Global Teacher Empowerment Network GTEN

Saturday 22 JANUARY 2022 15.00 – 17.00 London Time

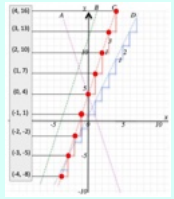
FROM MULTIPLICATION TABLES TO FUNCTIONS

5, 10, 15, 20, 25,...
7, 12, 17, 22, 27,...

Toni Beardon

Caroline Ainslie

Cynthia Fries

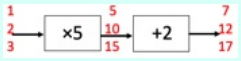


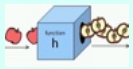
$a \rightarrow [+5] \rightarrow a+5$

$b \rightarrow [\times 3] \rightarrow [-2] \rightarrow 3b-2$

$c \rightarrow [+2] \rightarrow [\times 3] \rightarrow [+5] \rightarrow 3c/2+5$

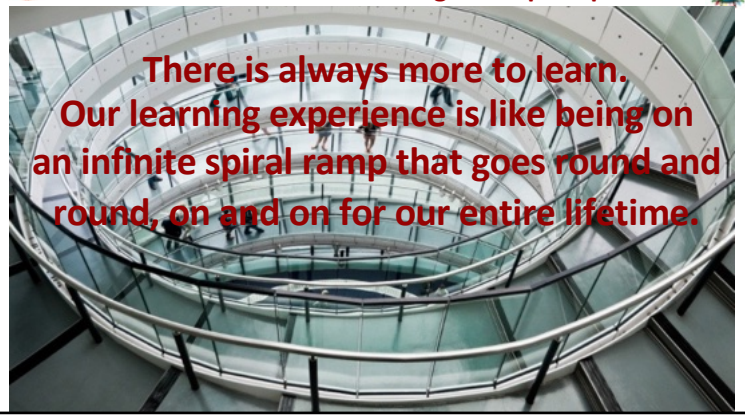
$d \rightarrow [-2] \rightarrow [\times 3] \rightarrow [+2] \rightarrow [+5] \rightarrow 3(d-2)/2+5=3d/2+2$





1

Build skills and understanding on a spiral path



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AIMS | African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

MATHS TOYS

GTEN

Global Teacher Empowerment Network (GTEN)

PROGRAMME: FROM MULTIPLICATION TABLES TO FUNCTIONS

Learning Spiral

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:

- Multiplication Tables & Multiples
- Factors
- Linear functions and line graphs
- Mapping diagrams
- Gradient
- Composite Functions
- Inverse Functions

UPPER SECONDARY	9. Undoing - Inverse Functions
	8. Building Composite Functions
LOWER SECONDARY	7. Steps, Gradients and Straight Lines
	6. Function Machine
UPPER PRIMARY	5. Function Game
LOWER PRIMARY	4. Multiple Patterns
	3. Shifting Times tables
EARLY YEARS	2. Orchestra
STARTER ACTIVITY	1. Clap Counting and Skip Counting

3

DESCRIBING PATTERNS AND RULES

MATHS TOYS

Do the activities.

Questions are in green with this icon.

Please participate even if you make guesses.
That way you'll get more benefit
out of the workshop.

4

CLAP COUNTING AND MULTIPLES STARTER FOR ALL

Count up to 50 and clap on the multiples.

Caroline – multiples of 2
GROUP A: 1 clap 3 clap 5 clap 7 clap 9 clap 11 clap 13 clap 15 …

Toni – multiples of 3
GROUP B: 1 2 clap 4 5 clap 7 8 clap 10 11 clap 13 14 clap …

Cynthia – multiples of 5
GROUP C: 1 2 3 4 clap 6 7 8 9 clap 11 12 13 14 clap …

When will all the groups clap together?

WARNING
This works wonderfully well when you are all in one room but not so well online because of time delays.

5

SKIP COUNTING AND MULTIPLES

Only say the multiples.

Find different ways to do skip counting as a class for different multiples.

The teacher conducts the timing.

Either skip count all together as a class with everyone just saying the multiples.

Or skip count with your group only saying aloud multiples of your group number.

Counting in 2's
- 2 - 4 - 6 - 8 - 10 - 12 - 14…

Counting in 3's
- - 3 - - 6 - - 9 - - 12 - - 15…

Counting in 5's
- - - - 5 - - - - 10 - - - - 15…

What are we learning?

6

ORCHESTRA

Everyone is given one of the numbers 1 to N (start with 1 to 6).

They stand in rows facing the conductor of the orchestra.

Everyone counts 1, 2, 3, 4, …. (or they can sing!)

People clap their hands above their heads when a multiple of their number is called.

ANY SIZE GROUP:

1 2 3 4 5 6

Number 1 claps on every beat 1 2 3 4 5 6

Number 2 claps on 2, 4, 6, 8, … 1 2 3 4 5 6

Number 3 claps on 3, 6, 9, … 1 2 3 4 5 6

Number 4 claps on 4, 8, 12, … 1 2 3 4 5 6

Number 5 claps on 5, 10, 15, … 1 2 3 4 5 6

Number 6 claps on 6, 12, 18, … **Conductor**

Alternatives:

(1) Have 1 to 7 or more in the row.

(2) The people can sit in rows of chairs and stand up when the number called is a multiple of their number.

(3) The class splits into groups and each group rehearses their show. Then the groups perform for the whole class, with actions, music, song or noises.

(4) The whole school can line up outside in ranks with the 1s behind each other, all the 2s behind each other…

Everyone counts in unison and jumps and claps when the number called is a multiple of their number.

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ORCHESTRA

Questions to ask:

Who claps or stands up every time?
Who claps or stands up alternately, that is every other time?
When do only two people clap or stand up ?
When will there be an odd number of people clapping or standing up?

A question for the audience (more difficult).
The orchestra secretly decide to start counting at some number other than one (e.g. start at 7) and they do all the actions but count SILENTLY.
What number did the orchestra start counting from?

ANY SIZE GROUP

1 2 3 4 5 6
1 2 3 4 5 6
1 2 3 4 5 6
1 2 3 4 5 6
1 2 3 4 5 6

Notes for Teachers
There are many variations on this activity and the class can repeat it in different lessons.
It gives learners practice for learning multiplication tables and about factors and multiples.
Keep using the words **MULTIPLE** and **FACTOR** so the learners get to know the words and their meaning.
Write these words on the board.
Participants should notice who is clapping (or standing up).
Good follow up activities are: colouring multiples on a 1 to 100 number square and The Sieve of Eratosthenes for finding prime numbers.

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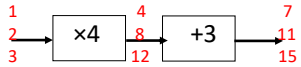
SHIFTING TIMES TABLES

Count in 4s saying the 4 times table: 4, 8, 12, 16, 20, … 80, 84, 88, …

Now shift the 4 times table up 3 and count in 4s again: 7, 11, 15, 19, 23, … 83, 87, 91, …

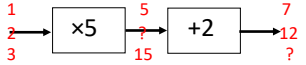
This diagram shows a Function Machine, also called a mapping diagram.



You input numbers into the boxes and the machine follows a rule and outputs a number.

How does the diagram above represent shifting the 4 times table up by 3?

What happens if you put the number 20 into the machine?



What is happening in this diagram?

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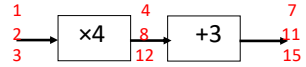
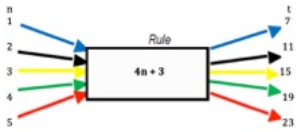
COMBINING 2 OPERATIONS INTO ONE FUNCTION

Input numbers *n* : 1, 2, 3, 4, 5, … 20, 21, 22, …

Multiples of 4: 4, 8, 12, 16, 20, … 80, 84, 88, …

Output 4*n* + 3: 7, 11, 15, 19, 23, … 83, 87, 91, …

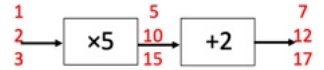


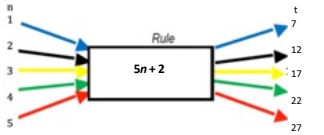


Input numbers n: 1, 2, 3, 4, 5, … 20, 21, 22, …

Multiples of 5: 5, 10, 15, 20, 25, … 100, 105, 110, …

Output 5n + 2: 7, 12, 17, 22, 27, …102, 107, 112, …





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SHIFTING TIMES TABLES MATCH SEQUENCE WITH RULES

Which multiplication tables were shifted to give the sequences below?

Match the rules to the sequences.

Sequences	Rules / Formulas
(a) 7, 12, 17, 22, 27, … 82, 87, 92, …	(1) $n \rightarrow 7n + 6$
(b) 9, 11, 13, 15, 17, … 89, 91, 93, …	(2) $n \rightarrow 5n + 2$
(c) 13, 20, 27, 34, 41, … 83, 90, 97, …	(3) $n \rightarrow 3n + 1$
(d) 4, 7, 10, 13, 16, … 79, 82, 85, …	(4) $n \rightarrow 6n - 1$
(e) 5, 11, 17, 23, 29, … 71, 77, 83, …	(5) $n \rightarrow 2n + 7$

Make up your own sequence and rule.

https://aiminghigh.aimssec.ac.za/shifting-times-tables/

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FUNCTION GAME

HOW TO PLAY THE GAME AS A LARGE GROUP

This game must be played in silence.

The teacher asks different people in the class to give her numbers which she writes in the INPUT column and then she writes number beside it in the OUTPUT column and asks **'What am I doing?'**

INPUT	OUTPUT
6	40
2	16
4	28 …

When someone thinks they know what rule the teacher is using they must put up their hand. The teacher will give them an INPUT and they must tell her the OUTPUT.

The game continues until most players have guessed the rule. **Then those who guessed it explain the rule to the others who did not guess it.**

First the presenters will play the game so everyone understands the rules.

Then everyone should find the answer to the 'What am I doing?' question. The rule can be described in words or given as a formula.

The game can be played as a pre-algebra learning activity with Year 6 or 7.

For older learners more complicated functions can be used.

The game be played by the whole class led by the teacher, **or in smaller groups** taking it in turns to ask the 'What am I doing ' question again and again until someone guesses the rule.

PLAYING THE FUNCTION GAME IN PAIRS OR SMALL GROUPS

First the players must agree on what numbers are allowed as inputs, and what types of function are allowed.

For example, players might agree on whole number inputs and +, -, × and ÷ or choose a different set of input numbers and allow more functions.

Players take turns to create their own function for the other player to guess.

The winner is the player who guesses the rule correctly with the smallest number of clues.

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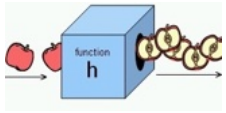
FUNCTION MACHINE

https://aiminghigh.aimssec.ac.za/function-game/



What is happening in this function machine? Perhaps it's labelled as h, for HALF, because the fruit is cut in half.
Don't put your hand in this machine! 😂
With your learners make your own function machine using any old cardboard box.

Make holes in two opposite faces, label the hole on the left INPUT and label the other hole OUTPUT.
Suppose your machine halves numbers rather than fruit. Cut some scrap paper into small pieces. Ask a learner to write a number on a scrap of paper and put it into the INPUT hole. You should take it out through the OUTPUT hole, look at it and write the output on another scrap of paper. For example, if 20 is the input then your output should be 10.
You could use the function machine to play the 'What Am I doing?' Game instead of writing numbers on the board.

FOR OLDER LEARNERS - VARIATION TO INVERSE FUNCTIONS (whole group game)
The game can be varied by sometimes using the numbers suggested by learners as inputs and sometimes as outputs, the teacher writing the numbers in the appropriate input and output columns. This can lead to a class discussion of inverse functions.
For example, for the function $x \rightarrow 3x+5$, if the output is 41 how do you find the input? You undo the "multiply the number by 3 then add 5" by first subtracting 5, then dividing by 3 which gives the input 12. The inverse function is $x \rightarrow (x-5)/3$.

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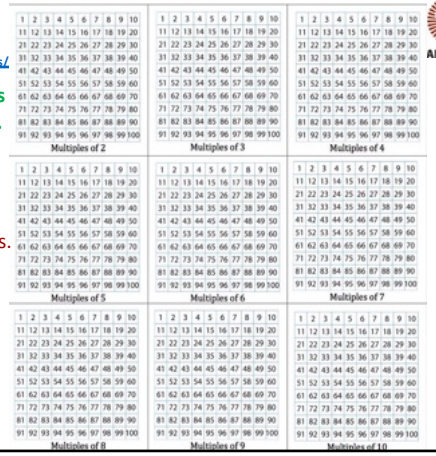
MULTIPLE PATTERNS

https://aiminghigh.aimssec.ac.za/multiple-patterns/

Choose a grid and shade the squares with multiples of the given number.

If this is done in groups each person can choose a different set of multiples. The group can discuss and compare the different patterns.

Groups could work together to produce wall posters for their classroom.



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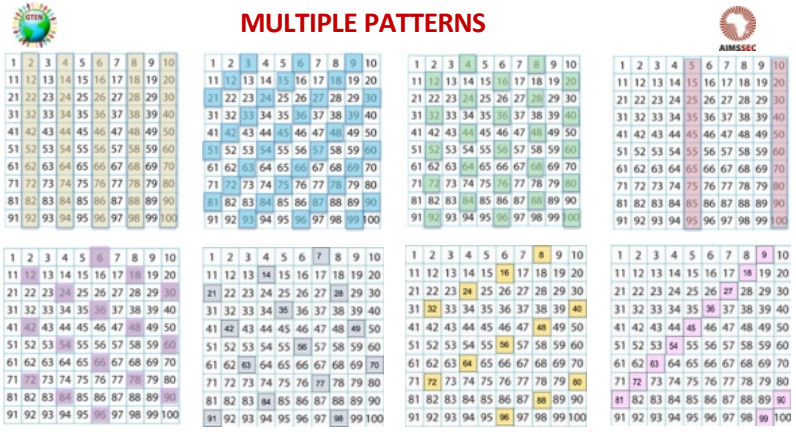
MULTIPLE PATTERNS



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MULTIPLE PATTERNS

https://aiminghigh.aimssec.ac.za/multiple-patterns/

Why do this activity?
By shading in the multiples and seeing the patterns learners can gain practice until they have instant recall of the multiplication facts and, at the same time, develop their number sense and gain insights into the concepts of multiples and common multiples. Learners usually enjoy this activity and it provides **visual learning** which will help many of them learn and remember the multiplication tables.

This activity paves the way for shading the multiples on one grid and producing the Prime Numbers Sieve (also known as The Sieve of Eratosthenes). https://aiminghigh.aimssec.ac.za/prime-sieve/

Learning objectives
In doing this activity students will have an opportunity to:
- practise multiplication tables;
- gain understanding of the concepts of multiples and common multiples;
- learn the vocabulary associated with multiplication, division, factors, multiples and common multiples.

Generic competences
In doing this activity students will have an opportunity to:
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts;
- **persevere and work systematically** to investigate all possible cases;
- **work collaboratively in a team** to produce a wall poster.

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MIND READER

Another pre-algebra game.

Learners should try to explain the answer.
They can give their explanation in words or as a formula.

Input x	x
Multiply by 3	$3x$
Add 6	$3x + 6$
Take away x	$2x + 6$
Divide by 2	$x + 3$
Take away x	**3**

Invent your own number tricks.

MIND READER

https://aiminghigh.aimssec.ac.za/mind-reader/

Choose any number to start with
- **multiply it by 3**
- **add 6**
- **take away your number**
- **divide by 2**
- **take away your number.**

What answer do you get?

Explain what happens.

Create your own number tricks

5 → 15 → 21 → 16 → 8 → 3

7 → 21 → 27 → 20 → 10 → 3

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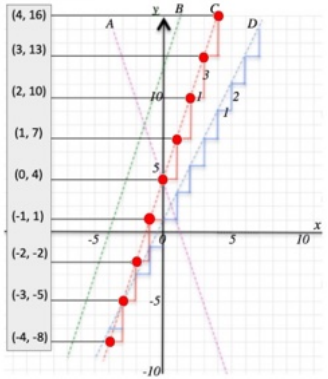
STEPS TO LINEAR FUNCTIONS AND THEIR GRAPHS



Continue the following sequences for the next 3 terms and continue the sequences backwards to the previous 3 terms:

-2, 1, 4, 7, 10, 13, 16, 19, 22, 25, 28	Line C $t = 3n + 4$
6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36	Line B $t = 3n + 12$
10, 7, 4, 1, -2, -5, -8, -11, -14, -17, -20	Line A $t = -3n + 4$

How do line C and the red steps in the diagram relate to the first sequence?

The first 2 sequences come from a multiplication table shifted up.

Which multiplication table is it?

What do you notice about the third sequence?

How do these sequences relate to the lines A, B and C?

Which sequence relates to which line?

What about Line D? Line D $t = 2n + 1$

https://aiminghigh.aimssec.ac.za/steps/

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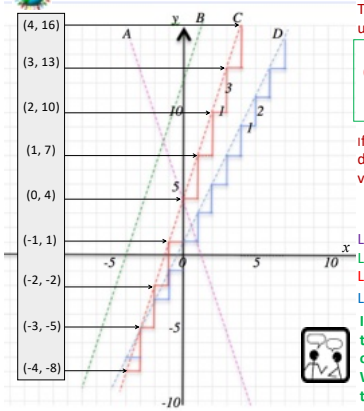
STEPS * LINEAR FUNCTIONS * GRAPHS * GRADIENTS



The variables n and t (for the terms in the sequence) are used for the points with WHOLE NUMBER coordinates (n, t).

-2, 1, 4, 7, 10, 13, 16, 19, 22, 25, 28	Line C $t = 3n + 4$
6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36	Line B $t = 3n + 12$
10, 7, 4, 1, -2, -5, -8, -11, -14, -17, -20	Line A $t = -3n + 4$
-3. -1, 1, 3, 5, 7, 9, 11, 13, 15, 17,...	Line D $t = 2n + 1$

If ALL the points on the line are included, and it is extended in both directions, the equation is written using (x, y) coordinates, but any variables can be used a long as axes are labelled

	Points from sequence (Grid points)	Equation of line
Line A	$t = -3n + 4$	$y = -3x + 4$
Line B	$t = 3n + 12$	$y = 3x + 12$
Line C	$t = 3n + 4$	$y = 3x + 4$
Line D	$t = 2n + 1$	$y = 2x + 1$

Imagine you are climbing the red steps from the point (-3 , -5) to the point (3, 13). Now imagine you climb the same number of the blue steps from the point (-3, -5) to the point (3, 7). Which is the steeper climb? Why? How could you measure the steepness of the climb?

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BUILDING FUNCTIONS

https://aiminghigh.aimssec.ac.za/building-functions/

Amy and her friends have built some functions.

If they all input the number 1 into their functions, Amy's output would be 6 and Busi's 1.

What would Chris's and Dudu's outputs be?

−2 | × 3 | ÷2 | +5

Amy's function $a \rightarrow [+5] \rightarrow a+5$

Busi's function $b \rightarrow [\times 3] \rightarrow [-2] \rightarrow 3b-2$

Chris's function $c \rightarrow [\div 2] \rightarrow [\times 3] \rightarrow [+5] \rightarrow 3c/2 + 5$

Dudu's function $d \rightarrow [-2] \rightarrow [\times 3] \rightarrow [\div 2] \rightarrow [+5] \rightarrow 3(d-2)/2 + 5 = 3d/2 + 2$

How many different functions can you make using 2 of the 4 operations without repetition.

Build some of your own functions using the operators:
[subtract 2],
[multiply by 3],
[divide by 2]
and [add 5].
Choose inputs and give the corresponding outputs to show how your functions work. Your inputs can be numbers or variables.

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BUILDING FUNCTIONS POSTER

https://aiminghigh.aimssec.ac.za/building-functions/

HOW MANY DIFFERENT FUNCTIONS CAN YOU FIND USING 2 OF THESE 4 OPERATIONS?			
−2 first	×3 first	+2 first	+5 first
-2 ×3 6 → 4 → 12 $x \rightarrow x-2 \rightarrow 3(x-2)$			

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BUILDING FUNCTIONS POSTER

https://aiminghigh.aimssec.ac.za/building-functions/

There are 12 functions that combine 2 of the 4 given operations

−2 first	×3 first	+2 first	+5 first
$6 \rightarrow 4 \rightarrow 12$ $x \rightarrow x-2 \rightarrow 3(x-2)$	$6 \rightarrow 18 \rightarrow 16$ $x \rightarrow 3x \rightarrow 3x-2$	$6 \rightarrow 3 \rightarrow 1$ $x \rightarrow \frac{1}{2}x \rightarrow \frac{1}{2}x-2$	$6 \rightarrow 11 \rightarrow 9$ $x \rightarrow x+5 \rightarrow 3(x+5)$
$6 \rightarrow 4 \rightarrow 2$ $x \rightarrow x-2 \rightarrow \frac{1}{2}(x-2)$	$6 \rightarrow 18 \rightarrow 9$ $x \rightarrow 3x \rightarrow \frac{3}{2}x$	$6 \rightarrow 3 \rightarrow 9$ $x \rightarrow \frac{1}{2}x \rightarrow \frac{3}{2}x$	$6 \rightarrow 11 \rightarrow 33$ $x \rightarrow x+5 \rightarrow 3(x+5)$
$6 \rightarrow 4 \rightarrow 9$ $x \rightarrow x-2 \rightarrow x+3$	$6 \rightarrow 18 \rightarrow 23$ $x \rightarrow 3x \rightarrow 3x+5$	$6 \rightarrow 3 \rightarrow 8$ $x \rightarrow \frac{1}{2}x \rightarrow \frac{1}{2}x+5$	$6 \rightarrow 11 \rightarrow \frac{11}{2}$ $x \rightarrow x+5 \rightarrow \frac{1}{2}(x+5)$

There are 24 functions that combine all 4 operations.
To find all 24 would be a good challenge for a group of high flyers.

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UNDOING : INVERSE FUNCTIONS

https://aiminghigh.aimssec.ac.za/undoing/

Amy and her friends have built some functions and they are challenging each other to find the input when they know an output. They think the inputs to their functions that all give the output 10 are:

5 for Amy's function, 4 for Busi's, 2½ for Chris's and 5⅓ for Dudu's.

Do you agree? Why or why not?

Amy's function $a \rightarrow$ [+5] $\rightarrow a+5$

Busi's function $b \rightarrow$ [×3] $\rightarrow$ [-2] $\rightarrow 3b-2$

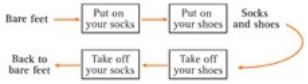
Chris's function $c \rightarrow$ [÷2] $\rightarrow$ [×3] $\rightarrow$ [+5] $\rightarrow 3c/2+5$

Dudu's function $d \rightarrow$ [-2] $\rightarrow$ [×3] $\rightarrow$ [÷2] $\rightarrow$ [+5] $\rightarrow 3(d-2)/2+5=3d/2+2$

Busi says that she goes back in the other direction to find inverses undoing the functions one by one. For her function, to find the input that gives the output 10 she works out 10+2=12 and 12÷3=4.

Amy says that inverse functions undo the operation of a function like undoing your shoelaces. She says that + and – undo each other and × and ÷ undo each other.

Find the inputs for Amy's, Busi's, Chris's and Dudu's functions corresponding to an output of 20?



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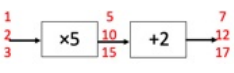
SUMMARY AND Q&A

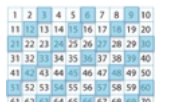
MULTIPLES, SEQUENCES, MAPPINGS, LINES, FUNCTIONS

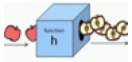
Clap Counting - Skip Counting - Patterns of Multiples - Shifting Times Tables - Steps - Graphs - Gradients - Building Functions - Undoing

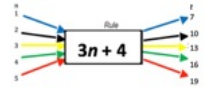
5, 10, 15, 20, 25,...
7, 12,, 17, 22, 27,...

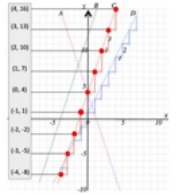
Orchestra









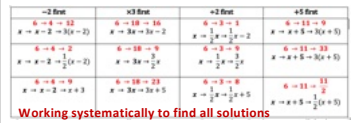


$a \rightarrow$ [+5] $\rightarrow a+5$

$b \rightarrow$ [×3] $\rightarrow$ [-2] $\rightarrow 3b-2$

$c \rightarrow$ [÷2] $\rightarrow$ [×3] $\rightarrow$ [+5] $\rightarrow 3c/2+5$

$d \rightarrow$ [-2] $\rightarrow$ [×3] $\rightarrow$ [÷2] $\rightarrow$ [+5] $\rightarrow 3(d-2)/2+5=3d/2+2$



Working systematically to find all solutions

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

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