**Slide 1**

# Global Teacher Empowerment Network GTEN
Saturday 20 July 2024   16:00 – 18:00 London Time
## TESSELLATION REPTILES AND FRACTALS

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AIMS | African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

MATHS TOYS

## Global Teacher Empowerment Network (GTEN)
### PROGRAMME: TESSELLATION AND REPTILES

IMPROVE SKILLS, KNOWLEGE AND UNDERSTANDING OF

Transformations:
Translation,
Reflection and Rotation
Tiling and Tessellation
Similarity and Enlargement

**Learning Spiral**

UPPER SECONDARY — 11. Create your own Escher type artwork
10. Reptile puzzles
LOWER SECONDARY — 9. Fractals
8. Reptiles
UPPER PRIMARY — 7. Some examples of tessellations
6. The 17 two dimensional wallpaper patterns
5. Difference between tessellation and tiling
LOWER PRIMARY — 4. Transformations that define the 7 frieze patterns
3. One dimensional strip (or frieze) patterns
EARLY YEARS — 2. Some of the work of Escher
1. Tiling starter

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During this session you need to wear 2 hats.

Do the activities as if you were a learner

to appreciate how a learner

at a particular stage would do them,

and then reflect as a teacher

on what they could learn

from doing the activity.

Teacher

Learner

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## TILING FOR YOUNG CHILDREN

Show how the dinosaur and the camel can be tiled using tiles made of 4 squares.

Tiles made from 4 squares are called Tetrominoes.

**MORE ACTIVITIES TO TRY:**
*Make your own animal from a tiling of tetrominoes.
*Make tetrominoes of different shapes.
*How many different tetrominoes can you find?
*How about pentominoes (5 squares) and hexominoes (6 squares)?
*Either make a tiling on squared paper with an H-shape made up of 7 squares or cut out an H-shape made from 7 squares and use it as a template to make a tiling. What do you notice?

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## TILING FOR YOUNG CHILDREN



The elephant can be tiled with L shapes.
Which tetrominoes can you use to tile the penguin and the robot giant?

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## ESCHER MATHS & ART



We are going to do **TESSELLATION** activities.

**Maurits Cornelius Escher**

(1898 - 1972) was a Dutch graphic artist who made mathematically inspired woodcuts, lithographs, and mezzotints.

His work explored **TESSELLATION, that is** the filling of space by regularly repeating patterns.

He was greatly influenced by the Islamic art that he saw in Spain which was brought there from North Africa.

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## ESCHER MATHS, ART & ALHAMBRA



TILING FROM ALHAMBRA

INSIDE ALHAMBRA PALACE

At the Alhambra Palace in Granada, Spain, Escher saw all 17 patterns that fill two-dimensional space (the wallpaper patterns).

Islamic art provides evidence of the sophisticated culture of the Moors in Spain in the 14th century, who had a good knowledge of mathematics.

EXAMPLES OF ESCHER'S ART

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## FRIEZE PATTERNS



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**Persian frieze designs at Persepolis**

https://en.wikipedia.org/wiki/Persepolis

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## Slide 10

**Frieze from Delphi, Greece UNESCO World Heritage site** https://en.wikipedia.org/wiki/Delphi

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## Slide 11

**FRIEZE PATTERNS**

TIRE TRACKS

**FOOTPRINTS**

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## Slide 12
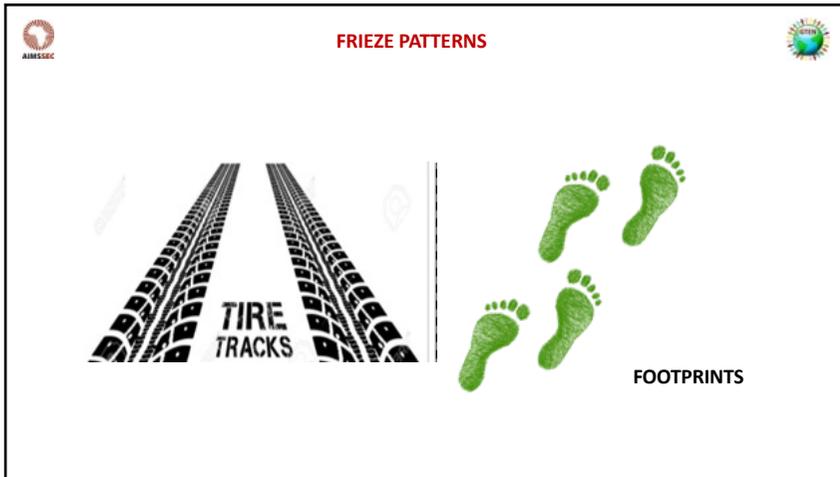
**7 FRIEZE PATTERNS**

It can be proved that there are only 7 ways that transformations can be combined to give frieze patterns.

There are countless different motifs
(for example, motifs could be letters of the alphabet)

The transformations involved define the patterns, **not the motifs.**

pppppppp

dbdbdbdb

HHHHHHHH

EEEEEEEEEE

pbpbpbpb

pdpdpdpd

pqbdpqbd

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**FRIEZE PATTERNS**

# ppppppp frieze pattern

Repeated by **TRANSLATIONS**

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**FRIEZE PATTERNS**

# EEEEEEEE frieze pattern
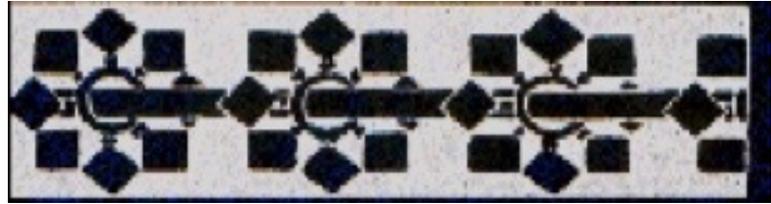
Repeated by **TRANSLATIONS** and **REFLECTION IN A HORIZONTAL MIRROR LINE**

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**FRIEZE PATTERNS**

# dbdbdbdb frieze pattern

Repeated by **TRANSLATIONS** and **REFLECTIONS IN A VERTICAL MIRROR LINE**

15

**FRIEZE PATTERNS**

# pdpdpdpd frieze pattern
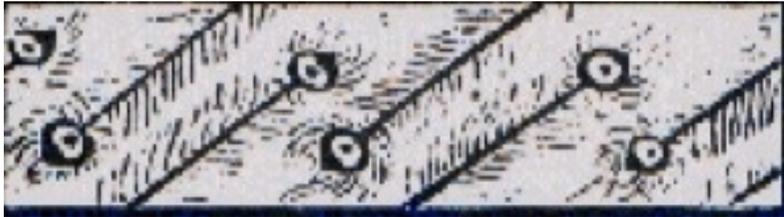
Repeated by **TRANSLATIONS** and **ROTATIONS**

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**FRIEZE PATTERNS**

# HHHHHHHH frieze pattern



Repeated by **TRANSLATIONS, REFLECTION IN HORIZONTAL & VERTICAL MIRROR LINES** and **ROTATIONS**

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**HAPPY MATHS HOUR * FRIEZE PATTERNS**

# pbpbpbpb frieze pattern



Repeated by **TRANSLATIONS, GLIDE (FOOTSTEPS) REFLECTIONS** and **ROTATIONS**

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**HAPPY MATHS HOUR * FRIEZE PATTERNS**

# pqbdpqbd frieze pattern



Repeated by **TRANSLATIONS, REFLECTION IN VERTICAL MIRROR LINES, GLIDE (FOOTSTEPS) REFLECTIONS** and **ROTATIONS**

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# Tessellations and Tilings
Tessellations by M C Escher



Tiling is a covering of the plane by copies of a geometric shape with no overlaps or gaps.

Tessellation is a special case of tiling that fills space with a *regularly repeating* pattern.

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## Slide 21

**THE 17 WALLPAPER PATTERNS**
**TESSELLATIONS OF THE PLANE**



$C_1$   $C_2$   $C_3$   $C_4$   $C_6$

$D_1kk$   $D_1gg$   $D_1kg$   $D_2kkkk$   $D_2gggg$

$D_2kkgg$   $D_2kgkg$   $D_4$   $D_4^\circ$

$D_3'$   $D_3^\circ$   $D_6$

1924Polya

There are 230 different crystal structures in 3D.

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**TESSELLATION & TILING**



What do you see in this picture?

It has been drawn by hand but let's assume that all the bulls' heads are identical but not symmetrical.

Imagine adding another bull's head, and another.

Could this pattern go on to cover a large wall, and beyond that?

What do you notice about the orange and green lines?

Do the lines help you to imagine the pattern repeating itself?

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## Slide 23

**TESSELLATION & TILING**



On squared paper, draw a tessellation made with the H shape.

Can you see how to make the pattern using a half-H shape?

Can you see how to make the pattern using a quarter-H shape?

Join the dots. What do you notice?

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## Slide 24

**TESSELLATION & TILING   *   TRANSLATION & REFLECTION**



These two tessellations are based on repeating a parallelogram by translations.

The H-tessellation could be based on reflection of a half-H (in the dotted line) followed by parallel translations or it could be based on rotations and reflections of a quarter-H followed by translations.

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## TESSELLATION & TILING  H-PUZZLE

**Cut an H shape into 5 pieces as shown.**
**Use the 5 pieces to make a parallelogram**.



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## What is a reptile?



This reptile is called
**The Sphinx**.

A reptile is a shape that can be dissected into smaller identical copies of the same shape.

Identical copies of a reptile can be put together to make an **enlargement** of the shape with linear scale factor 2 and area scale factor 4

The Sphynx can be made up of 4 smaller copies of itself so it is called a rep-4.

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## POLYOMINOES

Polyominoes are shapes made up of squares. Here are some examples.



DOMINO

TRIOMINO

TETROMINO

PENTOMINO

HEXOMINO

TESSELLATION OF SEPTOMINOES

The table shows the total number of each type.

Can you find them all?

Which pentominoes tessellate?

Which pentominoes are reptiles?

| Name | Number of variations |
|---|---|
|  | 1 |
| Domino | 1 |
| Triomino | 2 |
| Tetromino | 5 |
| Pentomino | 12 |
| Hexomino | 35 |

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## PENTOMINO PUZZLE

Make all 12 pentominoes and use them to make a rectangle.



Start with 3 pentominoes and complete the rectangle.
Each piece of the jig-saw is a different shape.

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## Pentomino Puzzle

The solution to the pentomino puzzle.
Cut out the pieces, mix them, then solve the puzzle.
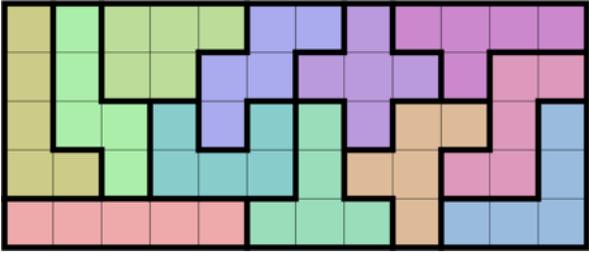


Which of these pentominoes are reptiles?

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## Reptiles and Fractals * Triomino or Trisquare
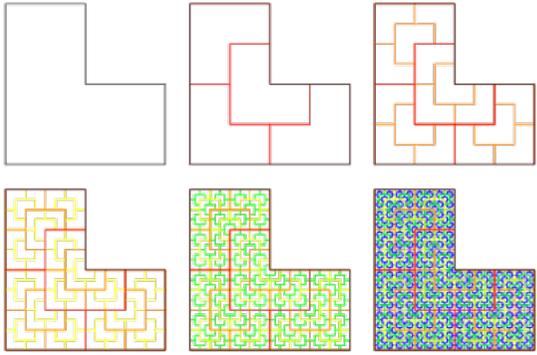


Making smaller copies of the shape inside every copy of the shape, and repeating this process infinitely often, produces a fractal.

The diagrams show stages 0 to 5 of the fractal process based on the shape made up of 3 squares.

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## Polyomino Reptile Fractals
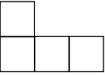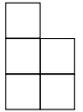


A tetromino

Put 4 copies of these shapes together to make enlarged copies of the same shape.

A pentomino

This can be repeated again and again, infinitely often to produce a fractal.

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## Four different Octomino Reptiles

Can you make 4 tessellations with these octominoes?
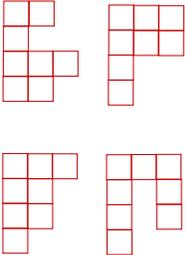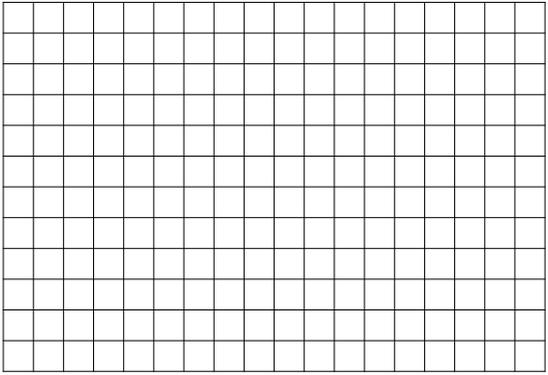


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## Reptile Trapezia * all rep-4s

Fill in 4 smaller similar copies of the same shape inside each tetrahedron.

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## Selection of Reptiles

Can you spot a rep-2?

And some rep-9s?

How would you classify the other reptiles?

In some tilings (e.g. the 2 fish) the shapes intersect but still fill the plane without gaps or overlapping.

By MagistraMundi - CC BY-SA 3.0

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## Sierpinski Triangle
## Reptile fractal construction

3 stage 5 triangles like this make the stage 6 Sierpinski triangle.

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## Create your own Escher type tilings

Cut a piece from one edge of a square (or parallelogram) and attach it to the opposite edge. This can be done on both pairs of opposite edges.
If you do this on card then you can use it as a template and draw around it to create a tessellation.

Be creative with your design. Perhaps turn the shape into a cat or fish or …

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**Watch recordings of GTEN workshops on the AIMSSEC YouTube channel**
https://youtube.com/c/mathstoys

**DAILY MATHS STARTER AIMSSEC Facebook**
https://facebook.com/aimssecsa

**REPTILES & FRACTALS RESOURCES ON THE AIMING HIGH TEACHER NETWORK**

| | |
|---|---|
| Fractals Collection | https://aiminghigh.aimssec.ac.za/30-minute-fractals.../ |
| Trisquares | https://aiminghigh.aimssec.ac.za/trisquares/ |
| Enlargement | https://aiminghigh.aimssec.ac.za/enlargement/ |
| Tessellating Triangles | https://aiminghigh.aimssec.ac.za/tessellating-triangles/ |
| Tessellating Quadrilaterals | https://aiminghigh.aimssec.ac.za/tessellating-quadrilaterals/ |
| Tessellation and Tiling | https://aiminghigh.aimssec.ac.za/tessellation-and-tiling/ |

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Thanks for coming to this workshop.

Use the AIMSSEC ideas on AIMING HIGH and add comments.

Share what you have learned with other teachers.

Try to help all your learners to have a

**'YES I CAN'**

attitude to mathematics.

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Enquire about signing up for an AIMSSEC course as a self-funding student admin@aimssec.ac.za

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