Global Teacher Empowerment Network GTEN

CHECKIT GAMES - COMBINING 3 NUMBERS

Saturday 12 February 15:00 – 17:00 London Time


Toni Beardon


Caroline Ainslie


Mary Achieng

Put your name, country and the age group you teach on the chat

1

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

MATHS TOYS

Global Teacher Empowerment Network (GTEN)

PROGRAMME: FROM ADDITION OF 2 NUMBERS TO GEOMETRY OF CIRCLES AND TRIANGLES

Learning Spiral

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:

Addition and subtraction as inverse operations
Fractions. Exponents
Factorising quadratic expressions
Simple circle geometry
Solving simultaneous equations
Similar and congruent triangles
Parallelograms

UPPER SECONDARY
9. Triangle midpoints: some pure and some analytic geometry.
8. Solving simultaneous equations

LOWER SECONDARY
7. Polycircles Puzzle: Simple circle geometry for touching circles.
6. Checkit with Exponents.

UPPER PRIMARY
5. Checkit Algebra game
4. Checkit Word Puzzle – invent your own.

LOWER PRIMARY
3. Checkit with Fractions
2. Checkit with different sets of numbers and different rules

EARLY YEARS STARTER ACTIVITY
1. Checkit with small children.

2

MATHS TOYS

Do the activities.

Answer questions and comment on the chat.

Questions are in green with this icon.

PARTICIPATE even if you make guesses.

You'll get more benefit out of the workshop.

Remember to put your name, country and the age group you teach on the chat

3

MATHS TOYS

CHECKIT FOR SMALL CHILDREN

https://aiminghigh.aimssec.ac.za

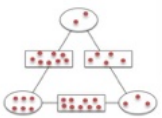
You need some counters or other small objects.

Draw a Checkit frame.

Take turns to put counters in the boxes.

The number of counters in the middle boxes must be the sum of the numbers at the vertices.

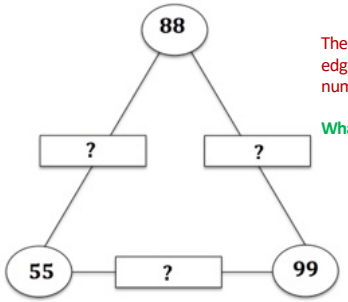
Check. If you make a mistake you lose a point.



4

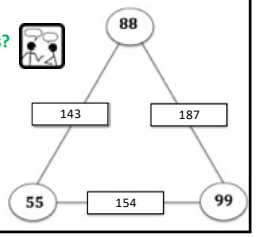
Global Teacher Empowerment Network (GTEN)

NEW SKILLS NEW HOPES NEW HORIZON for
teachers and learners worldwide
CHECKIT PUZZLES & GAMES COMBINING 3 NUMBERS



The numbers in the middle of the edges are the total of the numbers at the vertices.

What are the missing numbers?



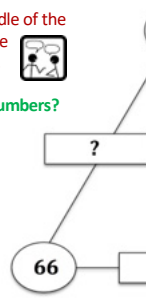
5

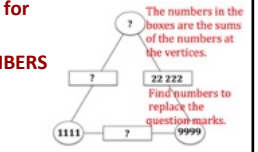
Global Teacher Empowerment Network (GTEN)

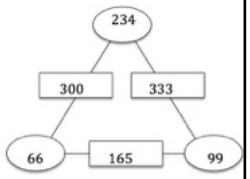
NEW SKILLS NEW HOPES NEW HORIZON for
teachers and learners worldwide
CHECKIT PUZZLES & GAMES COMBINING 3 NUMBERS

The numbers in the middle of the edges are the total of the numbers at the vertices.

What are the missing numbers?





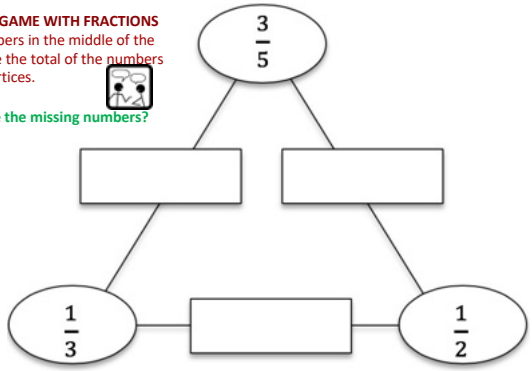


6

Global Teacher Empowerment Network (GTEN)

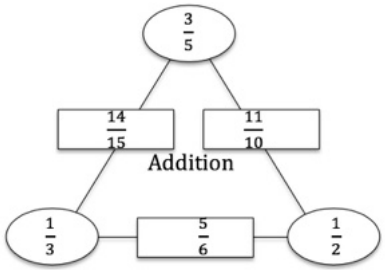
CHECKIT GAME WITH FRACTIONS
The numbers in the middle of the edges are the total of the numbers at the vertices.

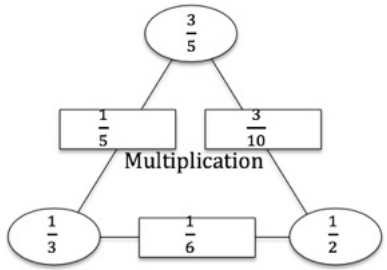
What are the missing numbers?



7

Players agree the rules at the start.
Checkit can be played with whole numbers, with integers, with fractions, even with exponents, algebraic expressions or words.





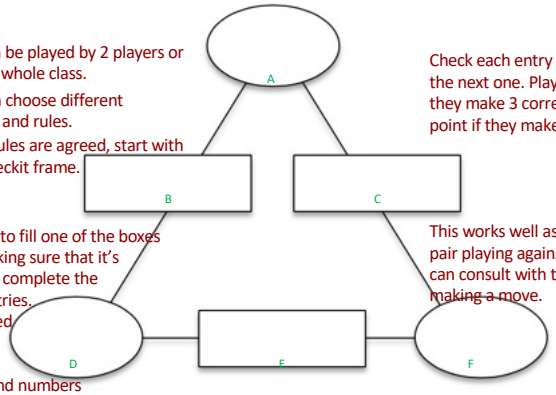
8

TAILOR YOUR OWN CHECKIT MAKE IT UP AS YOU GO

Checkit can be played by 2 players or teams or a whole class.

Players can choose different operations and rules.

Once the rules are agreed, start with a blank Checkit frame.



Check each entry before moving on to the next one. Players score a point if they make 3 correct entries and lose a point if they make a mistake.

Take turns to fill one of the boxes always making sure that it's possible to complete the set of 6 entries.

If challenged a player must show they can find numbers to complete all 6 entries.

This works well as a game for 4 with one pair playing against another pair. They can consult with their partner before making a move.

9

CHECKIT WITH EXPONENTS AND GREATEST COMMON DIVISOR

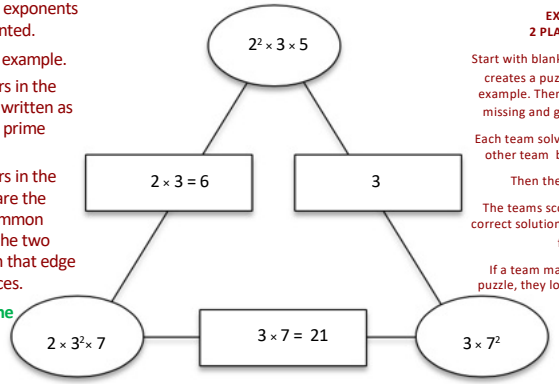
Many different Checkit games with exponents can be invented.

Here is one example.

The numbers in the ellipses are written as products of prime factors.

The numbers in the rectangles are the greatest common divisors of the two numbers on that edge at the vertices.

What are the missing numbers?



EXPONENT GAME
2 PLAYERS OR 2 TEAMS

Start with blank Checkit frames. Each player creates a puzzle with 6 entries as in this example. Then they draw it with 3 entries missing and give that to the other team.

Each team solves the puzzle created by the other team by completing all 6 entries.

Then they check the solutions.

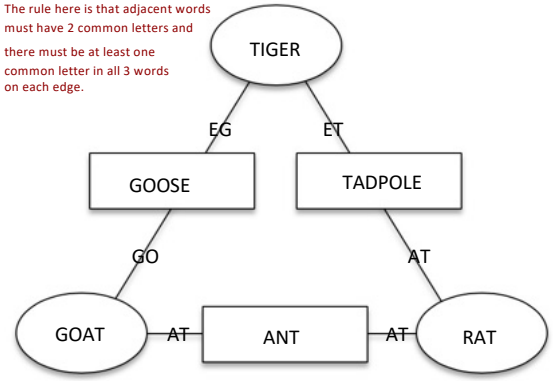
The teams score 3 points for a complete correct solution but all mistakes lose a point for that team.

If a team makes mistakes creating the puzzle, they lose a point for each mistake.

10

CHECKIT WORD PUZZLE

The rule here is that adjacent words must have 2 common letters and there must be at least one common letter in all 3 words on each edge.



Either: Make up your own Checkit word puzzles.

Then you can give another person the puzzle with 3 words missing

Or: Take turns to fill one of the boxes always making sure that it's possible to complete the set of 6 entries.

If challenged, players must show they can find words to complete all 6 entries.

You can set a time limit if the players agree at the start.

11

POLYCIRCLES

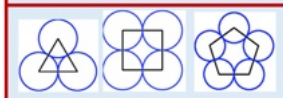
STATEMENT: Given a collection of discs they can be arranged to touch each other.

CONVERSE OF STATEMENT: Given a polygon can you find circles with centres at the vertices of the polygon with each circle just touching both it's neighbours?

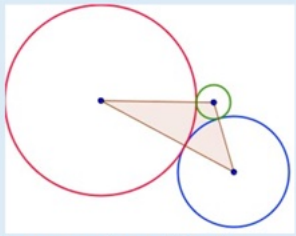
12

POLYCIRCLES PUZZLE

https://aiminghigh.aimssec.ac.za/polycircles/

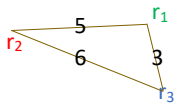


Starting with a polygon, find circles with centres at the vertices of the polygon such that each circle just touches its neighbouring circles, like beads on a necklace. The simplest cases are regular polygons.



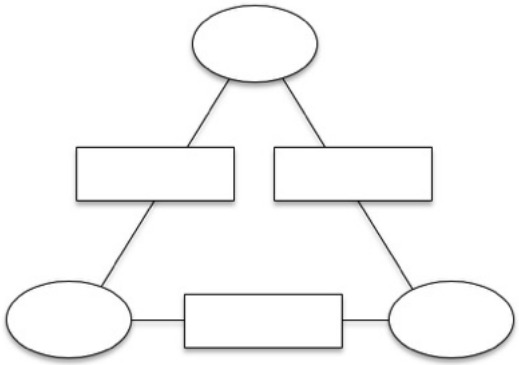
More generally, solving the puzzle depends on the number of edges of the polygon. When is it possible to find solutions?

Start with any triangle. Can you draw touching circles with the vertices of the triangle as centres?



13

TAILOR YOUR OWN CHECKIT

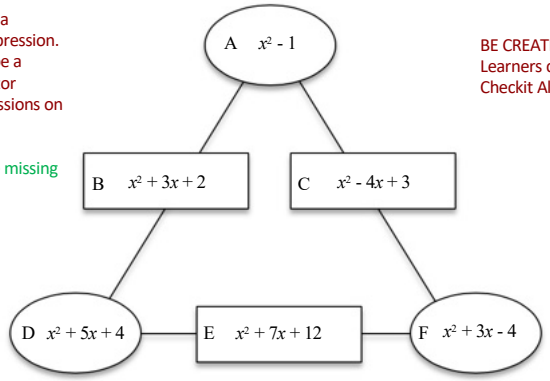


14

CHECKIT ALGEBRA GAME

Each entry is a quadratic expression. There must be a common factor in all 3 expressions on each edge.

What are the missing expressions?

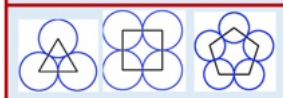


BE CREATIVE
Learners can invent their own Checkit Algebra games

15

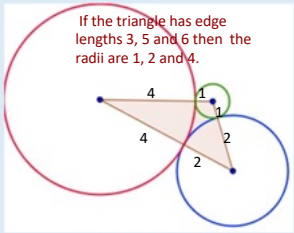
POLYCIRCLES PUZZLE

https://aiminghigh.aimssec.ac.za/polycircles/



Starting with a polygon, find circles with centres at the vertices of the polygon such that each circle just touches its neighbouring circles, like beads on a necklace. The simplest cases are regular polygons.

If the triangle has edge lengths 3, 5 and 6 then the radii are 1, 2 and 4.



More generally, solving the puzzle depends on the number of edges of the polygon. When is it possible to find solutions?

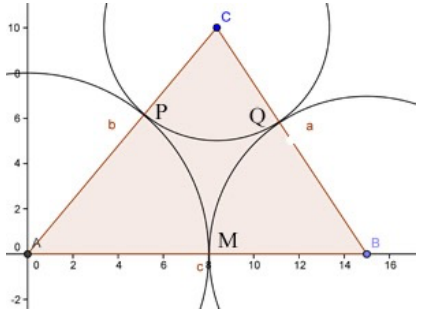
Find some solutions for 3 circles and draw them.

Can you develop a method for finding solutions?

Start with any triangle. Can you find a solution?

16

POLYCIRCLES



If you and your learners have access to Geogebra then learners can experiment to find solutions.

They can use Geogebra to demonstrate that once they have found one solution then the shape and size of the triangle can be changed and the circles change correspondingly.

17

POLYCIRCLES AND SIMULTANEOUS EQUATIONS

TO FIND RADII AND A SOLUTION FOR A TRIANGLE WITH EDGES 4, 6 AND 8

If the 3 circles have radii a, b and c then

$a + b = 6$ (1)

$b + c = 4$ (2)

$c + a = 8$ (3) **Solve these 3 equations**

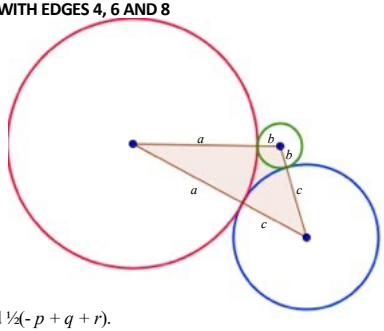
From (1) and (2)

$a - c = 2$ (4)

Adding (3) and (4) $a = 5$ and $c = 3$

So $b = 1$.

The radii are 1, 3 and 5 units.



Using the method above if

$a + b = p$

$b + c = q$

$c + a = r$

then the solutions are: $\frac{1}{2}(p - q + r)$, $\frac{1}{2}(p + q - r)$ and $\frac{1}{2}(- p + q + r)$.

To find solutions for quadrilaterals you need 4 equations in 4 unknowns, for pentagons you need 5… for n-gons you need n equations in n unknowns and so on. A more challenging question is 'when is it possible to find solutions?'.

18

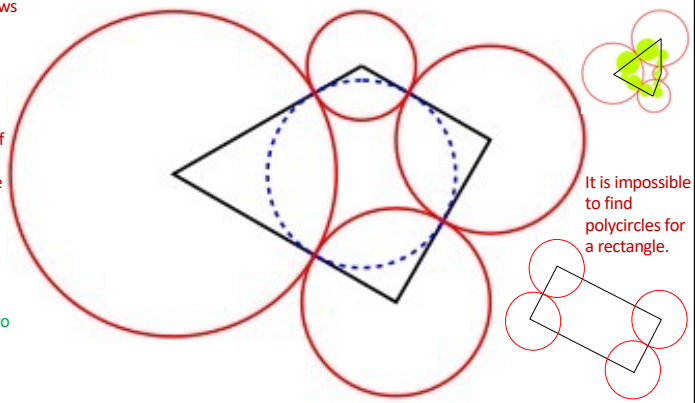
POLYCIRCLES FOR QUADRILATERALS

The diagram shows a quadrilateral with polycircles.

Notice that the pairs of radii are tangents to the inscribed circle of the quadrilateral and that they are equal in length.

Label the radii r_1, r_2, r_3 and r_4.

What do you notice about the lengths of the two pairs of opposite edges of the quadrilateral?

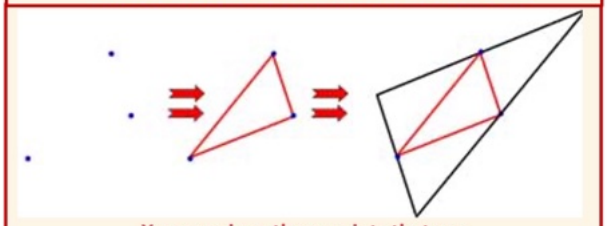


It is impossible to find polycircles for a rectangle.

19

TRIANGLE MIDPOINTS

https://aiminghigh.aimssec.ac.za/triangle-midpoints/



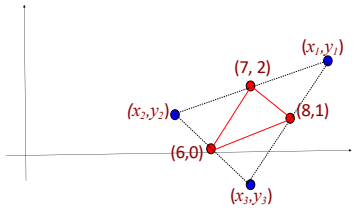
You are given three points that are the midpoints of the edges of a triangle.
How can you construct the original triangle?
Is there more than one answer?

What does the diagram on the right suggest?

Can we use the Triangle Midpoint Theorem to provide information so that, given the red triangle, we can construct the black triangle?

20

TRIANGLE MIDPOINTS



This method involves solving 3 equations in 3 unknowns but they are not difficult to solve because all the coefficients are 1.

If your learners can do this then you can praise them highly and tell them they will not have to do anything so difficult in their exams.

METHOD 1

If the triangle has MIDPOINTS (6,0), (7, 2) and (8,1) and the coordinates (x_1,y_1), (x_2,y_2) and (x_3,y_3) of the larger triangle to be found,then $(x_1 + x_2)/2 = 7$ etc.

$x_1 + x_2 = 2\times7=14$ (1)

$x_2 + x_3 = 2\times6=12$ (2)

$x_3 + x_1 = 2\times8=16$ (3) Solve these 3 equations

Solving these equations simultaneously

equation (1) - equation (2) gives

$x_1 - x_3 = 2$ (4)

So from equations (3) and (4)

we get $x_1 = 9$, $x_2 = 5$ and $x_3 = 7$.

Use the same method to find the y coordinates.

The solutions are: $y_1 = 3$, $y_2 = 1$ and $y_3 = -1$.

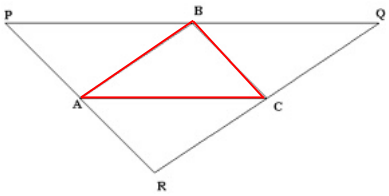
So the larger triangle has vertices (9, 3), (5, 1) and (7, -1).

21

TRIANGLE MIDPOINTS – PROPORTIONAL INTERCEPT THEOREM

When you join the midpoints of the edges of any triangle the large triangle is made up of four congruent triangles one of which is the triangle joining the midpoints.

What do you notice about triangles PAB and PRQ?
What do you notice about triangles QBC and QPR?



Method 2 PROPORTIONAL INTERCEPT TRIANGLE MIDPOINT THEOREM

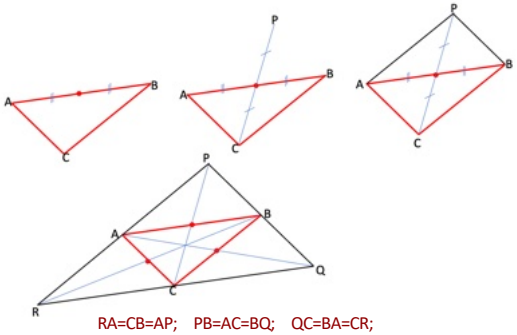
Call the mid points A, B and C
Join A, B and C to form a triangle ABC.

The original triangle PQR can be drawn by constructing a line through A parallel to BC, a line through B parallel to AC and a line through C parallel to AB.

You should then be able to prove that the points A, B and C are the mid-points of PR, PQ and QR using the fact that triangles PAB and PRC are similar.

22

TRIANGLE MIDPOINTS – CONSTRUCTING PARALLELOGRAMS



RA=CB=AP; PB=AC=BQ; QC=BA=CR;
A, B, C are midpoints of the edges RP, PQ, QR
Triangle PQR is the required triangle.

Method 3

Construction: Take the three midpoints and call them A, B and C. Draw triangle ABC.

Take the midpoint of AB and measure the distance from it to C. Draw a line CP twice that distance.

What do you notice about the quadrilateral APBC?

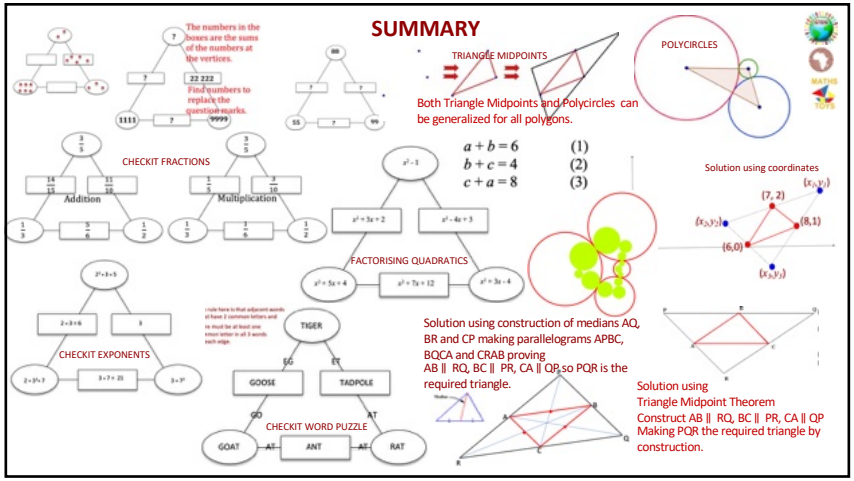
What do you notice about triangles ABC and APB?

Observe that the lines AB and PC bisect each other so **APBC is a parallelogram,** triangles ABC and APB are congruent.

Repeat this for the other edges forming the parallelograms BQCA and CRAB.

23

SUMMARY



24

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

AIMSSEC Website: http://aimssec.ac.za
AIMING HIGH Free lesson resources: http://aiminghigh.aimssec.ac.za
AIMSSEC APP: https://aimssec.app **to download the resources for use offline**

Checkit Game	https://aiminghigh.aimssec.ac.za/checkit-game/
Checkit Challenge	https://aiminghigh.aimssec.ac.za/checkit-challenge/
Polycircles	https://aiminghigh.aimssec.ac.za/polycircles/
Triangle Midpoints	https://aiminghigh.aimssec.ac.za/triangle-midpoints/
Download Geogebra Apps	https://www.geogebra.org/download
Learn Geogebra Classic	https://www.geogebra.org/m/XUv5mXTm

COLLABORATIVE PROFESSIONAL DEVELOPMENT https://aiminghigh.aimssec.ac.za/category/cpd
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS
https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/

25

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

Global Teacher Empowerment Network (GTEN)

For teachers in primary and secondary schools, colleges and universities

MATHS TOYS

AIMSSEC GTEN YouTube Channel
https://www.youtube.com/c/MathsToys/videos

AIMSSEC FACEBOOK https://www.facebook.com/aimssecsa/
HAPPY MATHS HOUR Weekly on Mondays 17:00 - 18:00 UK time

To apply to join the GTEN Teachers WhatsApp Group and to get information about GTEN or to apply for an AIMSSEC course write to <admin@aimssec.ac.za>

26

LET'S PLAY MATHEMATICALLY AND LEARN

Order from AMAZON or TARQUIN https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically

Play Mathematically
- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

27

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

Toni Beardon LAB11@cam.ac.uk
Caroline Ainslie caroline@bubblymaths.co.uk
Enquire about signing up for an AIMSSEC course as a self-funding student admin@aimssec.ac.za

Thankyou for coming to this workshop.

Use the AIMSSEC ideas on AIMING HIGH and add comments.

Share what you have learned with other teachers.

Try to help all your learners to have a 'YES I CAN' attitude to mathematics.

28