

Global Teacher Empowerment Network GTEN
 Saturday 23 September 2023 16.00 – 18.00 London Time
BELTWAY ROUND THE WORLD

Toni Beardon Caroline Ainslie James Tanton

The outer fence is 1 metre longer than the inner fence.
 Is the path wide enough for a cow to walk along it? Or a mouse? ..

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AIMS African Institute for Mathematical Sciences
 SCHOOLS ENRICHMENT CENTRE

MATHS TOYS GTEN

Global Teacher Empowerment Network (GTEN)
 PROGRAMME: BELTWAY ROUND THE WORLD

Learning Spiral

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:

- Perimeters
- Circumference of a circle
- Problem solving
- The importance of connections and generalisation in mathematics

UPPER SECONDARY 9. Generalisations and similar challenges.
 8. Belt Around the World. Find the gap again.

LOWER SECONDARY 7. The Circle and The Square
 6. Generalising Roundabout

UPPER PRIMARY 5. Not-So-Square Fence. Find the gap again.
 4. Roundabout locus – Disc around a square.

LOWER PRIMARY 3. People Maths – Square Fence.
 2. Square Fence – Mind/find the gap

EARLY YEARS 1. Find the distance around the room.

2

GTEN **AIMSSEC**

During this session you should wear 2 hats. Do the activities, as if you were a learner, to appreciate how a learner would do them. Reflect as a teacher on what they could learn.

Use the worksheet to do your working out.

When you see this icon answer the questions. Use the worksheet and pause the video when you need time.

3

FIND THE DISTANCE AROUND THE ROOM - PERIMETER

This is an activity for a whole class. Find a room where you can stand against the walls all the way around. All the learners should stand with their backs to the wall and their arms outstretched touching fingertips.

You can do this along two walls if the room is rectangular but, if there are enough learners, then they can line up against the wall as described all the way around the room.

Count the learners.
 Talk about how you can use the number of learners to find the distance around the room.
 Learners could work in pairs and measure, for each learner, the distance from fingertip to fingertip with their arms outstretched (arm span).
 See the AIMING HIGH Learning packs;
<https://aiminghigh.aimssec.ac.za/estimate-my-girth/>
 and <https://aiminghigh.aimssec.ac.za/metre-measures/>


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MIND THE GAP

A fence is built around a square field. Another fence is built exactly one metre longer so the path between the two fences is the same width along the edges of the field.

- How wide would this path be?
- Would a mouse be able to run along this path?
- Could a farmer drive a herd of cows along the path between the fences?
- Suppose you were the farmer and you wanted to make the path a certain width, how would you work out how much fencing to buy?
- Work it out and put responses on the chat.

The outer fence is 1 metre longer than the inner fence



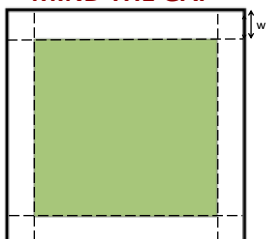
Not to scale

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This is a question about perimeter. The key to a quick and easy solution is to understand that:

- the edges of the fence parallel to the edges of the field will always be the same
- WHATEVER THE SIZE OF THE FIELD;
- extra fence is only needed at the corners.

If the width of the path is w , and you think about the extra fence at each corner, it forms two edges of length w at right angles.

This means that the extra fence makes a square and it has length $8 \times w$ metres for any sized field.

We know that the extra length of fence is 1 metre so the width of the path is one eighth of a metre, that is 12.5 centimetres.

The path is too narrow for cows, but a mouse could easily run along it.

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SQUARE FENCE - PEOPLE MATHS

Find the perimeter of a square, indoors or outdoors.

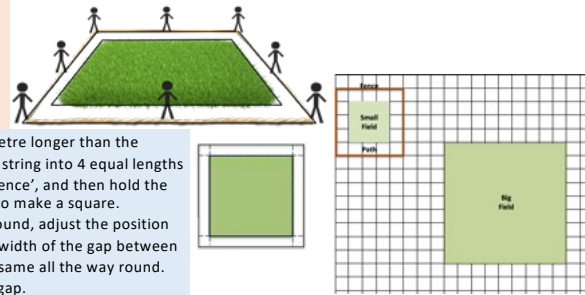
ACTIVITY: Use string to mark out the line of a fence (string-fence) around the square that is exactly one metre longer than the perimeter of the square.

METHOD: Make your string 1 metre longer than the perimeter of the field, mark the string into 4 equal lengths for the 4 corners of the 'string-fence', and then hold the 4 corners, stretching the string to make a square.

Lower the string close to the ground, adjust the position of the 'string-fence' so that the width of the gap between the square and the string is the same all the way round. Then measure the width of the gap.

Talk about what would happen to the gap if the experiment is repeated for a smaller square, and again with the string one metre longer than the perimeter of the square.

Vote as to whether the gap between the square and the string will be less, the same, or more. You can think of this as making the path between the square and the fence narrower, the same, or wider. Which will it be? Carry out the experiment with the smaller square to find the answer.



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WHY DO THIS LEARNING ACTIVITY?

Learning objectives

In doing this activity students will have an opportunity to:

- reinforce **understanding of perimeter**
- develop **problem solving skills**
- develop the ability to **use and apply knowledge to a real-life context.**

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically**, reason logically
- develop **communication skills by giving explanations**
- visualize, develop the **skill of interpreting and creating visual images** to represent concepts and situations
- apply visualization skills** to interpreting information and solving problems.

See the Learning Pack on AIMING HIGH <https://aiminghigh.aimssec.ac.za/square-fence/>

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ROUNDAABOUT THOUGHT EXPERIMENT

A disc rolls around the outside edge of a square so that its circumference always touches the edge of the square.

Describe the path (or locus) of the **centre** of the disc and its length.

Describe the locus of the centre when a disc rolls around other polygons.

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ROUNDAABOUT THOUGHT EXPERIMENT

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Describe the locus of the centre when a disc rolls around other polygons.

SOLUTION
As the red disc rolls along each edge, the centre moves parallel to the edge, a distance equal to the length of the edge of the square. At the corners, the point of contact on the disc stays still, and the disc turns through 90° with the centre describing a quarter circle. The locus of the centre of the disc is like a square with rounded corners, and the length of the locus is equal to the perimeter of the square plus the circumference of the circle.

NOTE ON THE LANGUAGE: A **CIRCLE** is, strictly speaking, the one-dimensional set of points around the perimeter (outline) of a **DISC**. The **CIRCUMFERENCE** is the distance around a circle (the perimeter of the disc). The **DIAMETER** is the distance across the circle through the centre.

A disc is the set of points inside the circle. The terms circle and disc are used interchangeably in schools. But be careful, a disc is not a cylinder, it is two-dimensional and a cylinder is three-dimensional.

See the Learning Pack on AIMING HIGH <https://aiminghigh.aimssec.ac.za/roundabout/>

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Is the path wide enough for you to walk along it?
<https://aiminghigh.aimssec.ac.za/not-so-square-fence/>

A path around a square field between two fences is the same width all the way round including the corners, the outer fence exactly one metre longer than the inner.

How wide would the path be?

Not-So-Square Fence

Does the width of the path depend on the size of the field?

What is the difference between this problem and the Square Fence problem?

What is the connection between this problem and Roundabout?

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The outer fence is one metre longer than the inner fence. The path between the two fences is the same width all the way round including at the corners.

1. How wide would this path be?

2. If you were the farmer and you had already decided on the width that you wanted to make the path, how would you work out how much fencing to buy for the outer fence?


For any sized field, the fence must be curved at the corners around a quarter circle. The extra fence has length 4 times the length of a quarter circle, which is the length of the circumference of a full circle, that is $2\pi w$ metres. The extra fence has length one metre so the width of the path (the radius) must be 1 metre divided by 2π , that is 0.159 metres or 15.9 centimetres (to 3 sig. figs).

This path is a little wider than the path for the Square Fence problem, and it is wide enough for a family of mice to run along, but not nearly wide enough for cows.

If the farmer wants the width of the path to be say x metres, then he must buy sufficient extra fencing to go round the circumference of a circle radius x , that is $2\pi x$ metres of extra fencing (plus a little more for fixing it in place).

See the Learning Pack on AIMING HIGH <https://aiminghigh.aimssec.ac.za/not-so-square-fence/>


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KEY QUESTIONS

These are questions teachers ask when learners are stuck, to help them to solve problems for themselves without the teacher showing them how to do it.

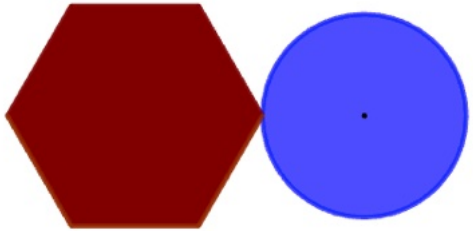
1. Have you drawn a diagram?
2. When you make the fence longer and the path wider, where does the extra fence go?
3. What shape is the fence in this question?
4. Where is the fence straight?
5. Where is the fence curved?




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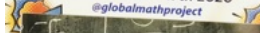

GENERALISING ROUNDABOUT



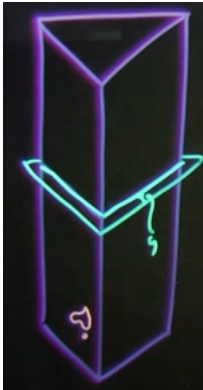
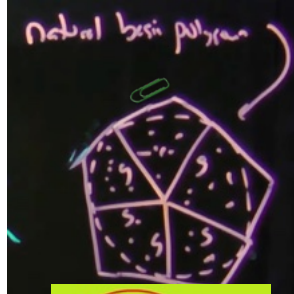

See the NRICH interactivity Rolling Around <https://rich.maths.org/2159>



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JAMES' SECTION 1

<https://www.youtube.com/@JamesTantonMath>

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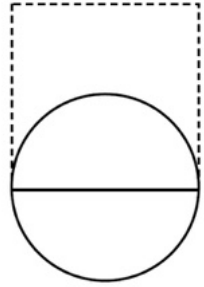


**5 MINUTE BREAK
FOLLOWED BY CAROLINE & TONI'S SECTION 2**


THE CIRCLE AND THE SQUARE

Which is longer, the dashed portion of the square or the circumference of the circle?


Explain your reasoning. 🤖




See the Learning Pack on AIMING HIGH <https://aiminghigh.aimssec.ac.za/the-circle-and-the-square/>




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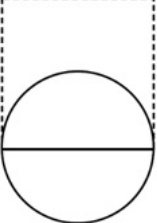


THE CIRCLE AND THE SQUARE



Which is longer, the dashed portion of the square or the circumference of the circle?

Explain your reasoning. 



SOLUTION


The dashed portion of the square is $3 \times D$
 The circumference of the circle is $\pi \times D$
 π is approximately 3,14 (or 3.14 if you prefer the decimal point)

Therefore, the circumference of the circle is bigger than the dashed portion of the square.


Note: $\pi = 3.14159265358979\dots$
 π is an *irrational number* corresponding to an infinite decimal with no recurring pattern. It is sometimes approximated by the *rational number* $\frac{22}{7}$

See the Learning Pack on AIMING HIGH <https://aiminghigh.aimssec.ac.za/the-circle-and-the-square/>

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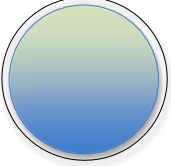
BELT AROUND THE WORLD



A wire belt is tied tightly around the Earth at the equator.

Suppose the belt is made exactly one metre longer and held around the Earth at the equator so that it is the same distance away from the Earth everywhere.

1. Would a mouse be able to crawl under the new belt? How do you know?
2. Would the answer be the same for the moon?
3. What simplification would you need to make, so that it is possible to solve the problem? 





MODELLING PROCESS: Imagine a slice cut through the equator, and model this by taking the land surface to be a sphere, and the circumference of the slice to be a circle.


The equator is not a perfect circle but this is a good enough approximation.

The belt is modelled by a circle just one metre longer than the circumference of the earth.



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BELT AROUND THE WORLD



Let the radius of the Earth be given by R metres and the height of the belt above the ground be given by h metres. Consider the circumference of the Earth $2\pi R$, and the length of the belt in metres:





$2\pi R + 1 = 2\pi(R + h)$
 This gives $2\pi h = 1$ metre for all values of R.
 $h = \frac{1}{2\pi}$ metre, that is 0.159 metres or **15.9 centimetres** (correct to 3 significant figures), which is just over 6 inches.
 This gap under the belt is plenty high enough for a mouse to walk underneath with his head held high, or even for a small cat stalking the mouse with his head down low.


This is the black footed cat *Felis Nigripes*, the smallest African wild cat. The adult male shoulder height is 25 cm. It lives in dry, open grassland and Karoo semidesert. If the belt around the Earth was on a great circle crossing the Karoo this cat could get under it to catch a mouse, but only by crouching low.

Because the calculation **does not use the radius R** the answer would be the same for the moon or even for a sphere as small as a soccer ball or even a golf ball.


See the Learning Pack on AIMING HIGH <https://aiminghigh.aimssec.ac.za/belt-round-the-world/>



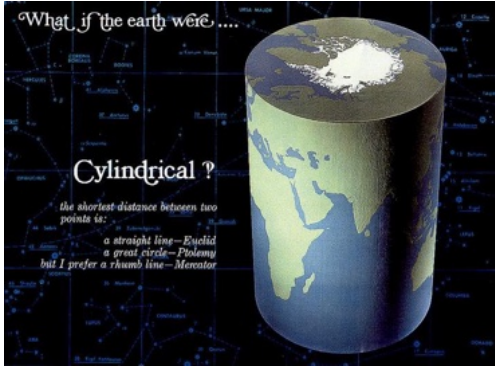
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WHAT IF




See 'Earth Shapes' on the NRICH website <http://nrich.maths.org/1363>. This is an imaginative science fiction idea, but it provides some further food for thought.



Another extension problem could be 'Watching the Wheels Go Round and Round' on the NRICH website. <http://nrich.maths.org/1039>.

The front wheel on the penny-farthing bicycle has a circumference of 2 metres and the back wheel 50 centimetres.

How many times would the wheels turn if the bicycle travels one kilometre? Would the large or the small tyre get more wear and tear on a long journey? 

What if the earth were

Cylindrical ?

the shortest distance between two points is:

a straight line – Euclid
a great circle – Ptolemy
but I prefer a rhumb line – Mercator

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WATCHING THE WHEELS GO ROUND AND ROUND
<http://nrich.maths.org/1039>

The front wheel on the penny-farthing bicycle has a circumference of 2 metres and the back wheel 50 centimetres.

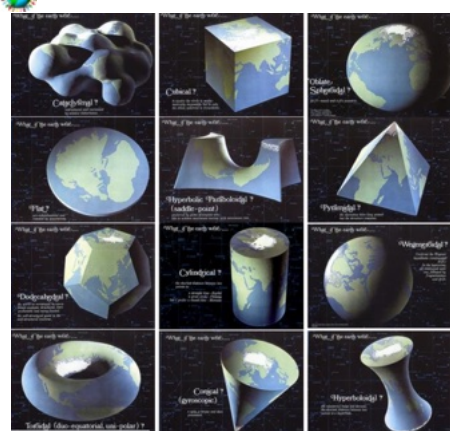
How many times would the wheels turn if the bicycle travels one kilometre?
 Would the large or the small tyre get more wear and tear on a long journey?

Seven pupils of Wrenbury Primary School's Extension Maths Club had the following solutions:

First you need to know how many centimetres in a metre (100).
 Then you take 200cm and convert them into metres, which gives you 2 metres.
 Therefore, the circumference is 2 metres.
 Then you need to know how many metres in a kilometre (1000).
 So, you halve it to find out how many times the wheel turns (500 times).
 Then you do the same calculations for the back wheel,
 which is 50cm in circumference or 0.5m.
 This means that the back wheel would turn 4 times more than the front wheel (2000 times).
 So, the back wheel must get more wear and tear because it goes round
 4 times more often than the front wheel.

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WHAT IF



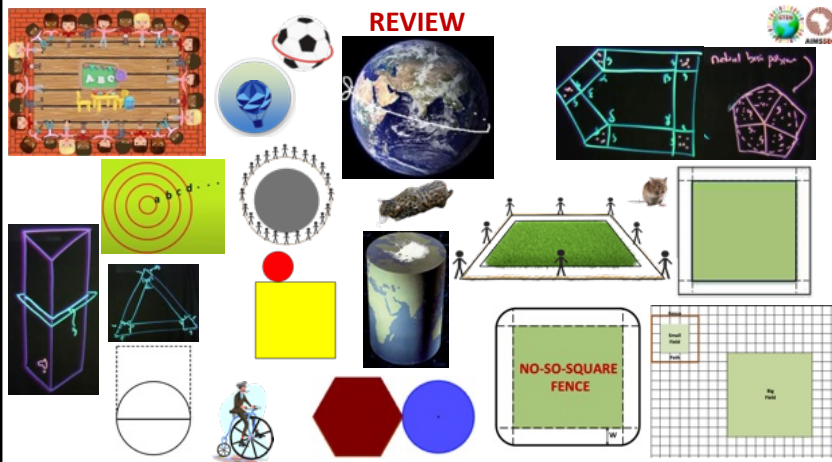
The Earthshapes were invented by Joseph Portney about 40 years ago when flying over the North pole on board a U.S. Air Force KC-135. These odd shaped models were the result of Joe looking down on the icy scene below and thinking: "What if the Earth were...?"

Joe graduated from the US Naval Academy, served as a navigator bombardier in the US Air Force and then has worked for many years for Litton Guidance & Control Systems on navigation problems concerned with the safe guidance and control of high altitude, long range, aircraft (both military and commercial).

By the application of mathematics to re-programming the guidance system's computer, a plane could be guided safely over and around the odd curves and corners of these hypothetical worlds. **NOW JAMES' SECTION 2**

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REVIEW



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AIMINGHIGH TEACHER NETWORK

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See the James Tanton Youtube Channel <https://www.youtube.com/@JamesTantonMath>

See the Learning Packs on AIMING HIGH

SQUARE FENCE <https://aiminghigh.aimssec.ac.za/square-fence/>

ROUNDBOUT <https://aiminghigh.aimssec.ac.za/roundabout/>

THE CIRCLE AND THE SQUARE <https://aiminghigh.aimssec.ac.za/the-circle-and-the-square/>

NOT-SO-SQUARE FENCE <https://aiminghigh.aimssec.ac.za/not-so-square-fence/>

BELTWAY ROUND THE WORLD <https://aiminghigh.aimssec.ac.za/belt-round-the-world/>

METRE MEASURES <https://aiminghigh.aimssec.ac.za/metre-measures/>

ESTIMATE MY GIRTH <https://aiminghigh.aimssec.ac.za/estimate-my-girth/>

See the NRICH learning activities:

The interactivity ROLLING AROUND <https://nrich.maths.org/2159>

EARTH SHAPES <http://nrich.maths.org/1363>

WATCHING THE WHEELS GO ROUND AND ROUND <http://nrich.maths.org/1039>

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On the AIMING HIGH website there are freely downloadable worksheets, Inclusion and Home Learning Guides with activities for learners of all ages and attainment levels, Notes for Teachers with solutions, Key Questions and Diagnostic Quizzes for formative assessment.

AIMSSEC GTEN YouTube Channel 
<https://www.youtube.com/c/MathsToys/videos>



AIMSSEC FACEBOOK <https://www.facebook.com/aimssecsa/>

To apply to join the GTEN Teachers WhatsApp Group and to get information about GTEN or to apply for an AIMSSEC course write to [<admin@aimssec.ac.za>](mailto:admin@aimssec.ac.za)

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LET'S PLAY MATHEMATICALLY AND LEARN 

Order from AMAZON or TARQUIN <https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically>



Play Mathematically

- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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Thanks for coming to this workshop.

Use the AIMSSEC ideas on AIMING HIGH and add comments.

Share what you have learned with other teachers.

Try to help all your learners to have a **'YES I CAN'** attitude to mathematics.



Toni Beardon LAB11@cam.ac.uk
 Caroline Ainslie caroline@bubblymaths.co.uk

Enquire about signing up for an AIMSSEC course as a self-funding student admin@aimssec.ac.za

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