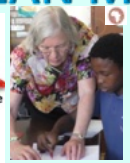
Global Teacher Empowerment Network GTEN

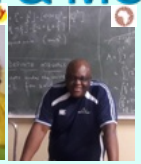
Saturday 6 May 2023 16.00 – 18.00 London Time

MEAN MEDIAN & MODE

MMM GAME

Find all the sets of five positive whole numbers that have:
a mean of 4
a median of 3
and a mode of 3.





Toni Beardon Caroline Ainslie Kwethemba Moyo

FOR YOUNGER FOR OLDER

Abe	Buk	Chris	Dudu	Erin
2	8	10	14	16

Sam	Mzu	Bulie
10	12	14

Can you find a way for one child to leave one group and join another so that the mean age drops in the group left and rises in the group joined?

PROBLEM SOLVING

FOR RICHER FOR POORER

Mdu's move from Cape Town to Durban caused the average salary to go up in both. How?

CARD SORTING

FORMATIVE ASSESSMENT

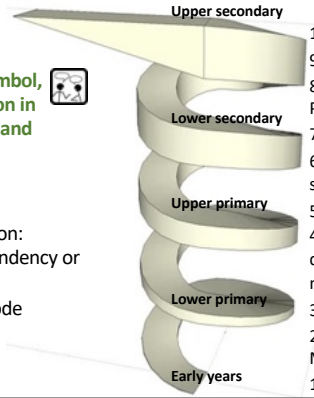
1

MEAN MEDIAN & MODE

Programme for the GTEN workshop Saturday 6 May 2023

When you see this symbol, and you see a question in green, do the activity and answer the question.

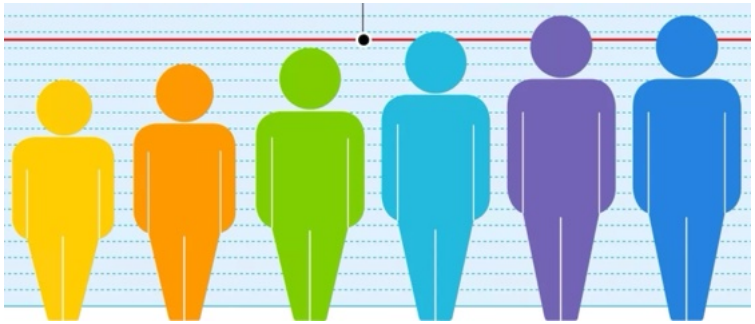
Today we are working on:
Measures of central tendency or averages
Mean, Median and Mode
Range



10. Review
9. Match the Matches
8. Problem solving: 'For Richer for Poorer' and 'For Younger for Older'.
7. MMM & R card sorting
6.Systematic method for finding all solutions for MMMR problem
5. Playing the MMM Game
4. Histograms for normal and skewed distributions showing, mean, median and mode.
3. Why different averages?
2. Definitions of Mean, Median, Mode & Range
1. Finding the median

2

FINDING THE MEDIAN – AN ACTIVITY FOR ALL AGES



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MEAN, MEDIAN, MODE & RANGE of a set of n numbers $x_1, x_2, x_3, \ldots x_n$

3

The ARITHMETIC MEAN is the sum of the terms divided by the number of terms.
This is given by the formula $\frac{\sum_{i=1}^{n} x_i}{n}$

3

The MEDIAN is the middle value or 50th percentile when the numbers are arranged in order.
The median is the middle term if n is odd and between the two middle terms if n is even.

3
5

For example, the median height of 6 people is halfway between the heights of the 3rd and 4th person.

The MODE is the value that appears most often.

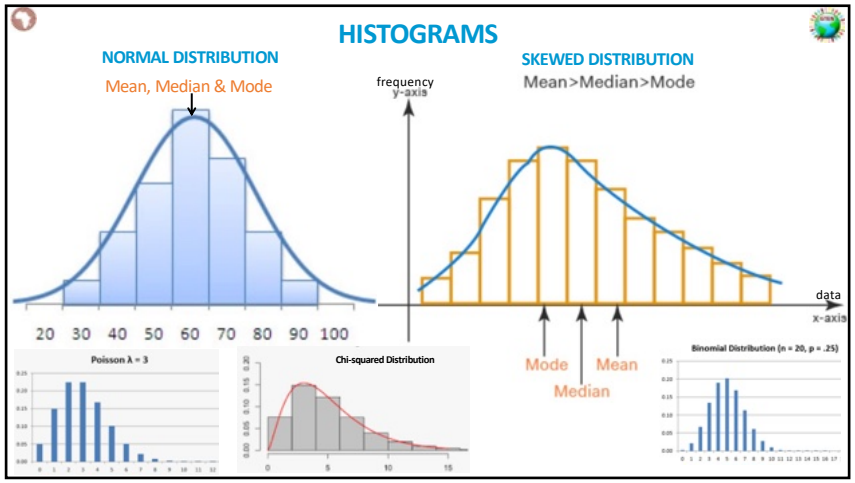
6

The RANGE is the difference between the largest and smallest value.

Work out the mean, median, mode and range of 3, 3, 3, 5 and 6.

4

HISTOGRAMS



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MEAN, MEDIAN, MODE & RANGE

Why different averages? In what situations are they used?

The **arithmetic mean** is better when the data follow a symmetric distribution. When the data are skewed, the **median** is more useful because outliers distort the mean.

For example, the table below shows the annual salary of 10 workers. The mean is 30.7k but this is not typical as most workers earn between 12k and 18k. The median 15.5k is the more appropriate average for describing the workforce. But, if workers are asking for a pay rise, they would use the mean.

Staff	1	2	3	4	5	6	7	8	9	10
Salary	15k	18k	16k	14k	15k	15k	12k	17k	90k	95k

In an opinion poll the **mode** is of most interest as it indicates what is likely to be the consumer's or voter's first choice, or the judge's choice of a winner in a competition.

Shopkeepers use the mode in deciding on which item to stock in the greatest quantity. For ordering stock, they prefer to see all the data rather than just an average.

Other averages are used in statistics such as the harmonic mean and the geometric mean.

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MEAN, MEDIAN & MODE GAME

Find all the sets of five positive whole numbers that have:
a mean of 4
a median of 3
and a mode of 3.

RULES OF THE MMM GAME

1. The teacher thinks of any 5 positive numbers and tells the class their mean, median and mode.
2. The class are given 2 minutes to find sets of 5 numbers with the given mean, median & mode.
3. After 2 minutes players score a point for each solution that they have found.

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MEAN, MEDIAN & MODE GAME

Mean 4, Median 3, Mode 3.

1, 2, 3, 3, 11
1, 3, 3, 3, 10
1, 3, 3, 4, 9
1, 3, 3, 5, 8
1, 3, 3, 6, 7
2, 3, 3, 3, 9
2, 3, 3, 4, 8
2, 3, 3, 5, 7
3, 3, 3, 3, 8
3, 3, 3, 4, 7
3, 3, 3, 5, 6

Bi-modal

1, 1, 3, 3, 12
2, 2, 3, 3, 10
2, 3, 3, 6, 6

Mean 7 Median 6 Mode 5

5, 5, 6, 7, 12
5, 5, 6, 8, 11
5, 5, 6, 9, 10
and 5, 5, 6, 6, 13

Mean 8 Median 7 Mode 7

There are 45 uni-modal and 9 bi-modal solutions.
Here are a few
1, 2, 7, 7, 23
1, 3, 7, 7, 22
1, 4, 7, 7, 21
1, 5, 7, 7, 20
1, 6, 7, 7, 19
1, 7, 7, 7, 18
1, 7, 7, 8, 17
1, 7, 7, 9, 16
1, 7, 7, 10, 15
1, 7, 7, 11, 14
1, 7, 7, 12, 13

For Notes for Teachers and a complete set of solutions see https://aiminghigh.aimssec.ac.za/m-m-and-m/

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CARD SORTING FOR REVISION AND FORMATIVE ASSESSMENT

3	3,5	5

Cut out the cards.

Work out the mean, median, mode or range as on the card.

The answer will be 3 or 3,5 or 5.

Place the card in the corresponding column.

A1 Mean -1 5 4 3 4	A2 Mean 5 3,5 3 5 1	A3 Mean 7 7 5 -1 7
B1 Mode 3 5 7 3 3 5	B2 Mode 5 3,5 -1 3,5 3	B3 Mode 5 4 5 5 3,5
C1 Median 1 7 1 3 4 5	C2 Median 7 5 7 1 -1 -3	C3 Median 7 7 3 3 5
D1 Range 4 3 1 4 3	D2 Range 1 1 3,5 3 0	D3 Range 1 -1 3,5 4 3

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CARD SORTING FOR REVISION AND FORMATIVE ASSESSMENT

3	3,5	5
A1 Mean 3 -1 5 4 3 4	A2 Mean 3,5 5 3,5 3 5 1	A3 Mean 5 7 7 5 -1 7
B1 Mode 3 3 5 7 3 3 5	B2 Mode 3,5 5 3,5 -1 3,5 3	B3 Mode 5 5 4 5 5 3,5
C2 Median 3 7 5 7 1 -1 -3	C1 Median 3,5 1 7 1 3 4 5	C3 Median 5 7 7 3 3 5
D1 Range 3 4 3 1 4 3	D2 Range 3,5 1 1 3,5 3 0	D3 Range 5 1 -1 3,5 4 3

MEAN MEDIAN MODE & RANGE (card sorting) https://aiminghigh.aimssec.ac.za/mean-median-mode-and-range/

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PROBLEM SOLVING

FOR RICHER FOR POORER

Mdu's move from Cape Town to Durban caused the average salary to go up in both. How?

A simple case

FOR YOUNGER FOR OLDER

	Abe	Buk	Chris	Dudu	Erin
1	2	8	10	14	16

	Sam	Mzu	Bulie
2	10	12	14

Can you find a way for one child to leave one group and join another so that the mean age drops in the group left and rises in the group joined?

Work out the mean age of the two groups, Group 1 and Group 2.
Can you suggest solutions to the 'For younger for older' problem?

FOR YOUNGER FOR OLDER https://aiminghigh.aimssec.ac.za/for-younger-for-older/

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PROBLEM SOLVING

FOR YOUNGER FOR OLDER

	Abe	Buk	Chris	Dudu	Erin
1	2	8	10	14	16

	Sam	Mzu	Bulie
2	10	12	14

Can you find a way for one child to leave one group and join another so that the mean age drops in the group left and rises in the group joined?

The mean age of Group 1 is 50/5 = 10 years.
The mean age of Group 2 is 36/3 = 12 years.

Notice that the mean can stay the same, go down or up depending on whether the age of the child leaving is the same as, less than or greater than the mean of the group. This is the focus of the main problem.

Group	Mean age	Dudu moved Group 1 to 2	Erin moved Group 1 to 2	Bulie moved Group 2 to 1
1	50/5 = 10 years.	36/4 = 9 years ↘	34/4=8y 6m ↘	64/6=10y 8m ↗
2	36/3 = 12 years	50/4 = 12y 6m ↗	52/4= 13 years ↗	22/2=11y ↘

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FOR RICHER FOR POORER
https://aiminghigh.aimssec.ac.za/for-richer-for-poorer/

The mean can stay the same, go up or go down depending on whether Mdu's salary is the same as, or greater or less than the mean of the city.

For the mean to go up in Cape Town when Mdu leaves he must earn LESS than the mean for Cape Town.

For the mean to go up in Durban when Mdu moves there he must earn MORE than the mean for Durban.

It is possible that his move caused the mean salary to go up in both if the mean salary in Cape Town is higher than in Durban.

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MATCH THE MATCHES

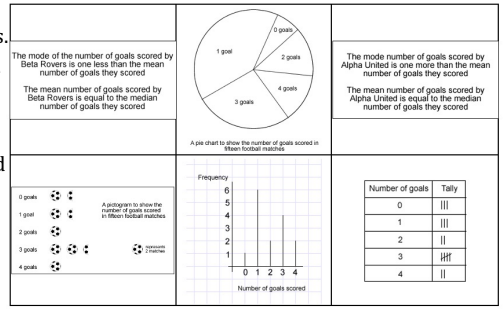
Two teams Alpha United and Beta Rovers have each played 15 matches.

The data shows how many goals the teams scored in their matches.

There are six sets of data, three showing the results for Alpha United and three for Beta Rovers.

Work out the Mean, Median and Mode in each set of data.

Match the data to the teams and explain your answers.



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MATCH THE MATCHES

Work out the Mean, Median and Mode in each set of data.

The mode of the number of goals scored by Beta Rovers is one less than the mean number of goals they scored

The mean number of goals scored by Beta Rovers is equal to the median number of goals they scored

Match the data to the teams and explain your answers.

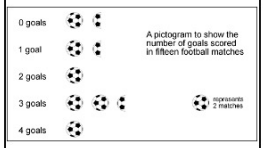
The mode number of goals scored by Alpha United is one more than the mean number of goals they scored

The mean number of goals scored by Alpha United is equal to the median number of goals they scored

BETA ROVERS

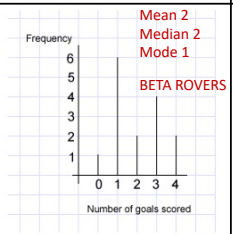
A pie chart to show the number of goals scored in fifteen football matches

ALPHA UNITED



Mean 2
Median 2
Mode 1

BETA ROVERS



Mean 2
Median 2
Mode 3

ALPHA UNITED

Number of goals	Tally
0	III
1	III
2	II
3	IIII
4	II

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MATCH THE MATCHES

Results for Alpha United Represented in the tally chart and the pictogram

Number of goals	Frequency - number of matches	Total number of goals	Angle in pie chart
0	3	0	72
1	3	3	72
2	2	4	48
3	5	15	120
4	2	8	48
TOTAL	15	30	360

Mode 3 goals Mean 2 Median 2

Results for Beta Rovers Represented in the pie chart and the frequency graph

Number of goals	Frequency - number of matches	Total number of goals	Angle in pie chart
0	1	0	24
1	6	6	144
2	2	4	48
3	4	12	96
4	2	8	48
TOTAL	15	30	360

Mode 1 goal Mean 2 Median 2

MATCH THE MATCHES https://aiminghigh.aimssec.ac.za/match-the-matches/

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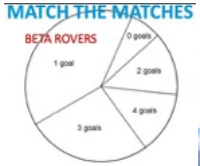
MATCH THE MATCHES

BETA ROVERS



A pie chart to show the number of goals scored in fifteen football matches

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M, M and M, ... GAME

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AIMSSEC

AIMS | African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

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AN AIMING HIGH LEARNING PACK IS A WEBPAGE CONTAINING

A learning activity with links to:

a. PDF of the worksheet
b. Templates and instructions for making resources
c. Videos
d. Notes for Teachers with
   - solutions
   - curriculum links and learning objectives
   - diagnostic quizzes
   - suggestions for teaching
   - key questions to guide learning
   - follow up ideas and links

https://aiminghigh.aimssec.ac.za/

e. Inclusion Guides for School and Home Learning with
   - a starter activity for a mixed–age group to do together
   - a collection of learning activities to suit all ages from 4 to 18+
   - Solutions with suggestions for teaching and assessment.

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LINKS FOR LEARNING ACTIVITIES INVOLVING MEAN, MEDIAN, MODE AND RANGE

GTEN website https://gtenmaths.org/

M, M and M https://aiminghigh.aimssec.ac.za/m-m-and-m/

MEAN MEDIAN MODE & RANGE (card sorting)
https://aiminghigh.aimssec.ac.za/mean-median-mode-and-range/

FOR YOUNGER FOR OLDER https://aiminghigh.aimssec.ac.za/for-younger-for-older/

FOR RICHER FOR POORER https://aiminghigh.aimssec.ac.za/for-richer-for-poorer/

MATCH THE MATCHES https://aiminghigh.aimssec.ac.za/match-the-matches/

AIMSSEC YouTube Channel https://www.youtube.com/@MathsToys

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AIMSSEC

LET'S PLAY MATHEMATICALLY AND LEARN

Order from AMAZON or TARQUIN https://www.tarquingroup.com/products/aiming-...

GTEN

Play Mathematically
- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

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Thanks for coming to this workshop.

Use the AIMSSEC ideas
on AIMING HIGH and add comments.

Share what you have learned
with other teachers.

Try to help all your learners to have a

'YES I CAN'

attitude to mathematics.

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Caroline Ainslie caroline@bubblymaths.co.uk

Enquire about signing up for an AIMSSEC course
as a self-funding student admin@aimssec.ac.za

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