

Global Teacher Empowerment Network GTEN

SQUARE GAMES, PUZZLES AND PYTHAGORAS THEOREM

Saturday 24 February 2024 15:00 – 17:00 London Time

SQUARE IT GAME
 Game for 2 players or teams.
 Take turns to claim one of dots with your marker. The winner is the first player to make a square.

Area 5

$(a + b)^2 = a^2 + b^2 + 2ab$

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Global Teacher Empowerment Network (GTEN)

PROGRAMME: Square games, puzzles and Pythagoras Theorem – from Games to Mathematical Proof

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:

- Playing games strategically
- Visualisation
- Working systematically
- Areas by counting squares
- Conservation of area
- Discovery by learners of Pythagoras Theorem
- Reflections and rotations
- Algebra / geometry link

Learning Spiral

UPPER SECONDARY

LOWER SECONDARY

UPPER PRIMARY

LOWER PRIMARY

EARLY YEARS

STARTER ACTIVITY

12. Summary.
11. Learning objectives.
10. Algebra by areas.
9. Reflected solutions – transformations.
8. Comparing areas in different solutions to find a proof of Pythagoras Theorem.
7. Making your own Make Squares Jigsaw pieces.
6. Make Squares Jigsaw from given pieces.
5. Finding all solutions through systematic work.
4. Areas of squares by dissection.
3. Squares on a geoboard.
2. Folding paper to find properties of a square.
1. Square It Game, on paper, against a computer and as People Maths.

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INVESTIGATING A SQUARE

What do you notice?

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PROPERTIES OF SQUARES

1. A square is a regular quadrilateral with 4 equal edges and 4 equal angles of 90° .
2. A square is a rectangle with adjacent edges of equal length.
3. The diagonals of a square are equal and bisect each other at right angles.
4. The diagonal of a square bisects its internal angle, forming two angles of 45° .
5. Opposite edges of a square are parallel.
6. The external angle of a square is equal to 90° .

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MAKE A SQUARE IN YOUR COLOUR

PLAY for fun **THINK** to win **PLAY** to learn
<https://aiminghigh.aimssec.ac.za/square-it-game/>

SQUARE IT GAME

Game for 2 players or teams.

Take turns to claim one of dots with your marker. The winner is the first player to make a square.

A game for all ages

THIS IS E3 ●

THIS IS D4 ●

Join Toni's team or Elizabeth's team
 Where should we put our markers?

Claim all 4 corners of a square to win

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SQUARE IT GAME

Play against the NRICH computer
<https://nrich.maths.org/squareit>
 Suggest the next move and try to beat the computer.

PEOPLE MATHS

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GEOBOARDS

Make your own

How many squares can you find on a 3 by 3 geoboard?
 You can do this on paper.

See the Learning Pack on the AIMING HIGH website with worksheets, dotty grids, Inclusion Guide, Notes for Teachers, solutions and more.

<https://aiminghigh.aimssec.ac.za/how-many-squares/>

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GEOBOARD ACTIVITIES

Look at this geoboard.

How many pins are there on the geoboard in the diagram? **25**

The small square in the diagram shows one square unit.

How many square units are there on the geoboard? **16**

The red square is made by joining the middle pins on the outer edges.

What is the area in square units of the red square? **8**

Explain your answer.

8

TILTED SQUARES

Area 4
A

Area 5
B

Area 8
C

These are **all** squares.
B and C are *tilted squares*.

What are the areas of these 3 squares?
Can you make a square of area 2 square units?

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SQUARES ON A GEOBOARD

How many squares can you make for 9 pins?
What are their areas?

There are 6 squares.
1 square has area 4 square units and
4 squares have area 1 square unit
1 square has area 2 square units.

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For 16 pins there are 20 squares.
Can you find them all?

For 25 pins there are 50 squares.
Can you find them all?
What are the areas of the squares?

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How many for 4 dots? How many for 9? then 16? then 25?
<https://aiminghigh.aimssec.ac.za/how-many-squares/>

HOW MANY SQUARES? ACTIVITY FOR ALL


How many types of square? Here are two types.
How many squares of each type?
What can you say about the squares?

Play the SQUARE IT GAME. Take turns to claim one of the dots. The first player to make a square with his/her dots is the winner.

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Young children
Give them the frame and show them one way the pieces fit into the frame. Then jumble up the pieces and ask them to put them back into the frame. When they can do that ask them to try to find different solutions.





MAKE SQUARES JIGSAW
2-challenges in one!
Fit 5 of these pieces together to make a square
Fit 6 of these pieces together to make a square

Upper Primary
Learners can cut out the pieces from a template to make the puzzles. When they have found solutions they should draw diagrams. Encourage the children to find all the solutions and variations. They can come back to it on other days if they don't find the solutions the first day.

11 year-olds
Learners could make their own puzzle pieces.

<https://aiminghigh.aimssec.ac.za/make-squares-jigsaw/>

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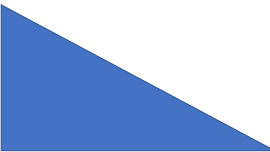



MAKE YOUR OWN 'MAKE SQUARES JIGSAW' PIECES 1

Use scrap paper or card but it must have right angles at the corners.

Start with **any right angled triangle**.



Give the edges names:
shortest edge b , base a , longest edge c .



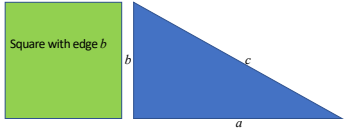
Make 4 identical (congruent) triangles.

Use the corners of a piece of paper to give you right angles.
To make the drawing more accurate make a pin prick to mark a point before drawing the line.

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MAKE YOUR OWN 'MAKE SQUARES JIGSAW' PIECES 2



Square with edge b



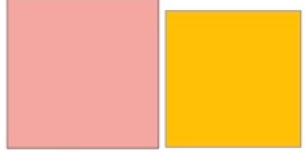
Square with edge length a

Square with edge length c

Use the first triangle that you make to give you the lengths for the edges of the 3 squares.

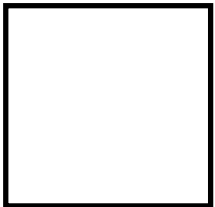
A pin prick to mark a point before drawing the line helps to make the drawing more accurate. You don't need to do any measuring. You only need a ruler to draw straight lines.

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MAKE SQUARES JIGSAW
2-challenges in one!
Fit 5 of these pieces together to make a square
Fit 6 of these pieces together to make a square

Put your pieces into the frame to make a square



<https://aiminghigh.aimssec.ac.za/make-squares-jigsaw/>

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MAKE SQUARES JIGSAW

5 piece solution

1

6 piece solution

2

KEY QUESTIONS

1. What do you notice about these 2 solutions?
2. Knowing the angles in the triangles, how do you know the frame has right angles at the corners?
3. What do you know about the lengths of the edges of the squares and of the edges of the triangles?

Both solutions fit exactly into the same square frame.

Put the 4 triangles in position for solution 1

4. What can you say about the space left uncovered?
5. What is the area of that space?

Move the 4 triangles into position for solution 2

6. What can you say about the space left uncovered?
7. What is the area of that space?

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3 SOLUTIONS, 1, 2 AND 3

1

5 PIECE SOLUTION

2

SIX PIECE SOLUTIONS

3

SIX PIECE SOLUTIONS

What do you notice about the 6 diagrams?

What transformations can you see between one diagram and another?

Why does the heading say that there are only 3 solutions?

What is the same and what is different about the solutions?

Draw different diagrams showing other transformations of the three solutions.

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14 VARIATIONS

Solution 1

Solution 1 reflected either in a vertical mirror or in a horizontal mirror

Solution 2

Solution 2 rotated 90° anti-clockwise

Solution 2 reflected in vertical mirror

Solution 3

Solution 3 rotated 90° anti-clockwise

Solution 3 reflected in vertical mirror

Solution 2 rotated 90° clockwise

Solution 2 rotated 180°

Solution 2 reflected in horizontal mirror

Solution 3 rotated 90° clockwise

Solution 3 rotated 180°

Solution 3 reflected in horizontal mirror

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ALGEBRA AND GEOMETRY

Length of edges of outer frame = ? $a + b$

Area of outer frame = ? $(a + b)^2$

Area of yellow square = ? a^2

Area of green square = ? b^2

Total area of 4 triangles = ? $2ab$

Conclusion

$(a + b)^2 = a^2 + b^2 + 2ab$

From this solution to the puzzle, use the fact that the area of the 4 squares is $2ab$. What is the remaining area of the pink square?

$c^2 = a^2 + b^2$

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LEARNING OBJECTIVES

In doing this activity students will have an opportunity to:

- learn or review the properties that define a square by paper folding;
- develop understanding of area from counting squares and half-squares;
- for learners who have never heard of Pythagoras Theorem to engage in thinking mathematically about triangles, squares and their areas and discover a proof of Pythagoras Theorem for themselves;
- develop a deeper understanding of symmetry, reflection and rotation.

GENERIC COMPETENCES

In doing this activity students will have an opportunity to:

- **develop visualization** skills;
- **work systematically** and **collaboratively** to find **all possible solutions** to a given problem.

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WHY DO THIS ACTIVITY?

These activities provide a visual proof of Pythagoras Theorem that is suitable for learners from upper primary to the school leaving year.

The activities should be used early to lay the foundation for work on Pythagoras Theorem **BEFORE learners meet the theorem**.

These are **Low Entry Point High Ceiling** learning activities providing challenges to allow for differentiation and inclusion in lessons.

The proof **only** requires a little simple reasoning and the knowledge that

- the angles of a triangle add up to 180°
- angles on a straight line add up to 180°
- the area of a square is equal to the square of the length of one edge.

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SUMMARY AND Q&A
PYTHAGORAS AND THE SQUARE PUZZLES
FROM THE GAME TO THE MATHEMATICAL PROOF

SQUARE IT GAME
Game for 2 players or teams.
Take turns to claim one of the dots with your marker. The winner is the first player to make a square.

MAKE SQUARES JIGSAW
2 challenges to solve. Fit all these pieces together to make a square. Fit all these pieces together to make a square.

Area 5

$(a + b)^2 = a^2 + b^2 + 2ab$

1 c^2

2 b^2 a^2

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

AIMSSEC Website: <http://aimssec.ac.za>

AIMING HIGH Free lesson resources: <http://aiminghigh.aimssec.ac.za>

AIMSSEC APP: <https://aimssec.app> to download the resources for use offline

SQUARE IT GAME
<https://aiminghigh.aimssec.ac.za/square-it-game/>

PLAY SQUARE IT AGAINST A COMPUTER
<https://nrich.maths.org/squareit>

HOW MANY SQUARES
<https://aiminghigh.aimssec.ac.za/how-many-squares/>

MAKE SQUARES JIGSAW
<https://aiminghigh.aimssec.ac.za/make-squares-jigsaw/>

PYTHAGORAS SIMILARLY includes the proof by similarity in CAPS and a collection of jigsaw proofs
<https://aiminghigh.aimssec.ac.za/pythagoras-similarly/>

COLLABORATIVE PROFESSIONAL DEVELOPMENT <https://aiminghigh.aimssec.ac.za/category/cpd>
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS
<https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/>

PYTHAGORAS JIGSAW COLLECTION

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AIMS African Institute for
Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE



Global Teacher Empowerment Network (GTEN)
For teachers in primary and secondary schools, colleges and universities


MATHS
TOYS

AIMSSEC GTEN YouTube Channel
<https://www.youtube.com/c/MathsToys/videos>




AIMSSEC FACEBOOK <https://www.facebook.com/aimsseca/>

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LET'S PLAY MATHEMATICALLY AND LEARN

Order from **AMAZON** or **TARQUIN** <https://www.tarquingroup.com/products/aiming-high-play-mathematically>



Play Mathematically

- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE




Thankyou for coming to this workshop.

Use the AIMSSEC ideas
on AIMING HIGH and add comments.

Share what you have learned
with other teachers.

Try to help all your learners to have a
'YES I CAN'
attitude to mathematics.



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