + 12 + 8 = 30$
 $30 / 15 = 2$

Mode 1

Median Goals: 0 1 1 1 1 1 1 2 2 3 3 3 3 4 4
2




These are results for Beta Rovers because the mode is 1 less than the mean and the median is the same as the mean.

The mode of the number of goals scored by Beta Rovers is one less than the mean number of goals they scored

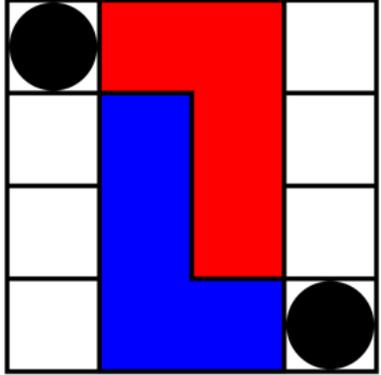
The mean number of goals scored by Beta Rovers is equal to the median number of goals they scored

Number of goals	Frequency - number of matches	Total number of goals	Angle in pie chart
0	1	0	24
1	6	6	144
2	2	4	48
3	4	12	96
4	2	8	48
		30	360

Mode 1 goal Mean 2 Median 2

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THE L GAME



The first player chooses one L shape and moves it on the board, then moves one black counter if they wish.

The second player moves the other L followed by either one of the counters if they wish.

Pieces cannot overlap.
The first player who is unable to move loses the game.

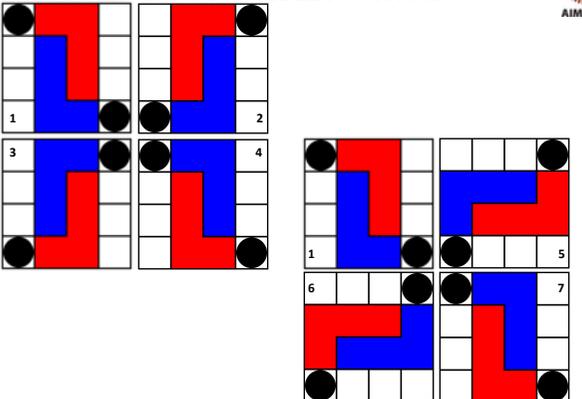
The game was invented by Edward de Bono.

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THE L GAME ANALYSIS OF POSSIBLE POSITIONS

There are fewer positions than you might think because of symmetry.

Describe the transformations of arrangement 1 that give the other arrangements numbered 2 to 7.



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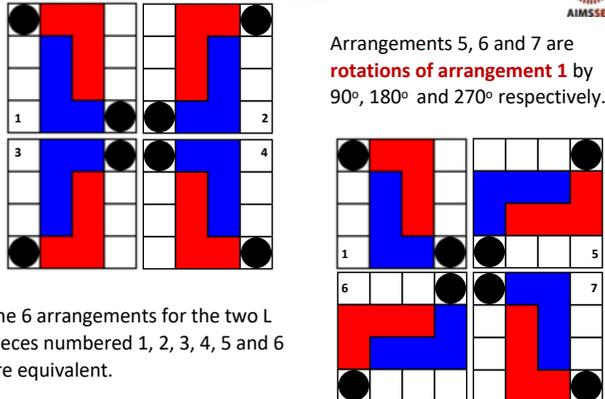
THE L GAME ANALYSIS OF POSSIBLE POSITIONS

Arrangements 2, 3 and 4 are reflections of arrangement 1 in vertical and horizontal mirror lines.

Arrangement 4 (2 reflections) is identical to arrangement 7 (a rotation of 180°.)

The 6 arrangements for the two L pieces numbered 1, 2, 3, 4, 5 and 6 are equivalent.

Arrangements 5, 6 and 7 are rotations of arrangement 1 by 90°, 180° and 270° respectively.



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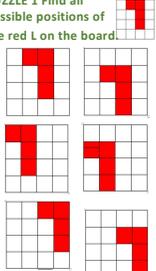
L GAME PUZZLES




CLASS GAME STRATEGY INVESTIGATION AND PUZZLE SOLVING

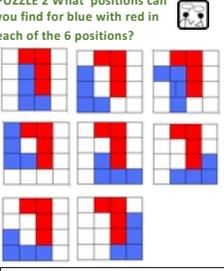
- (1) Play the game with the whole class in two teams, then the students can play in pairs.
- (2) The whole class solves Puzzle 1 and shares results.
- (3) To share the work, six groups each tackle one of the 6 positions for red, solve Puzzle 2 for these 6 positions and then present their findings to the class.
- (4) The groups then decide where the black pieces can be placed so that one or other of the players is in a winning position.
- (5) The class discusses winning strategies and how computers are programmed to beat the best human players.
- (6) The class studies the winning positions and then runs a league competition to find the champion.

PUZZLE 1 Find all possible positions of the red L on the board.



Here are all positions of the red L apart from rotations and reflections of these positions.

PUZZLE 2 What positions can you find for blue with red in each of the 6 positions?

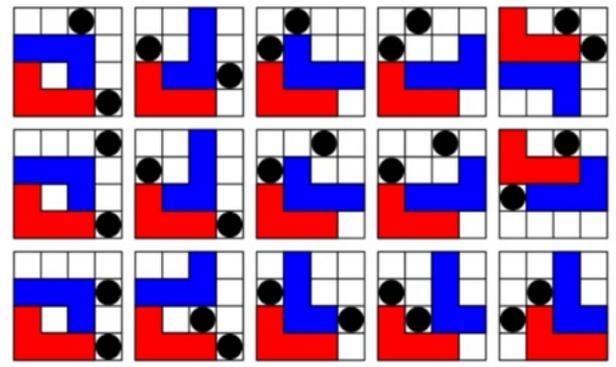


With red in the starting position here are some of the possible positions for blue) providing the position is not blocked in one of the black pieces). Can you find others?

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THE L GAME WINNING POSITIONS



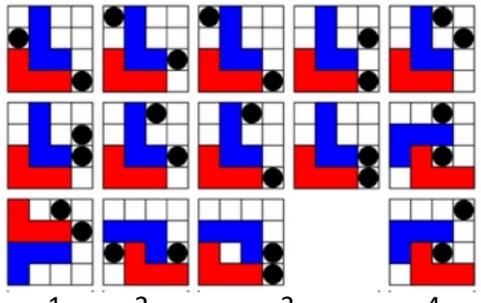



Possible final positions. Blue has won. **Can you find others?**

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THE L GAME WINNING POSITIONS



Winning positions with red to move next.
Red will lose in 1, 2, 3 or 4 moves if blue plays correctly.

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RIGHT ANGLE MAKER



THE CONVERSE OF PYTHAGORAS THEOREM

If the sum of the areas on the two shorter edges of a triangle add up to the area of the square on the longest edge
then the angle opposite the longest edge is a right angle (90°).

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SUMMARY
Lesson Activities during the World Cup

Starting position

Blue has won Red can't move

FOOT + BALL CHALLENGE

ACTIVITIES WITH STRING

MATCH THE MATCHES

These are results for 15 football matches played by Alpha United and Beta Rovers.

Can you say whose results are shown in the each chart?

The mode of the number of goals scored by Beta Rovers is one less than the mean number of goals they scored.

The mean number of goals scored by Beta Rovers is equal to the median number of goals they scored.

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AIMINGHIGH TEACHER NETWORK

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On the AIMING HIGH website there are freely downloadable worksheets, Inclusion and Home Learning Guides with activities for learners of all ages and attainment levels, Notes for Teachers with solutions, Key Questions and Diagnostic Quizzes for formative assessment.

Icosahedron Puzzle <https://aiminghigh.aimssec.ac.za/icosahedron-puzzle/>

Construct with Circles <https://aiminghigh.aimssec.ac.za/construct-with-circles/>

Construct Geometrical Patterns <https://aiminghigh.aimssec.ac.za/construct-geometrical-patterns/>

Flower of Life <https://aiminghigh.aimssec.ac.za/flower-of-life/>

Football Challenge <https://aiminghigh.aimssec.ac.za/football-challenge/>

In a soccer stadium <https://aiminghigh.aimssec.ac.za/in-a-soccer-stadium/>

To apply to join the GTEN Teachers WhatsApp Group and to get information about GTEN or to apply for an AIMSSEC course write to <admin@aimssec.ac.za>

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AIMSSEC GTEN YouTube Channel
<https://www.youtube.com/c/MathsToys/videos>

AIMS BRINGING MATHS & SCIENCE TO AFRICA

AIMSSEC FACEBOOK <https://www.facebook.com/aimssecsa/>

HAPPY MATHS HOUR Weekly on Mondays 17:00 - 18:00 UK time

MATHS TOYS

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DATES FOR GTEN WORKSHOPS NEXT YEAR

21st January 2023 Shadows Similarity and Enlargement
What shapes could cast these shadows? ● ■ ▲ ▬

11th February 2023 Place Value, Number Bases & Logic
10000, ??, 100, 31, 24, 22, 20, 17, 16, 15, 14, 13, 12, 11, 10
The numbers in this list look different because they are written in different bases but all the numbers are the same. What does the ?? represent?

25th March 2023 Reptiles

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Thanks for coming to this workshop.

Use the **AIMSSEC** ideas
on **AIMING HIGH** and add comments.

Share what you have learned
with other teachers.

Try to help all your learners to have a
'YES I CAN'
attitude to mathematics.



Toni Beardon LAB11@cam.ac.uk
 Caroline Ainslie caroline@bubblymaths.co.uk

Enquire about signing up for an **AIMSSEC** course
as a self-funding student admin@aimssec.ac.za

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INVESTIGATE THE L GAME




How many distinctly different positions of the two L pieces can you find? Colour the squares.

Which of these arrangements can be made into winning positions by placing the black pieces
in strategic places?

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