Slide 1

Global Teacher Empowerment Network GTEN
MAGIC NUMBERS
Saturday 11 October   16.00 – 18.00 London Time

Toni Beardon   Caroline Ainslie   Tejumade Ogundipe

Put your name, country and the age group you teach on the chat

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Slide 2

AIMS | African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

MATHS TOYS

Global Teacher Empowerment Network (GTEN)
Programme for Magic Numbers Workshop 11 October 2021

Improve knowledge and understanding of:
Division or sharing
Number patterns
Linear equations

Put your name, country and the age group you teach on the chat

Learning Spiral

9. Enrichment: Euclid's Algorithm
8. Follow-up: Beautiful Numbers Pattern.
7. Upper Secondary: Inverse operations and algebra. Solving equations
6. Lower Secondary: Unpicking the pattern. Division by chunking
5. Upper Primary: Buried treasure and the division algorithm
4. Lower Primary: More about equal shares
3. Early Years: Equal shares
2. Sharing the treasure
1. Starter activity: Story about **Discovering Buried Treasure**

Slide 3

NUMBER PATTERNS AND SHARING

Do the activities.
Comment on the chat.
Please participate even if you make guesses.
That way you'll get more benefit
out of the workshop.

Slide 4

STARTER ACTIVITY FOR ALL – Discovering buried treasure

Listen to this story and answer the questions at the end.

You and your friends and family, 8 of you altogether, were playing on the beach and you noticed that the sand had shifted to reveal something made of wood. You all started digging and found a treasure chest full of gold coins. The grown-ups said that you could not keep the money as it had been buried for a long long time, but there would be no harm in counting the coins.

To count the gold coins you counted out piles of a hundred gold coins. There were 98 hundreds.

You separated them into piles of a thousand and smaller piles of a hundred. Then you counted how many were left and found there were 72.

You sorted them into piles of ten with 2 left over.

Read the story again before you answer the questions.

How many thousands? How many hundreds?
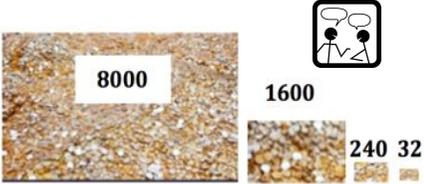How many tens? How many units? How many coins altogether?

Sharing the treasure

Imagine you are sharing the 9872 gold coins from the treasure chest between 8 people.

You split them into 4 piles, as shown in the picture chosen so that **each pile is a multiple of 8**.

Then you share each of these piles between the 8 people.

How many coins would each person get?

8000
1600
240 32

Explain this method of sharing the coins.

Explain how this method is used to find the answer to the division sum 9872 ÷ 8

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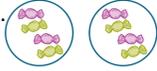
Early Years   All about equal shares

Talk about sharing and do some sharing with the children, **not** dividing up to make fractions, but sharing collections of objects into **equal shares** and perhaps having some left over (**the remainder**).

Don't make this into a lesson. Rather let it be a part of everyday life.
Share a bunch of grapes between a group of children so everyone gets the same number of grapes, or share some sweets, or whatever there is to share.

Make up up a story about this picture.

Make some play money and encourage the children to play at shopping.
Make up stories and play games that involve sharing money. If you play games as a family like Monopoly the children should share out the money at the start.

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Lower Primary   All about equal shares

To lay a good foundation for later work children should make up number stories that involve sharing situations and explain and illustrate their stories in their own ways. Ask questions like:
How many different ways can you describe what you see in this picture?

For 6 year olds do not introduce the formal way to write this down, it will come later.

If you ask children about what they see in this picture, what sort of answers would you expect them to give? In talking about the picture, without writing anything formally, try to connect the ideas that $6 = 2 + 2 + 2$ and $6 = 3 \times 2$ and $6 \div 3 = 2$.

Sharing is a normal activity for young children. Before starting school they understand that, if there are 6 sweets to share between 3 children, each child will get 2 sweets. When the division process is taught formally, if the teacher does not help them to connect it to what they already know, children will fail to understand the algorithm.
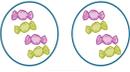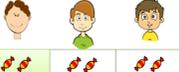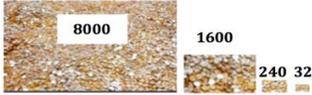
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Upper Primary   The Division Algorithm

Start the lesson with the story of the buried treasure and sharing 9872 coins between 8 people. Talk about the pros and cons of different ways of sharing.

8000   1600
240 32

Why would you choose to split the coins as shown in this picture?
How does this link to the division algorithm?

8|9872

		8	9872	How the money is shared:
a.	How many whole thousands can be shared?	8	9872	
b.	How many hundreds are left?	18	- 8000	8 × 1000
c.	How many whole hundreds can be shared?	16	1872	
d.	How many tens are left.	27	-1600	8 × 200
e.	How many whole tens can be shared?	24	272	
f.	How many whole units can be shared?	32	- 240	8 × 30
g.	How many units are left?	0	32	
			- 32	8 × 4

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96,…   **9872 ÷ 8 = 1234**

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Slide 9

Division   9872 ÷ 8 = 1234

Brunsviga Mechanical Calculator widely used in offices from 1890 until 1960s

To do this division 4 levers are pulled down the slots to set 9 thousands, 8 hundreds, 7 tens and 2 units.

Then the carriage at the bottom is moved to line up with the thousands column.

To subtract thousands the handle is turned anticlockwise until the bell rings to indicate the negative zone.

Next the carriage is moved to line up with the hundreds and hundreds are subtracted by anticlockwise turns of the handle. The carriage is moved into alignment with the tens column and anticlockwise turns subtract tens. Finally, the handle is moved to the units column and the handle is turned to subtract units. The answer 1234 is read from the machine.

This operation showed clearly that division is repeated subtraction.

Turning the handle in the opposite direction for multiplication added the numbers showing that multiplication is repeated addition.

Decimal points could be set for decimal calculations.

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Lower Secondary

$9 - 1 =$
$98 - 2 =$
$987 - 3 =$
$9876 - 4 =$
$98765 - 5 =$
$987654 - 6 =$
$9876543 - 7 =$
$98765432 - 8 =$
$987654321 - 9 =$

Write down this pattern. Copy the pattern and work out the subtractions. What do you notice?

Lesson Starter for Learners: Read the Buried Treasure story.

Main lesson: This lesson focusses on division.

Talk about the gold coins sharing story. Ask the learners to talk with their

partners, or in small groups, to find the best

way to use the coin sharing story to explain the division sum:

9872 ÷ 8 = 1000 + 200 + 30 + 4 = 1234.

After a few minutes have a whole class discussion about this.

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Lower Secondary Lesson continued

$(9 - 1) \div 8 =$
$(98 - 2) \div 8 =$
$(987 - 3) \div 8 =$
$(9876 - 4) \div 8 =$
$(98765 - 5) \div 8 =$
$(987654 - 6) \div 8 =$
$(9876543 - 7) \div 8 =$
$(98765432 - 8) \div 8 =$
$(987654321 - 9) \div 8 =$

Then copy out this list of calculations.

Tell the learners that they must do the calculation in the brackets first.

Ask them to complete the calculations without using a calculator.

Do this now and write your answers in the chat

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Lower Secondary Lesson continued

$(9 - 1) \div 8 = 1$
$(98 - 2) \div 8 = 12$
$(987 - 3) \div 8 = 123$
$(9876 - 4) \div 8 = 1234$
$(98765 - 5) \div 8 = 12345$
$(987654 - 6) \div 8 = 123456$
$(9876543 - 7) \div 8 = 1234567$
$(98765432 - 8) \div 8 = 12345678$
$(987654321 - 9) \div 8 = 123456789$

To end the lesson: The teacher or one of the learners should write the complete pattern on the board, with different learners telling them what to write line by line.

Key questions to guide learning

1. What do you notice?
2. Why have the brackets been put in those calculations?
3. If one number is divided by another, how does it help to split the first number into separate parts when you divide one number (e.g. number of coins) by a second number (e.g. number of people)?
4. What's the connection between division and subtraction?
5. What's the connection between multiplication and division?

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Years 10 - 12

This challenge is an extension to MAGIC NUMBERS

https://aiminghigh.aimssec.ac.za/more-magic-numbers/

Find the numbers to put in the boxes to make the calculations in LIST 1 correct.

Complete the calculations in LIST 2

LIST 1

$\square \times 8 + 1 = 9$
$\square \times 8 + 2 = 98$
$\square \times 8 + 3 = 987$
$\square \times 8 + 4 = 9876$
$\square \times 8 + 5 = 98765$
$\square \times 8 + 6 = 987654$
$\square \times 8 + 7 = 9876543$
$\square \times 8 + 8 = 98\ 765432$
$\square \times 8 + 9 = 987654321$

LIST 2

$(9 - 1) \div 8 =$
$(98 - 2) \div 8 =$
$(987 - 3) \div 8 =$
$(9876 - 4) \div 8 =$
$(98765 - 5) \div 8 =$
$(987654 - 6) \div 8 =$
$(9876543 - 7) \div 8 =$
$(98765432 - 8) \div 8 =$
$(987654321 - 9) \div 8 =$

Write down list 1 with numbers to fill in the boxes.

What do you notice about the connections between the equations: $\square \times 8 + 1 = 9$ etc. and the calculations: $(9 - 1) \div 8 = 1$ etc. ?

You have been doing algebra, $\square \times 8 + 1 = 9$ etc. can be written as $8x + 1 = 9$ etc.

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Years 10 - 12   MAGIC NUMBERS SOLUTION

$9 - 1 = 8$
$98 - 2 = 96$
$987 - 3 = 984$
$9876 - 4 = 9872$
$98765 - 5 = 98760$
$987654 - 6 = 987648$
$9876543 - 7 = 9876534$
$98765432 - 8 = 98765424$
$987654321 - 9 = 987654312$

The numbers in this triangle are all multiples of 8.

$(9 - 1) \div 8 = 1$ $x \, 8 + 1 = 9$
$(98 - 2) \div 8 = 12$ $x \, 8 + 2 = 98$
$(987 - 3) \div 8 = 123$ $x \, 8 + 3 = 987$
$(9876 - 4) \div 8 = 1234$ $x \, 8 + 4 = 9876$
$(98765 - 5) \div 8 = 12345$ $x \, 8 + 5 = 98765$
$(987654 - 6) \div 8 = 123456$ $x \, 8 + 6 = 987654$
$(9876543 - 7) \div 8 = 1234567$ $x \, 8 + 7 = 9876543$
$(98765432 - 8) \div 8 = 12345678$ $x \, 8 + 8 = 98765432$
$(987654321 - 9) \div 8 = 123456789$ $x \, 8 + 9 = 987654321$

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 98765$$
$$123456 \times 8 + 6 = 987654$$
$$1234567 \times 8 + 7 = 9876543$$
$$12345678 \times 8 + 8 = 98765432$$
$$123456789 \times 8 + 9 = 987654321$$

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WHY DO THE MAGIC NUMBERS ACTIVITY?

Discovering the pleasing patterns gives this activity an appeal for learners.

They will get practice in subtraction and division in an interesting way.

The activity helps learners to understand 3 fundamental mathematical ideas:

1. the process of division and how it is equivalent to 'sharing';
2. inverse operations;
3. how solving linear equations involves the use of inverse operations.

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BEAUTIFUL NUMBERS Lesson for secondary

$$? \times 9 + 7 = 88$$
$$? \times 9 + 6 = 888$$
$$? \times 9 + 5 = 8888$$
$$? \times 9 + 4 = 88888$$
$$? \times 9 + 3 = 888888$$
$$? \times 9 + 2 = 8888888$$
$$? \times 9 + 1 = 88888888$$
$$? \times 9 + 0 = 888888888$$

Work out the different numbers to replace the question marks.

What do you notice about the patterns in these calculations?

Can you explain why this pattern occurs?

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BEAUTIFUL NUMBERS Lesson for secondary

9 x 9 + 7 = 88
98 x 9 + 6 = 888
987 x 9 + 5 = 8888
9876 x 9 + 4 = 88888
98765 x 9 + 3 = 888888
987654 x 9 + 2 = 8888888
9876543 x 9 + 1 = 88888888
98765432 x 9 + 0 = 888888888

If we look at the last two calculations we can see how the pattern arises. The other calculations are similar.

Think of 9 as 10 – 1,

$9876543 \times 9 + 1 = 9876543 \times (10-1) + 1$ which can be written as

```
  98 765 431
-  9 876 543
  88 888 888
```

Similarly

$98765432 \times 9 + 0 = 98765432 \times (10-1) + 0$ can be written as

```
  987 654 320
-  98 765 432
  888 888 888
```

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CURIOUS PATTERNS WITH NINES Lesson for secondary

Investigate the decimal expansion of the fractions

ONE DIVIDED BY SEVEN $\frac{1}{7}$

TWO DIVIDED BY THIRTEEN $\frac{2}{13}$

Divide the numerator by the denominator. There are only a few possible remainders so the division must produce a recurring decimal or it must terminate.

Investigate the fractions $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$, ... $\frac{10}{11}$

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CURIOUS PATTERNS WITH NINES

Observe $\frac{1}{7} = 0.142857 \ldots$ recurring

142 + 857 = 999

Look at the pattern in the decimal expansions for thirteenths and elevenths. Do you notice any similar patterns there?

Also $\frac{1}{13} = 0.076923 \ldots$ recurring

076 + 923 = 999

$\frac{2}{13} = 0.153846 \ldots$ recurring

153 + 846 = 999

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CURIOUS PATTERNS WITH NINES

The fractions $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$, ... $\frac{10}{11}$ have decimal expansions:

$\frac{1}{11} = 0.0909 \ldots$, $\frac{2}{11} = 0.1818 \ldots$, $\frac{3}{11} = 0.2727\ldots$, $\frac{4}{11} = 0.3636 \ldots$ etc

Pairs of digits add up to 9:

0 + 9 = 1 + 8 = 2 + 7 = 3 + 6 etc.

Similar patterns arise for denominators 101, 1001, 10001 etc.

For example, try $\frac{32}{101}$ and investigate other fractions with these denominators. $\frac{32}{101}\ldots= 0.31683168\ldots$ and 31 + 68 = 99

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UPPER SECONDARY ENRICHMENT EUCLID'S ALGORITHM

How can we solve equations like $13x+29y=42$ or $2x+4y=13$ or $83x + 19y = 3$ with the solutions x and y being integers?

Diophantus

Equations with integer solutions are called Diophantine equations after Diophantus who lived about 250 AD.

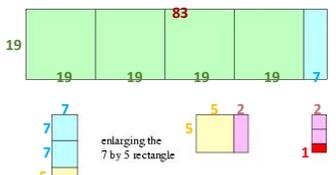The methods described here go back to Euclid (about 300 BC) and earlier.

When people hear the name Euclid they think of geometry, but the algorithm described here appeared as Proposition 2 in Euclid's Book 7 on Number Theory.

Euclid

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UPPER SECONDARY ENRICHMENT EUCLID'S ALGORITHM

This diagram shows Euclid's Algorithm for 83 ÷ 19.
It is used to find whole number solutions (x and y) for equations like $83x + 19y = 3$.

The diagram shows that $83 = 4 \times 19 + 7$
Continuing the same process:

$83 = 4 \times 19 + 7$
$19 = 2 \times 7 + 5$
$7 = 1 \times 5 + 2$
$5 = 2 \times 2 + 1$

Reversing this process and collecting like terms
$1 = (5) - 2(2)$
$= (5) - 2(7 - 5) = 3(5) - 2(7)$
Now replacing 5 gives:
$1 = 3[19 - 2(7)] - 2(7)$
$1 = -8(7) + (19 \times 3)$
$1 = -8(83 - 4 \times 19) + (19 \times 3)$
$1 = 83(-8) + 19(35)$

Multiply this equation by 3 to solve the equation $83x + 19y = 3$

Multiplying by 3
$3 = 83(3 \times -8) + 19(3 \times 35)$
$3 = 83(-24) + 19(105)$
The solutions are $x = -24$
and y = 105

enlarging the
7 by 5 rectangle

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Global Teacher Empowerment Network (GTEN)

NEW SKILLS   NEW HOPES   NEW HORIZONS
for teachers and learners worldwide
MAGIC NUMBERS RESOURCES

Magic Numbers   https://aiminghigh.aimssec.ac.za/magic-numbers/

More Magic Numbers   https://aiminghigh.aimssec.ac.za/more-magic-numbers/

Beautiful Numbers   https://aiminghigh.aimssec.ac.za/beautiful-numbers/

Euclid's Algorithm   https://nrich.maths.org/1357

To apply to join the GTEN Teachers WhatsApp Group and to get information about GTEN write to Layla Thompson <jjthompson@aimssec.ac.za>

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GLOBAL TEACHER EMPOWERMENT NETWORK (GTEN) MONTHLY WORKSHOPS
16:00 - 18:00h UK time
Toni Beardon and Caroline Ainslie with Guest Presenters
1st hour primary 2nd hour secondary
Each workshop has a recurring Mathematical Theme from age 5 to 18+
Broadcast to centres in The Caribbean, India, Nigeria, Kenya, Tanzania, Uganda and South Africa

GTEN Date CAIXA	GTEN workshop	Workshop Title	Learning Packs on AIMING HIGH	Guest Presenter
GLOBAL TEACHER EMPOWERMENT NETWORK (GTEN) 16:00-18:00h UK time.* 1st hour for primary * 2nd hour for secondary. Broadcast to centres in The Caribean, India, Nigeria, Kenya, Tanzania, Uganda and South Africa.				
EDU CAIXA WORKSHOPS broadcast tp Spain and Latin America 17:00-18:30h UK time.				
20-Feb-21 29-Apr-21	1	Handshakes, Bows and Roses Theme: Number Patterns	https://aiminghigh.aimssec.ac.za/handshakes/ https://aiminghigh.aimssec.ac.za/mystic-rose/	Cynthia Fries UK & AIMSSEC
20-Mar-21	2	Algebrarea Theme: Multiplication & Area	https://aiminghigh.aimssec.ac.za/algebrarea-product-of-two-brackets-and-area/	Kwethemba Moyo Zimbabwe & AIMSSEC
24-Apr-21 4-Mar-21	3	Path to the Stars Theme: Multiples and Factors	https://aiminghigh.aimssec.ac.za/path-to-the-stars/ https://aiminghigh.aimssec.ac.za/multiple-patterns/ https://aiminghigh.aimssec.ac.za/prime-sieve/	Marie Joubert Nottingham University
22-May-21 20-May-21	4	Tri-fold Theme: Paper Folding Geometry	https://aiminghigh.aimssec.ac.za/tri-fold/	Jonathan Shipp UK
19-Jun-21 10-Jun-21	5	Paper Sticks Puzzles Theme: Developing visualisation	https://aiminghigh.aimssec.ac.za/tets-and-octs-puzzles/ https://aiminghigh.aimssec.ac.za/cosahedron-puzzle/ https://aiminghigh.aimssec.ac.za/collapsible-cube/	Mary Achieng Kenya and Strathmore University
31-Jul-21	6	Target Games Collection Theme: Estimation, Mental Maths	https://aiminghigh.aimssec.ac.za/target-10-thousand/ https://aiminghigh.aimssec.ac.za/target-games-collection-suitable-for-all-ages/	Zach Mbasu Kenya & AMI
14-Aug-21	7	Symmetry Challenge Theme: Symmetry	https://aiminghigh.aimssec.ac.za/symmetry-challenge/	Sam Okoth Kenya & AMI
11-Sep-21	8	Odds and Evens Red and Blue Theme: Are these Fair Games?	https://aiminghigh.aimssec.ac.za/magic-numbers/ https://aiminghigh.aimssec.ac.za/more-magic-numbers/	Kwethemba Moyo Zimbabwe & AIMSSEC
23-Sep-21		30-minute GMSL FRACTION WALL GAME Theme: PLAY TO LEARN	https://gtenmaths.org Follow-up activities are on the AIMING HIGH website	This annual event, started in 2015, involves around
9-Oct-21	9	Magic Numbers Theme: Number Patterns	https://aiminghigh.aimssec.ac.za/odds-and-evens/	Tejumade Ogundipe Nigeria and AIMSSEC
6-Nov-21 25-Nov-21	10	Spot the Mistakes Puzzles Theme: Learning from Mistakes	https://aiminghigh.aimssec.ac.za/spot-the-mistakes/	Barrie Barnard SA & AIMSSEC
04-Dec-21	11	Learning Through Games Theme: Play for Fun, Think to Win, Play to learn	https://aiminghigh.aimssec.ac.za/checkit-game/ https://aiminghigh.aimssec.ac.za/fifteen-game-collection/ https://aiminghigh.aimssec.ac.za/some-mathematical-	Elizabeth Turok UK, SA & AIMSSEC

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Global Teacher Empowerment Network (GTEN)
For teachers in primary and secondary schools, colleges and universities

MATHS

TOYS

AIMSSEC GTEN YouTube Channel
https://www.youtube.com/c/MathsToys/videos

HAPPY MATHS HOUR
Weekly on Mondays 17:00 - 18:00 UK time

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AIMSSEC Website: http://aimssec.ac.za
AIMING HIGH Free lesson resources: http://aiminghigh.aimssec.ac.za
AIMSSEC APP: https://aimssec.app to download the resources for use offline

COLLABORATIVE PROFESSIONAL DEVELOPMENT
https://aiminghigh.aimssec.ac.za/category/cpd
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS
https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/

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Enquire about signing up for an AIMSSEC course
as a self-funding student admin@aimssec.ac.za
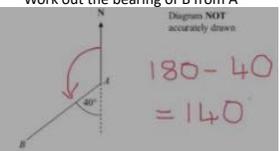
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Thanks for participating.
Goodbye until the GTEN workshop
on 6 November
SPOT THE MISTAKE

Work out the bearing of B from A

Diagram NOT accurately drawn

180 - 40
= 140°

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Slide 28

RECURRING DECIMALS WITH PATTERNS OF NINES

A fraction n with a decimal expansion of the special periodic form

$n = 0.a_1a_2a_3...a_kb_1b_2b_3...b_k$ recurring

where $(a_1a_2a_3...a_k) + (b_1b_2b_3...b_k) = 9...9 = 10^k - 1$

is given by the formula: $n = \dfrac{a_1a_2a_3...ak + 1}{10^k + 1}$.

For example, for $k = 4$, suppose $a_1a_2a_3a_4 = 3251$

then $\dfrac{3252}{10^4 + 1} = \dfrac{3252}{10001} = 0.32516748...$

and $3251 + 6748 = 9999$

This can be proved using the sum of geometric series.

See the Proof Sorter activity 'Recurring decimals with patterns of nines'
https://aiminghigh.aimssec.ac.za/magic-numbers/

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