

**Global Teacher Empowerment Network GTEN**  
 Saturday 20 July 2024 16:00 – 18:00 London Time

## TESSELLATION REPTILES AND FRACTALS

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**AIMS** African Institute for Mathematical Sciences  
 SCHOOLS ENRICHMENT CENTRE

**MATHS TOYS**

**Global Teacher Empowerment Network (GTEN)**  
 PROGRAMME: TESSELLATION AND REPTILES

**Learning Spiral**

**IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF**

Transformations:  
 Translation,  
 Reflection and Rotation  
 Tiling and Tessellation  
 Similarity and Enlargement

**UPPER SECONDARY** 11. Create your own Escher type artwork  
 10. Reptile puzzles

**LOWER SECONDARY** 9. Fractals  
 8. Reptiles

**UPPER PRIMARY** 7. Some examples of tessellations

**LOWER PRIMARY** 6. The 17 two dimensional wallpaper patterns  
 5. Difference between tessellation and tiling

**EARLY YEARS** 4. Transformations that define the 7 frieze patterns  
 3. One dimensional strip (or frieze) patterns  
 2. Some of the work of Escher  
 1. Tiling starter

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**GTEN**

**AIMSSEC**

During this session you need to wear 2 hats.

Do the activities as if you were a learner to appreciate how a learner at a particular stage would do them, and then reflect as a teacher on what they could learn from doing the activity.

**Teacher**

**Learner**

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**AIMSSEC**

## TILING FOR YOUNG CHILDREN

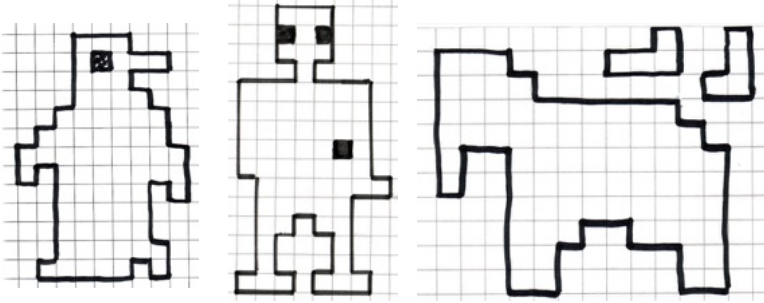
Show how the dinosaur and the camel can be tiled using tiles made of 4 squares. Tiles made from 4 squares are called Tetrominoes.

**MORE ACTIVITIES TO TRY:**

- \*Make your own animal from a tiling of tetrominoes.
- \*Make tetrominoes of different shapes.
- \*How many different tetrominoes can you find?
- \*How about pentominoes (5 squares) and hexominoes (6 squares)?
- \*Either make a tiling on squared paper with an H-shape made up of 7 squares or cut out an H-shape made from 7 squares and use it as a template to make a tiling. What do you notice?

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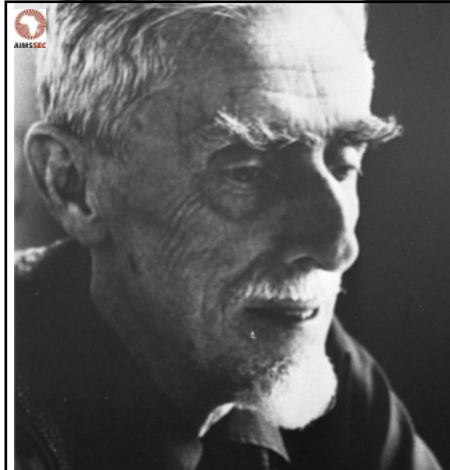
**TILING FOR YOUNG CHILDREN**



The elephant can be tiled with L shapes.  
Which tetrominoes can you use to tile the penguin and the robot giant?

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**ESCHER MATHS & ART**



We are going to do **TESSELLATION** activities.

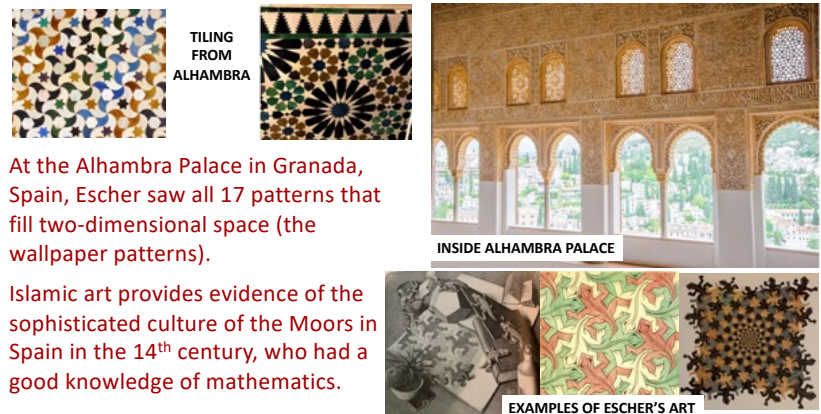
**Maurits Cornelius Escher** (1898 - 1972) was a Dutch graphic artist who made mathematically inspired woodcuts, lithographs, and mezzotints.

His work explored **TESSELLATION**, that is the filling of space by regularly repeating patterns.

He was greatly influenced by the Islamic art that he saw in Spain which was brought there from North Africa.

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**ESCHER MATHS, ART & ALHAMBRA**



**TILING FROM ALHAMBRA**

At the Alhambra Palace in Granada, Spain, Escher saw all 17 patterns that fill two-dimensional space (the wallpaper patterns).

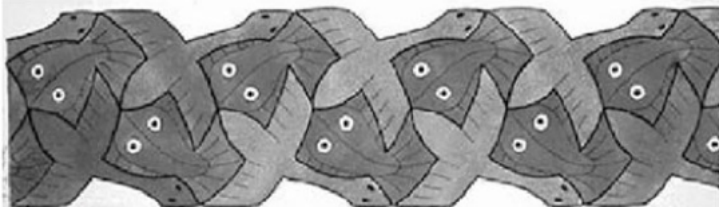
Islamic art provides evidence of the sophisticated culture of the Moors in Spain in the 14<sup>th</sup> century, who had a good knowledge of mathematics.

**INSIDE ALHAMBRA PALACE**

**EXAMPLES OF ESCHER'S ART**

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**FRIEZE PATTERNS**



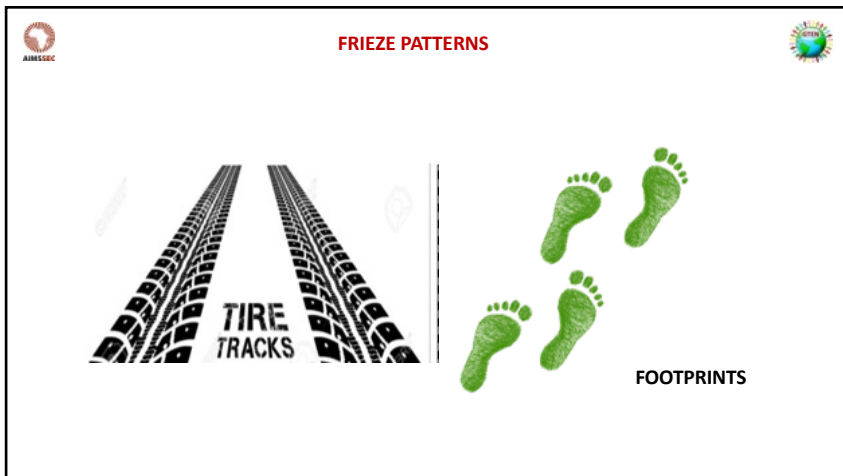
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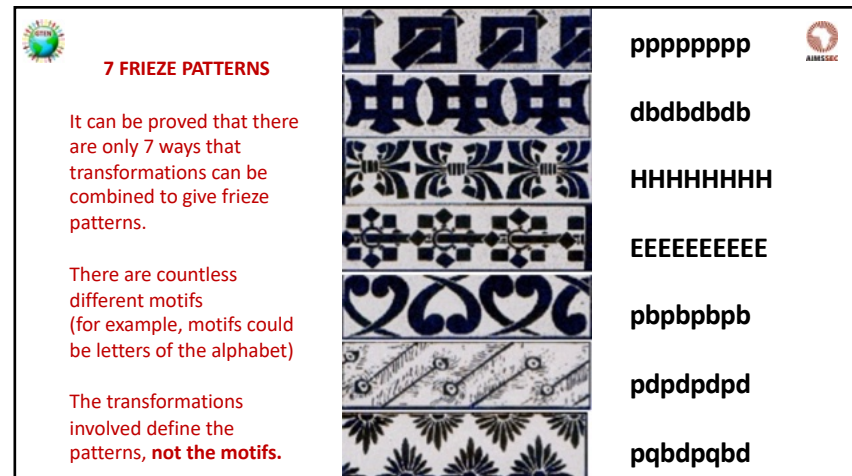
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11



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FRIEZE PATTERNS

pppppppp frieze pattern



Repeated by **TRANSLATIONS**

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FRIEZE PATTERNS

EEEEEEEE frieze pattern



Repeated by **TRANSLATIONS** and **REFLECTION IN A HORIZONTAL MIRROR LINE**

14

FRIEZE PATTERNS

dbdbdbdb frieze pattern



Repeated by **TRANSLATIONS** and **REFLECTIONS IN A VERTICAL MIRROR LINE**

15

FRIEZE PATTERNS

pdpdpdpd frieze pattern



Repeated by **TRANSLATIONS** and **ROTATIONS**

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**FRIEZE PATTERNS**

## HHHHHHHH frieze pattern




Repeated by **TRANSLATIONS, REFLECTION IN HORIZONTAL & VERTICAL MIRROR LINES** and **ROTATIONS**

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**HAPPY MATHS HOUR \* FRIEZE PATTERNS**

## pbpbpbpb frieze pattern



Repeated by **TRANSLATIONS, GLIDE (FOOTSTEPS) REFLECTIONS** and **ROTATIONS**

18

**HAPPY MATHS HOUR \* FRIEZE PATTERNS**

## pqbdpqbd frieze pattern

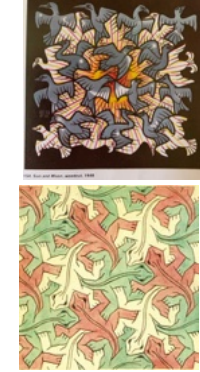


Repeated by **TRANSLATIONS, REFLECTION IN VERTICAL MIRROR LINES, GLIDE (FOOTSTEPS) REFLECTIONS** and **ROTATIONS**

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**Tessellations and Tilings**

Tessellations by M C Escher



Tiling is a covering of the plane by copies of a geometric shape with no overlaps or gaps.

Tessellation is a special case of tiling that fills space with a **regularly repeating** pattern.

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### THE 17 WALLPAPER PATTERNS TESSELLATIONS OF THE PLANE

1924Polya **There are 230 different crystal structures in 3D.**

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### TESSELLATION & TILING

What do you see in this picture?

It has been drawn by hand but let's assume that all the bulls' heads are identical but not symmetrical.

Imagine adding another bull's head, and another.

Could this pattern go on to cover a large wall, and beyond that?

What do you notice about the orange and green lines?

Do the lines help you to imagine the pattern repeating itself?

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### TESSELLATION & TILING

On squared paper, draw a tessellation made with the H shape.

Can you see how to make the pattern using a half-H shape?

Can you see how to make the pattern using a quarter-H shape?

Join the dots. What do you notice?

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### TESSELLATION & TILING \* TRANSLATION & REFLECTION

These two tessellations are based on repeating a parallelogram by translations.

The H-tessellation could be based on reflection of a half-H (in the dotted line) followed by parallel translations or it could be based on rotations and reflections of a quarter-H followed by translations.

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**TESSELLATION & TILING H-PUZZLE**

Cut an H shape into 5 pieces as shown.  
Use the 5 pieces to make a parallelogram.

The diagram illustrates the process of dissecting an H-shape into five pieces. On the left, a white outline of an H-shape is shown. In the middle, yellow lines indicate the cuts. On the right, the five pieces are shown separated. Below this, a 3D rendering shows the pieces being assembled into a parallelogram. A small icon of a person's head is visible in the bottom right corner of the diagram area.

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**What is a reptile?**

This reptile is called **The Sphinx**.

A reptile is a shape that can be dissected into smaller identical copies of the same shape.

Identical copies of a reptile can be put together to make an **enlargement** of the shape with linear scale factor 2 and area scale factor 4

The Sphinx can be made up of 4 smaller copies of itself so it is called a rep-4.

The slide features a photograph of the Great Sphinx of Giza on the left. To its right is a geometric diagram of the Sphinx shape, which is a right-angled triangle with a smaller right-angled triangle attached to its hypotenuse. This shape is composed of 16 smaller triangles of the same shape, arranged in a 4x4 grid of the basic shape's components.

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**Reptiles and Fractals \* Triomino or Trisquare**

A fractal is produced by making smaller copies of the shape inside every copy of the shape, and repeating this process infinitely often.

The diagrams show stages 0 to 5 of the fractal process based on the shape made up of 3 squares.

**END OF PART 1**

The diagrams show the iterative construction of a fractal from a 3-square L-shape. Stage 0 is a simple L-shape made of three squares. Stage 1 shows the L-shape with a smaller copy of itself inside. Stage 2 shows the L-shape with two smaller copies inside. Stage 3 shows the L-shape with four smaller copies inside. Stage 4 shows the L-shape with eight smaller copies inside. Stage 5 shows the L-shape with sixteen smaller copies inside, creating a complex, self-similar pattern.

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