

Global Teacher Empowerment Network GTEN
14 August 2021

MAGIC NUMBERS

SEQUENCES & SERIES

DIVISION & SHARING

8000 1600 240 32

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Work out the numbers to replace the ? marks

FRACTIONS AND DECIMALS

NUMBER PATTERNS

ALGEBRA

INVERSE OPERATIONS

$9 \times 9 + 7 = 88$
 $98 \times 9 + 6 = 888$
 $987 \times 9 + 5 = 8888$
 $9876 \times 9 + 4 = 88888$
 $98765 \times 9 + 3 = 888888$
 $987654 \times 9 + 2 = 8888888$
 $9876543 \times 9 + 1 = 88888888$
 $98765432 \times 9 + 0 = 888888888$

$7 \times 9 + 7 = 88$
 $7 \times 9 + 6 = 888$
 $7 \times 9 + 5 = 8888$
 $7 \times 9 + 4 = 88888$
 $7 \times 9 + 3 = 888888$
 $7 \times 9 + 2 = 8888888$
 $7 \times 9 + 1 = 88888888$
 $7 \times 9 + 0 = 888888888$

$(9 - 1) \div 8 =$
 $(98 - 2) \div 8 =$
 $(987 - 3) \div 8 =$
 $(9876 - 4) \div 8 =$
 $(98765 - 5) \div 8 =$
 $(987654 - 6) \div 8 =$
 $(9876543 - 7) \div 8 =$
 $(98765432 - 8) \div 8 =$
 $(987654321 - 9) \div 8 =$

$1 \times 8 + 1 = 9$
 $12 \times 8 + 2 = 98$
 $123 \times 8 + 3 = 987$
 $1234 \times 8 + 4 = 9876$
 $12345 \times 8 + 5 = 98765$
 $123456 \times 8 + 6 = 987654$
 $1234567 \times 8 + 7 = 9876543$
 $12345678 \times 8 + 8 = 98765432$
 $123456789 \times 8 + 9 = 987654321$

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AIMS African Institute for Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE

MATHS TOYS

Global Teacher Empowerment Network (GTEN)
Programme for Magic Numbers Workshop 11 October 2021

Improve knowledge and understanding of:
 Division or sharing
 Number patterns
 Linear equations

Learning Spiral

1. Starter activity: Story about **Discovering Buried Treasure**
2. Sharing the treasure
3. Early Years: Equal shares
4. Lower Primary: More about equal shares
5. Upper Primary: Buried treasure and the division algorithm
6. Lower Secondary: Unpicking the pattern. Division by chunking
7. Upper Secondary: Inverse operations and algebra. Solving equations
8. Follow-up: Beautiful Numbers Pattern.
9. Enrichment: Euclid's Algorithm

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STARTER ACTIVITY FOR ALL – Discovering buried treasure

Listen to this story and answer the questions at the end.

You and your friends and family, 8 of you altogether, were playing on the beach and you noticed that the sand had shifted to reveal something made of wood. You all started digging and found a treasure chest full of gold coins. The grown-ups said that you could not keep the money as it had been buried for a long long time, but there would be no harm in counting the coins.

To count the gold coins you counted out piles of a hundred gold coins. There were 98 hundreds.

You separated them into piles of a thousand and smaller piles of a hundred. Then you counted how many were left and found there were 72.

You sorted them into piles of ten with 2 left over.

Read the story again before you answer the questions.

How many thousands? How many hundreds?
How many tens? How many units? How many coins altogether?



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Sharing the treasure

Imagine you are sharing the 9872 gold coins from the treasure chest between 8 people.

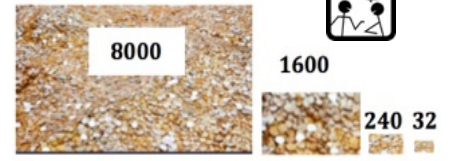
You split them into 4 piles, as shown in the picture chosen so that **each pile is a multiple of 8**.

Then you share each of these piles between the 8 people.

How many coins would each person get?

Explain this method of sharing the coins.

Explain how this method is used to find the answer to the division sum $9872 \div 8$



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Early Years All about equal shares




Talk about sharing and do some sharing with the children, **not** dividing up to make fractions, but sharing collections of objects into **equal shares** and perhaps having some left over (**the remainder**).



Don't make this into a lesson. Rather let it be a part of everyday life. Share a bunch of grapes between a group of children so everyone gets the same number of grapes, or share some sweets, or whatever there is to share.



Make up up a story about this picture. 

Make some play money and encourage the children to play at shopping. 


Make up stories and play games that involve sharing money. If you play games as a family like Monopoly the children should share out the money at the start.

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Lower Primary All about equal shares





To lay a good foundation for later work children should make up number stories that involve sharing situations and explain and illustrate their stories in their own ways. Ask questions like:

How many different ways can you describe what you see in this picture? 

For 6 year olds do not introduce the formal way to write this down, it will come later.



If you ask children about what they see in this picture, what sort of answers would you expect them to give? In talking about the picture, without writing anything formally, try to connect the ideas that $6 = 2 + 2 + 2$ and $6 = 3 \times 2$ and $6 \div 3 = 2$.



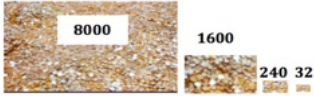
Sharing is a normal activity for young children. Before starting school they understand that, if there are 6 sweets to share between 3 children, each child will get 2 sweets. When the division process is taught formally, if the teacher does not help them to connect it to what they already know, children will fail to understand the algorithm.

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Upper Primary The Division Algorithm

Start the lesson with the story of the buried treasure and sharing 9872 coins between 8 people. Talk about the pros and cons of different ways of sharing.



Why would you choose to split the coins as shown in this picture?


How does this link to the division algorithm?

a. How many whole thousands can be shared?	8	9872	
b. How many hundreds are left?	18	<u>-8000</u>	
c. How many whole hundreds can be shared?	16	1872	
d. How many tens are left.	27	<u>-1600</u>	
e. How many whole tens can be shared?	24	272	
f. How many whole units can be shared?	32	<u>-240</u>	
g. How many units are left?	0	32	
		<u>-32</u>	

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96,...

How the money is shared:

8×1000

8×200 



8×30

8×4


$9872 \div 8 = 1234$

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Division $9872 \div 8 = 1234$

Brunsviga Mechanical Calculator widely used in offices from 1890 until 1960s



To do this division 4 levers are pulled down the slots to set 9 thousands, 8 hundreds, 7 tens and 2 units.

Then the carriage at the bottom is moved to line up with the thousands column.

To subtract thousands the handle is turned anticlockwise until the bell rings to indicate the negative zone.



Next the carriage is moved to line up with the hundreds and hundreds are subtracted by anticlockwise turns of the handle. The carriage is moved into alignment with the tens column and anticlockwise turns subtract tens. Finally, the handle is moved to the units column and the handle is turned to subtract units. The answer 1234 is read from the machine.

This operation showed clearly that division is repeated subtraction.


Turning the handle in the opposite direction for multiplication added the numbers showing that multiplication is repeated addition.

Decimal points could be set for decimal calculations.

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 **Lower Secondary** 

$9 - 1 =$
 $98 - 2 =$
 $987 - 3 =$
 $9876 - 4 =$
 $98765 - 5 =$
 $987654 - 6 =$
 $9876543 - 7 =$
 $98765432 - 8 =$
 $987654321 - 9 =$

Write down this pattern. Copy the pattern and work out the subtractions. What do you notice? 

Lesson Starter for Learners: Read the Buried Treasure story.



Main lesson: This lesson focusses on division.

Talk about the gold coins sharing story. Ask the learners to talk with their partners, or in small groups, to find the best way to use the coin sharing story to explain the division sum:

$$9872 \div 8 = 1000 + 200 + 30 + 4 = 1234.$$

After a few minutes have a whole class discussion about this.

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 **Lower Secondary Lesson continued** 

$(9 - 1) \div 8 =$
 $(98 - 2) \div 8 =$
 $(987 - 3) \div 8 =$
 $(9876 - 4) \div 8 =$
 $(98765 - 5) \div 8 =$
 $(987654 - 6) \div 8 =$
 $(9876543 - 7) \div 8 =$
 $(98765432 - 8) \div 8 =$
 $(987654321 - 9) \div 8 =$



Then copy out this list of calculations.

Tell the learners that they must do the calculation in the brackets first.

Ask them to complete the calculations without using a calculator.

Do this now and write your answers in the chat 

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 **Lower Secondary Lesson continued** 

$(9 - 1) + 8 = 1$
 $(98 - 2) + 8 = 12$
 $(987 - 3) + 8 = 123$
 $(9876 - 4) + 8 = 1234$
 $(98765 - 5) + 8 = 12345$
 $(987654 - 6) + 8 = 123456$
 $(9876543 - 7) + 8 = 1234567$
 $(98765432 - 8) + 8 = 12345678$
 $(987654321 - 9) + 8 = 123456789$



To end the lesson: The teacher or one of the learners should write the complete pattern on the board, with different learners telling them what to write line by line.

Key questions to guide learning

1. What do you notice?
2. Why have the brackets been put in those calculations?
3. If one number is divided by another, how does it help to split the first number into separate parts when you divide one number (e.g. number of coins) by a second number (e.g. number of people)?
4. What's the connection between division and subtraction?
5. What's the connection between multiplication and division?



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 **Years 10 - 12** 

This challenge is an extension to MAGIC NUMBERS
<https://aiminghigh.aimssec.ac.za/more-magic-numbers/>

Find the numbers to put in the boxes to make the calculations in LIST 1 correct.

Complete the calculations in LIST 2

LIST 1

$$\square \times 8 + 1 = 9$$

$$\square \times 8 + 2 = 98$$

$$\square \times 8 + 3 = 987$$

$$\square \times 8 + 4 = 9876$$

$$\square \times 8 + 5 = 98765$$

$$\square \times 8 + 6 = 987654$$

$$\square \times 8 + 7 = 9876543$$

$$\square \times 8 + 8 = 98765432$$

$$\square \times 8 + 9 = 987654321$$

LIST 2

$$(9 - 1) + 8 =$$

$$(98 - 2) + 8 =$$

$$(987 - 3) + 8 =$$

$$(9876 - 4) + 8 =$$

$$(98765 - 5) + 8 =$$

$$(987654 - 6) + 8 =$$


$$(9876543 - 7) + 8 =$$

$$(98765432 - 8) + 8 =$$

$$(987654321 - 9) + 8 =$$

Write down list 1 with numbers to fill in the boxes.

What do you notice about the connections between the equations: $\square \times 8 + 1 = 9$ etc. and the calculations: $(9 - 1) \div 8 = 1$ etc. ?

You have been doing algebra, $\square \times 8 + 1 = 9$ etc. can be written as $8x + 1 = 9$ etc. 

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Years 10 - 12 MAGIC NUMBERS SOLUTION

9 - 1 = 8
98 - 2 = 96
987 - 3 = 984
9876 - 4 = 9872
98765 - 5 = 98760
987654 - 6 = 987648
9876543 - 7 = 9876534
98765432 - 8 = 98765424
987654321 - 9 = 987654312

The numbers in this triangle are all multiples of 8.

(9 - 1) ÷ 8 = 1
(98 - 2) ÷ 8 = 12
(987 - 3) ÷ 8 = 123
(9876 - 4) ÷ 8 = 1234
(98765 - 5) ÷ 8 = 12345
(987654 - 6) ÷ 8 = 123456
(9876543 - 7) ÷ 8 = 1234567
(98765432 - 8) ÷ 8 = 12345678
(987654321 - 9) ÷ 8 = 123456789

x 8 + 1 = 9
x 8 + 2 = 96
x 8 + 3 = 987
x 8 + 4 = 9876
x 8 + 5 = 98765
x 8 + 6 = 987654
x 8 + 7 = 9876543
x 8 + 8 = 98765432
x 8 + 9 = 987654321

1 x 8 + 1 = 9
12 x 8 + 2 = 98
123 x 8 + 3 = 987
1234 x 8 + 4 = 9876
12345 x 8 + 5 = 98765
123456 x 8 + 6 = 987654
1234567 x 8 + 7 = 9876543
12345678 x 8 + 8 = 98765432
123456789 x 8 + 9 = 987654321

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WHY DO THE MAGIC NUMBERS ACTIVITY?

Discovering the pleasing patterns gives this activity an appeal for learners. They will get practice in subtraction and division in an interesting way. The activity helps learners to understand 3 fundamental mathematical ideas:

1. the process of division and how it is equivalent to 'sharing';
2. inverse operations;
3. how solving linear equations involves the use of inverse operations.

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BEAUTIFUL NUMBERS Lesson for secondary

? x 9 + 7 = 88
? x 9 + 6 = 888
? x 9 + 5 = 8888
? x 9 + 4 = 88888
? x 9 + 3 = 888888
? x 9 + 2 = 8888888
? x 9 + 1 = 88888888
? x 9 + 0 = 888888888

Work out the different numbers to replace the question marks.

What do you notice about the patterns in these calculations?

Can you explain why this pattern occurs?

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BEAUTIFUL NUMBERS Lesson for secondary

If we look at the last two calculations we can see how the pattern arises. The other calculations are similar.

Think of 9 as 10 - 1,
 $9876543 \times 9 + 1 = 9876543 \times (10-1) + 1$
 which can be written as

$$\begin{array}{r} 98\ 765\ 431 \\ -9\ 876\ 543 \\ \hline 88\ 888\ 888 \end{array}$$

Similarly
 $98765432 \times 9 + 0 = 98765432 \times (10-1) + 0$
 can be written as

$$\begin{array}{r} 987\ 654\ 320 \\ -98\ 765\ 432 \\ \hline 888\ 888\ 888 \end{array}$$

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CURIOUS PATTERNS WITH NINES Lesson for secondary

Investigate the decimal expansion of the fractions

ONE DIVIDED BY SEVEN $\frac{1}{7}$ Divide the numerator by the denominator. There are only a few possible remainders so the division must produce a recurring decimal or it must terminate.

TWO DIVIDED BY THIRTEEN $\frac{2}{13}$

Investigate the fractions $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \dots, \frac{10}{11}$



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CURIOUS PATTERNS WITH NINES

Observe $\frac{1}{7} = 0.142857 \dots$ recurring

$142 + 857 = 999$


Look at the pattern in the decimal expansions for thirteenthths and elevenths. Do you notice any similar patterns there?

Also $\frac{1}{13} = 0.076923 \dots$ recurring

$076 + 923 = 999$

$\frac{2}{13} = 0.153846 \dots$ recurring

$153 + 846 = 999$



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CURIOUS PATTERNS WITH NINES

The fractions $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \dots, \frac{10}{11}$ have decimal expansions:

$\frac{1}{11} = 0.0909 \dots, \frac{2}{11} = 0.1818 \dots, \frac{3}{11} = 0.2727 \dots, \frac{4}{11} = 0.3636 \dots$ etc


Pairs of digits add up to 9:

$0 + 9 = 1 + 8 = 2 + 7 = 3 + 6$ etc.

Similar patterns arise for denominators 101, 1001, 10001 etc.

For example, try $\frac{32}{101}$ and investigate other fractions with these denominators.

$\frac{32}{101} \dots = 0.31683168 \dots$ and $31 + 68 = 99$



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UPPER SECONDARY ENRICHMENT EUCLID'S ALGORITHM

How can we solve equations like $13x + 29y = 42$ or $2x + 4y = 13$ or $83x + 19y = 3$ with the solutions x and y being integers?


Equations with integer solutions are called Diophantine equations after Diophantus who lived about 250 AD.

The methods described here go back to Euclid (about 300 BC) and earlier.

When people hear the name Euclid they think of geometry, but the algorithm described here appeared as Proposition 2 in Euclid's Book 7 on Number Theory.



Diophantus



Euclid

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UPPER SECONDARY ENRICHMENT EUCLID'S ALGORITHM

This diagram shows Euclid's Algorithm for $83 \div 19$.
It is used to find whole number solutions (x and y) for equations like $83x + 19y = 3$.

The diagram shows that $83 = 4 \times 19 + 7$

Continuing the same process:

$$83 = 4 \times 19 + 7$$

$$19 = 2 \times 7 + 5$$

$$7 = 1 \times 5 + 2$$

$$5 = 2 \times 2 + 1$$

Reversing this process and collecting like terms

$$1 = (5) - 2(2)$$

$$= (5) - 2(7 - 5) = 3(5) - 2(7)$$

Now replacing 5 gives:

$$1 = 3[19 - 2(7)] - 2(7)$$

$$1 = -8(7) + (19 \times 3)$$

$$1 = -8(83 - 4 \times 19) + (19 \times 3)$$

$$1 = 83(-8) + 19(35)$$

Multiply this equation by 3 to solve the equation $83x + 19y = 3$

enlarging the 7 by 5 rectangle

Multiplying by 3

$$3 = 83(3 \times -8) + 19(3 \times 35)$$

$$3 = 83(-24) + 19(105)$$

The solutions are $x = -24$ and $y = 105$

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Global Teacher Empowerment Network (GTEN)

NEW SKILLS NEW HOPES NEW HORIZONS
for teachers and learners worldwide

MAGIC NUMBERS RESOURCES

Magic Numbers <https://aiminghigh.aimssec.ac.za/magic-numbers/>

More Magic Numbers <https://aiminghigh.aimssec.ac.za/more-magic-numbers/>

Beautiful Numbers <https://aiminghigh.aimssec.ac.za/beautiful-numbers/>

Euclid's Algorithm <https://nrch.maths.org/1357>

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AIMSSEC GTEN YouTube Channel
<https://www.youtube.com/c/MathsToys/videos>

MATHS TOYS

Global Teacher Empowerment Network (GTEN) videos
for teachers in primary and secondary schools, colleges and universities. Originally broadcast to centres in The Caribbean, India, Nigeria, Kenya, Tanzania, Uganda and South Africa

Each workshop has a recurring Mathematical Theme
from age 5 to 18+

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AIMSSEC Website: <http://aimssec.ac.za>



AIMING HIGH Free lesson resources: <http://aiminghigh.aimssec.ac.za>

COLLABORATIVE PROFESSIONAL DEVELOPMENT
<https://aiminghigh.aimssec.ac.za/category/cpd>


MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS
<https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/>

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 **LET'S PLAY MATHEMATICALLY AND LEARN** 

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Play Mathematically

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- to unlock knowledge
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

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Thanks for coming to this workshop.
Use the AIMSSEC ideas on AIMING HIGH and add comments.
Share what you have learned with other teachers.
Try to help all your learners to have a 'Yes I Can' attitude to mathematics.



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