Global Teacher Empowerment Network GTEN
DIGIT DETECTIVE

FREE AIMSSEC RESOURCES and learning activities for ages 5 to 18+
with a recurring Mathematical Theme to foster understanding and inclusion


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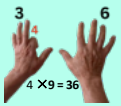
TWO + TWO = FOUR

FOOT-BALL GAME

A A A + A B A

9 × 9 + 7 = 88
98 × 9 + 6 = 888
987 × 9 + 5 = 8888
9876 × 9 + 4 = 88888
98765 × 9 + 3 = 888888
987654 × 9 + 2 = 8888888
9876543 × 9 + 1 = 88888888
98765432 × 9 + 0 = 888888888

Find the largest possible values for ?? + ?? = ?? and ?? × ?? = ???
Find the smallest possible values.
Explain.



Each ? is a missing digit from 0 to 9 either the same or different digits.
? + 8 = 2?
2? + 4 = 1?
33 + ? = 4?
What do you notice?

Think of 2 numbers a & b and $10a + b$

8?? ÷ 3 = 272

816 ÷ ? = 272
3?6 + 5? = 383
8?3 - 7? = 815

Have you noticed all the clues?
Have you found all the solutions?

1

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE
MATHS TOYS

Global Teacher Empowerment Network (GTEN)
PROGRAMME: DIGIT DETECTIVE

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:
Place value
Methods of calculation
Mathematical Thinking
Systematic planning and working to find all solutions.
Visualisation

Learning Spiral

UPPER SECONDARY
10. Number trick : I'll guess the two numbers in your mind.
9. Analysing and explaining number patterns involving multiples of 9

LOWER SECONDARY
8. Replace unknown digits puzzles with 1 then 2 then 3-digit numbers
7. Cryptarithm puzzles adding 4-digit numbers

UPPER PRIMARY
6. Cryptarithm puzzles adding 3-digit numbers
5. Cryptarithm puzzles adding 2-digit numbers

LOWER PRIMARY
4. Counting in 9s and multiplication tables – 'Using your hands'
3. A simple letter puzzle (cryptarithm)

EARLY YEARS
2. Number bonds for 10
1. Replacing shapes with digits to make sums correct

2

GTEN

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During this session you need to wear 2 hats.
Do the activities as if you were a learner
to appreciate how a learner
at a particular stage would do them,
and then reflect as a teacher
on what they could learn.

3

DO-TALK-RECORD
the basis of a good lesson

AIMSSEC

Replace the ? with digits to make the sum correct.
33 + ? = 4?

DO Solve the puzzle
TALK What do you notice?
RECORD Write down answer.

How do you tackle an activity when there might be more than one answer?

DO the activities pretending that you only know what your learners know.

TALK to other teachers in your group, or comment on the chat. Might learners discover new ideas by doing the activity? Or reinforce ideas they already know about.

RECORD for yourself as a teacher and also in a way that would help learners.

DO Study this flower picture
TALK What do you notice?
RECORD Draw your own flowers for other numbers.

An activity calling for creativity and collaborative learning.

4

DIGIT DETECTIVE - Getting there.... Start them young

Thinking algebraically

A

△ + 9 = 10

△ − △ = □

B

5 + △ = 9

7 − △ = □

Rules

1. Find a number between 0 and 9 that goes into each shape.
2. On each card the same number must go into the same shape.
3. The answer to the sum must be correct.

C

□ + △ + △ = 22 □ + △ = 13

What is △?

What is □?

Explain how you found your answers

Explain how you found your answer for Card C.

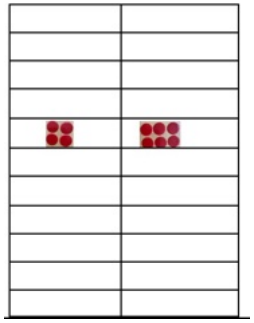
5

DIGIT DETECTIVE - Getting there.... Start them young

Put 10 counters into 2 boxes in different ways. You need 110 counters altogether; they can be stones, dried peas or beans.

One has been done for you.

4 + 6 = 10 is an example of a NUMBER BOND.



```
  A A
+ A
-----
B A
```

To solve this puzzle, put a number in place of A and another number in place of B to make this addition sum correct.

Can A be any other number? Explain why or why not.

6

TIMES FIVE & TIMES NINE - Getting started

5
10
15
20
25
30
35
40
45
50

Here are 14 handprints (or parts of handprints)
14 x 5 = 70

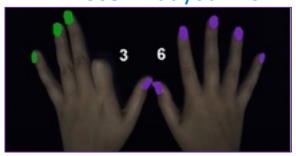
Count out some stones or dried peas and arrange them in sets of 9.

9	1 × 9 = 10 − 1 = 9
18	2 × 9 = 20 − 2 = 18
27	3 × 9 = 30 − 3 = 27
36	4 × 9 = 40 − 4 = 36
45	5 × 9 = 50 − 5 = 45
54	6 × 9 = 60 − 6 = 54
63	7 × 9 = 70 − 7 = 63
72	8 × 9 = 80 − 8 = 72
81	9 × 9 = 90 − 9 = 81

7

You can always count on a helping hand!

Use your digits... Use your eyes ... Use what you know...



9 Nine Multiplication Times Table Hand Trick - YouTube

9 x table on your fingers

1. Hold your hands in front of you with your fingers spread out.
2. For 9 x 4 bend your 4th finger down (like the picture).
3. You have 3 fingers in front of the bent finger and 6 after the bent finger. Thus the answer must be 36!
4. The technique works for the 9 times tables up to 10.

8

You can always count on a helping hand!

Use your digits... Do 'hands' only work with 9's?

Learning multiplication tables is in your own hands - 6, 7, 8, 9 & 10:

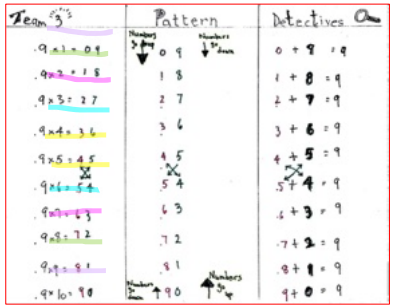
- https://www.youtube.com/watch?v=ovNRpka4JXU
- https://www.yestuition.co.uk/learning-times-tables-using-fingers/
- Always Remember Your Times Table (6 to 10) Finger Multiplication - Why It Works - YouTube

9

You can always count on a helping hand!

Use your digits... Use your eyes... Use what you know...

Some children's work

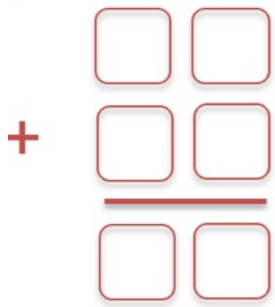


10

DIGIT DETECTIVE - Getting there.... Start them young

1. Put these numbers in the boxes to make this addition sum correct:

2 2 3 4 5 6



Put different numbers in place of the letters to make these addition sums correct.

2. B B + A = A C C

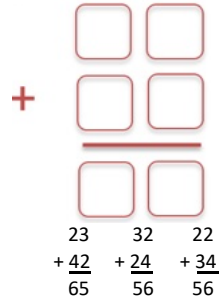
3. P Q + Q R = Q R Q

11

DIGIT DETECTIVE - Getting there.... Start them young

1. Put these numbers in the boxes to make this addition sum correct:

2 2 3 4 5 6



23 + 42 = 65

32 + 24 = 56

22 + 34 = 56

2. B B + A = A C C

3. P Q + Q R = Q R Q

A must be 1 so BB must be 99
Similarly, Q must be 1

2. 99 + 1 = 100

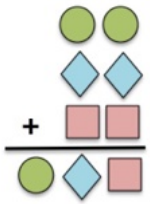
3. 91 + 10 = 101

Children can solve these puzzles when they can count and write numbers up to 100

12

FIND THE NUMBERS PUZZLE

The 3 symbols, green circle, blue rhombus and pink square, stand for 3 different digits that make the addition sum correct.



Replace the coloured shapes by digits.

You need to be a digit detective and find the clues that will help you to solve this puzzle.

What digit do you think has to go in the hundreds place? Why?

Looking at the pink squares in the units column what can you say about the circle and rhombus?

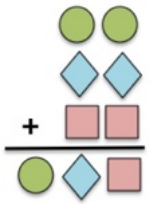
https://aimsinghigh.aimssec.ac.za/find-the-numbers/

13

FIND THE NUMBERS PUZZLE

Can you explain how you got this answer and why it is the only one?

CLUES



The digit in the hundreds place must be 1.

The squares in the units column are the same digit so the circle and rhombus must add up to 10.

From the tens column we see that the rhombus must be one more than the square.

```
 11
 99
+88
---
198
```

https://aimsinghigh.aimssec.ac.za/find-the-numbers/

14

MORE CRYPTARITHM PUZZLES – 3 DIGIT NUMBERS

7 possible solutions

```
 TWO
+TWO
----
FOUR
```

16 solutions

```
 ONE
+ONE
----
TWO
```

How many solutions can you find?

What clues could you give your learners here?

To help the learners the teacher might give them a clue (a partial solution) and ask them to find the rest of the digits or give them one solution and ask them to find others.

https://nrich.maths.org/twoandtwo

15

MORE CRYPTARITHM PUZZLES – 3 DIGIT NUMBERS

7 possible solutions

```
 TWO
+TWO
----
FOUR
```

```
938   928
+938  +928
----  ----
1876  1856

867   846
+867  +846
----  ----
1734  1692

836   765   734
+836  +765  +734
----  ----  ----
1672  1530  1468
```

One of the solutions is

```
 432
+432
----
 864
```

```
 ONE
+ONE
----
TWO
```

To help the learners the teacher might give them one solution and ask them to find other solutions.

Make up some more cryptarithm puzzles like this and find all the solutions.

https://nrich.maths.org/twoandtwo

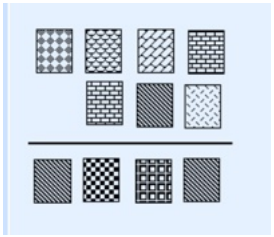
16

CRYPTARITHM PUZZLES 4 DIGIT NUMBERS - Getting started:

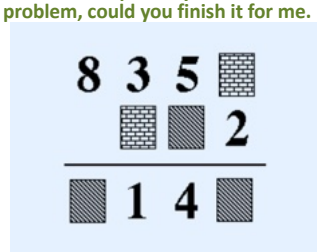
This is one for you to work on later

I have been working on this puzzle that a learner challenged me to solve.
Each type of sticker stands for a different number.
Wherever that same sticker occurs it is the same number.

This is the problem:



So far, I have partially solved the problem, could you finish it for me.



17

DIGIT DETECTIVE - Getting there.... Start them young

F	O	O	T
+B	A	L	L
G	A	M	E

Here is your key or code book:
A = 5, B = 6, E = 7, F = 1, G = 8,
L = 3, M = 2, O = 9, T = 4.

What does this reveal?

1	9	9	4
+6	5	3	3
8	5	2	7

This is one for you to work on later.
This is one of 224 solutions to
FOOT + BALL = GAME
when you replace each letter in by a different digit to make the sum correct.
How many solutions can you find?
https://aimsinghigh.aimssec.ac.za/football-challenge/

Replace B, A, L, L here for another solution:

2004
BALL +
5981

18

DIGIT DETECTIVE

What is the difference between a digit and a number?

We will start with simple calculations so learners will find it easy to explain their ways of thinking about the problems.

12 + ? = 19 1, 2 , and 9 are digits here. Twelve (12) and nineteen (19) are numbers.

The question marks ? are missing digits between 0 and 9.

How will you find the missing digit?

Where did you start? Why?

Let's share our methods with each other. Explain how you found your answer.

- Counting on from 12 to 19
- Just knew the answer
- Subtracting 12 from 19. *Why?*
- Because 2 + 7 = 9
- Some other method.

Now what about these examples? **? + 8 = ?4**

??? + 150 = 257

19

DIGIT DETECTIVE

Each ? is a missing digit from 0 to 9 either the same or different digits.

?+8=2?
2?+4=1?
33+?=4?

What do you notice?

https://aimsinghigh.aimssec.ac.za/digit-detective/

You are a detective and you have to discover what digits the question marks stand for. They can be any digit from 0 to 9 and they can be the same digit or different digits.

Explain why this happens?

How will you find the missing digits?

What clues did you use to help you work out your answers?

What do you notice?

How many solutions are there to the calculations?

20

DIGIT DETECTIVE SOLUTIONS

Each ? is a missing digit from 0 to 9 either the same or different digits.

?? + 8 = 2?

2? + 4 = 1?

33 + ? = 4?

What do you notice?

? + 8 = 2? is impossible.
There are no solutions because the largest number you can get from ? + 8 = 9 + 8 = 17 which is less than 20.

2? + 4 = 1? is impossible.
There are no solutions because 2? has to be at least 20 and by adding 4 you get a number greater than 20.

33 + ? = 4? has 3 solutions.
You do not get a solution if you add any digit less than 7 to 33 because you get an answer less than 40.
33 + 7 = 40
34 + 8 = 41
35 + 9 = 42

21

DIGIT DETECTIVE

Each ? is a missing digit from 0 to 9 either the same or different digits.

?? + 8 = 2?

2? + 4 = 1?

33 + ? = 4?

What do you notice?

The objective is not trying to teach facts or methods but trying to help learners to think mathematically and to find explanations.

Ask questions like:
How could you check you found all the solutions?
Why is there more than one solution?
Why is it you could ***not*** find the digits in **? + 8 = 2?**
What about **2? + 4 = 1?.**
How many solutions did you find to **33 + ? = 4?**

22

DIGIT DETECTIVE

MISSING DIGITS

(a) 3?6 + 5? = 383
(b) 8?3 – 7? = 815

What clues did you use? Explain how you solved the puzzles.

LARGEST AND SMALLEST

In the next two puzzles each of the digits must be different.

(c) Find the largest possible total for ?? + ?? =
Explain why it is the largest.
Find the smallest possible total and explain?

MULTIPLICATION AND DIVISION

(d) What about ?? × ??
(e) Now find the missing digit in this division: 816 ÷ ? = 272

23

DIGIT DETECTIVE – SOME SOLUTIONS

(a) 3?6 + 5? = 383 has the solution
326 + 57 = 383

The clues are:
- to get the units digit 3 you must add 7 to 6
- then 7 + 6 = 13 so look for 1 + 5 + ? = 8 to find the tens digit.

b) 8?3 - 7? = 815 has the solution
893 – 78 = 815

The clues are:
- you must find a number to subtract from 13 to get the units digit 5
- then use the inverse operation 815 + 78 gives the number 893.

24

DIGIT DETECTIVE – SOME SOLUTIONS

(c) The largest total for **?? + ?? =**

is **97 + 86 = 183**

or **96 + 87 = 183**

To get the largest total you need to make the tens digits as large as possible and then make the units digits as large as possible choosing from the remaining digits.

The smallest is **10 + 23 = 33** or **13 + 20 = 33**

25

DIGIT DETECTIVE – SOLUTIONS TO LARGEST AND SMALLEST

(d) The largest product for **?? × ??**

is **96 × 87 = 8352.**

(Remember you can't repeat a digit)

This is bigger than **97 × 86 = 8342**

×	90	7	
80	7200	**560**	7760
6	**540**	42	582
	7740	602	**8342**

×	90	6	
80	7200	**480**	7680
7	**630**	42	672
	7830	522	**8352**

To get the largest product we have to multiply the largest units digit by the largest tens digit as shown by the grid method of multiplication.

(e) The missing digit is 3 in this division:

816 ÷ ? = 272

816 ÷ 3 = 272

26

DIGIT DETECTIVE Why do these activities?

LEARNING OBJECTIVES

- To develop a deeper understanding of place value and inverse operations by challenging learners to act as detectives and to solve number puzzles.
- To get a feel for the size of numbers leading to the understanding of maximum and minimum values.
- To develop mathematical thinking and **communication skills**. Learners share ideas about their intuitive methods in simple addition and subtraction problems and progress to problems including multiplication and division.
- To develop **number sense**, **logical thinking, problem solving and application of knowledge and skills** by asking for reasons for what they do to solve the puzzles.
- To practise (a) calculations in what seems to be a game;
  (b) handling problems with no solutions and with multiple solutions;
  (c) flexible thinking, creativity and innovative
  (d) research, searching for, analysing, and interpreting information.

Generic skills are in red

27

MAGIC OF 37

Complete the number sentences by putting a number or × or = in the box:

37 × 3 □ 111
37 × □ = 222
37 □ 9 = 333
37 × □ = 444
□ × 15 = 555
37 □ 18 = 666
□ × 21 = 777
37 × 24 □ 888
□ × 27 = 999

First complete the number sentences and then explain why the pattern occurs.

Carry on these multiplications up to 37 × 54 and then explore the pattern that you get.

Talk about what you notice about this pattern.

Then write down everything that you notice.

https://aiminghigh.aimssec.ac.za/magic-of-37/magic-of-37/

28

BEAUTIFUL NUMBERS AND MULTIPLES OF 9

? × 9 + 6 = 888 Replace the ?s to complete these calculations.
? × 9 + 1 = 88 888 888 Don't use a calculator

Solutions **98 × 9 + 6 = 888**
9 876 543 × 9 + 1 = 88 888 888

Now complete these calculations with a calculator if you wish
? × 9 + 5 = 8 888
? × 9 + 0 = 888 888 888

Solutions **987 × 9 + 5 = 8 888**
98 765 432 × 9 + 0 = 888 888 888

What patterns do you see?

https://aiminghigh.aimssec.ac.za/beautiful-numbers/

29

BEAUTIFUL NUMBERS

How would you describe this pattern of calculations?
How do you find the missing digits?
Can you explain how this pattern arises?

? × 9 + 7 = 88
? × 9 + 6 = 888
? × 9 + 5 = 8888
? × 9 + 4 = 88888
? × 9 + 3 = 888888
? × 9 + 2 = 8888888
? × 9 + 1 = 88888888
? × 9 + 0 = 888888888

9 × 9 + 7 = 88
98 × 9 + 6 = 888
987 × 9 + 5 = 8888
9876 × 9 + 4 = 88888
98765 × 9 + 3 = 888888
987654 × 9 + 2 = 8888888
9876543 × 9 + 1 = 88888888
98765432 × 9 + 0 = 888888888

https://aiminghigh.aimssec.ac.za/beautiful-numbers/

30

BEAUTIFUL NUMBERS

How would you describe this pattern of calculations?

Can you explain how this pattern arises?

? × 9 + 7 = 88
? × 9 + 6 = 888
? × 9 + 5 = 8888
? × 9 + 4 = 88888
? × 9 + 3 = 888888
? × 9 + 2 = 8888888
? × 9 + 1 = 88888888
? × 9 + 0 = 888888888

9 × 9 + 7 = 88
98 × 9 + 6 = 888
987 × 9 + 5 = 8888
9876 × 9 + 4 = 88888
98765 × 9 + 3 = 888888
987654 × 9 + 2 = 8888888
9876543 × 9 + 1 = 88888888
98765432 × 9 + 0 = 888888888

Example – the bottom line:
98 765 432 × 9 = 88 888 888
is made up as follows:

	90 000 000 × 9 =	810 000 000
+	8 000 000 × 9 =	72 000 000
+	700 000 × 9 =	6 300 000
+	60 000 × 6 =	540 000
+	5 000 × 5 =	45 000
+	400 × 9 =	3 600
+	30 × 9 =	270
+	2 × 9 =	18

adding these subtotals:
= 98 765 432 × 9 = 88 888 888

https://aiminghigh.aimssec.ac.za/beautiful-numbers/

31

TIMES NINE

Complete the calculations in the picture.
What do you notice?

To explain how the patterns of numbers arise multiply by (10 – 1) instead of multiplying by 9.

https://aiminghigh.aimssec.ac.za/times-nine/

32

TIMES NINE

Complete the calculations in the picture.
What do you notice?
Multiply by (10 – 1) instead of multiplying by 9 and explain how the patterns of numbers arise.

Examples

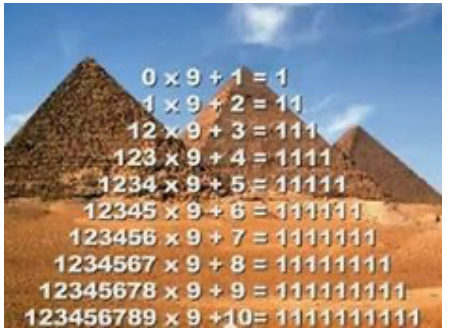
12 345 × (10 – 1) + 6
= 123 450 + 6 – 12 345
= 123 456 – 12 345
= 111 111

12 345 678 × (10 – 1) + 9
= 123 456 780 + 9 – 12 345 678
= 123 456 789 – 12 345 678
= 111 111 111

https://aiminghigh.aimssec.ac.za/times-nine/

33

TIMES NINE



The picture shows the pyramids in the background because this number pattern, and others like it, were known to the ancient Egyptians.

34

THINK OF TWO WHOLE NUMBERS

Here is an alternative version of the "Think of a Number" trick.
Think of two whole numbers under 10.
Take one of them and add 1.
Multiply by 5.
Add 1 again.
Double your answer.
Subtract 1.
Add your second number.
Add 2.
Double again.
Subtract 8.
Halve this number and tell me your answer.
I can work out both your numbers very quickly. How?
Choose some different pairs of numbers and repeat the process.
Can you find a good explanation of how the trick works?

https://aiminghigh.aimssec.ac.za/think-of-two-numbers/

35

THINK OF TWO WHOLE NUMBERS

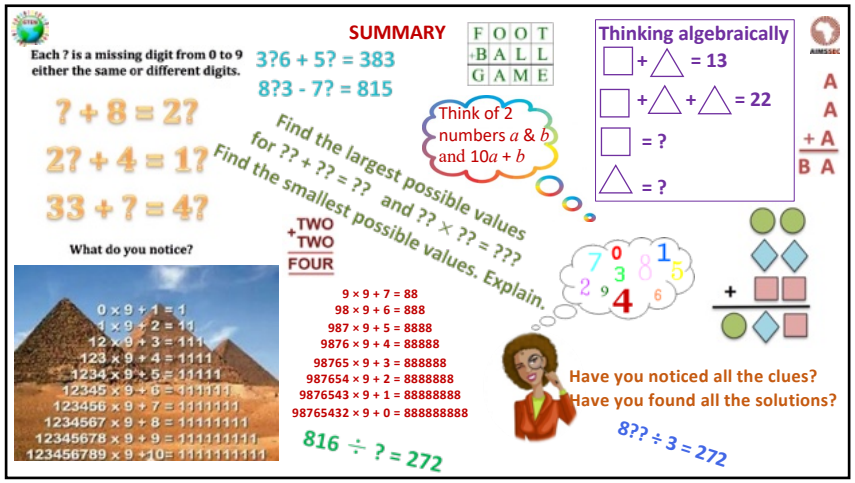
6 and 3	Think of two whole numbers under 10.	**Algebraically:**
6 + 1 = 7	Take one of them and add 1.	***a* and *b***
35	Multiply by 5.	***a* + 1**
36	Add 1 again.	**5*a* + 5**
72	Double your answer.	**5*a* + 6**
71	Subtract 1.	***10a* + 12**
74	Add your second number.	***10a* + 11**
76	Add 2.	**10*a* + *b* + 11**
152	Double again.	**10*a* + *b* + 13**
144	Subtract 8.	**20*a* + 2*b* + 26**
72	Halve this number and tell me your answer.	**20*a* + 2*b* + 18**
	I can work out both your numbers very quickly. How?	**10*a* + *b* + 9**
	Choose some different pairs of numbers and repeat the process.	**I subtract 9.**
	Can you find a good explanation of how the trick works?	**I know the number has digits *a* and *b***

72 - 9 = 63

10*a* + *b*

https://aiminghigh.aimssec.ac.za/think-of-two-numbers/

36



37

AIMINGHIGH TEACHER NETWORK

AIMSSEC

DIGIT DETECTIVE https://aiminghigh.aimssec.ac.za/digit-detective/

THINK OF TWO WHOLE NUMBERS https://aiminghigh.aimssec.ac.za/think-of-two-numbers/

BEAUTIFUL NUMBERS https://aiminghigh.aimssec.ac.za/beautiful-numbers

MAGIC OF 37 https://aiminghigh.aimssec.ac.za/magic-of-37/magic-of-37/

TIMES NINE https://aiminghigh.aimssec.ac.za/times-nine/

CRYPTARITHM – FIND TNUMBERS https://aiminghigh.aimssec.ac.za/find-the-numbers/

TWO AND TWO https://nrich.maths.org/twoandtwo

FOOT BALL CHALLENGE https://aiminghigh.aimssec.ac.za/football-challenge/

EGYPTIAN MATHEMATICS NUMBER HEIROGLYPHS

https://discoveringegypt.com/egyptian-hieroglyphic-writing/egyptian-mathematics-numbers-hieroglyphs/

BABYLONIAN SEXAGESIMAL NUMBER SYSTEM , OLD BABYLONIAN MATHEMATICS AND PLIMPTON 322

https://www.youtube.com/watch?v=J5Ug3Cr8RUE

38

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

On the AIMING HIGH website there are freely downloadable worksheets, Inclusion and Home Learning Guides with activities for learners of all ages and attainment levels, Notes for Teachers with solutions, Key Questions and Diagnostic Quizzes for formative assessment.

AIMSSEC GTEN YouTube Channel

https://www.youtube.com/c/MathsToys/videos

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39

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40

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Thanks for coming to this workshop.

Use the AIMSSEC ideas
on AIMING HIGH and add comments.

Share what you have learned
with other teachers.

Try to help all your learners to have a

'YES I CAN'

attitude to mathematics.

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Enquire about signing up for an AIMSSEC course
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41