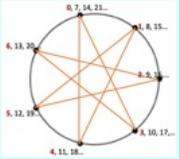
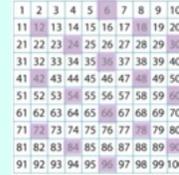


Global Teacher Empowerment Network GTEN
 Saturday 24 April 16.00 – 18.00 London Time

PATH TO THE STARS


Toni Beardon Caroline Ainslie Marie Joubert

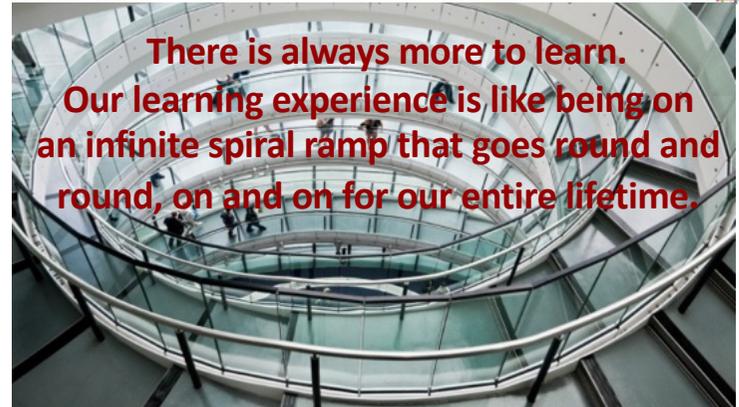




1

Build skills and understanding on a spiral path

**There is always more to learn.
 Our learning experience is like being on
 an infinite spiral ramp that goes round and
 round, on and on for our entire lifetime.**



2

CLAP COUNTING AND MULTIPLES

When will all the groups clap together?

Caroline
 GROUP A: 1 clap 3 clap 5 clap 7 clap 9 clap 11 clap 13 clap 15 ...

Toni
 GROUP B: 1 2 clap 4 5 clap 7 8 clap 10 11 clap 13 14 clap ...

Marie
 GROUP C: 1 2 3 4 clap 6 7 8 9 clap 11 12 13 14 clap ...



3

SKIP COUNTING AND MULTIPLES

Find different ways to do skip counting as a class for different multiples.
 The teacher conducts the timing.

Skip count all together as a class with everyone just saying the multiples.

Everyone has a different number. Skip count silently with only one person saying each number.

Counting in 2's
 - 2 - 4 - 6 - 8 - 10 - 12 - 14...

Counting in 3's
 - - 3 - - 6 - - 9 - - 12 - - 15...

Counting in 5's
 - - - 5 - - - 10 - - - 15...

What are we learning?

4



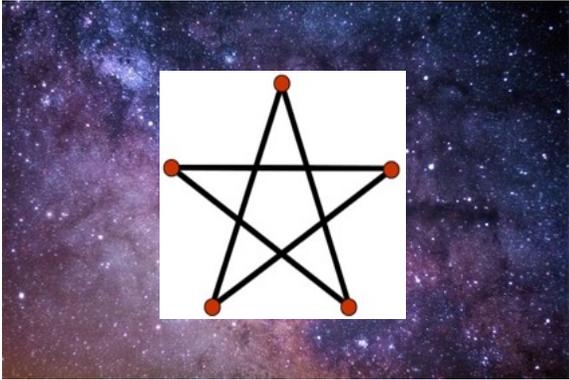
PATH TO THE STARS



Can you draw this pattern without taking your pencil off the paper.

Try it.

Comment on the CHAT



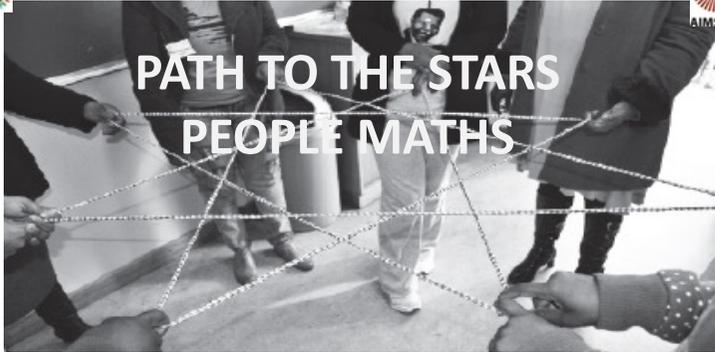
Another experience of multiples and factors

5



PATH TO THE STARS PEOPLE MATHS





<https://aiminghigh.aimssec.ac.za/path-to-the-stars/>

6



PATH TO THE STARS



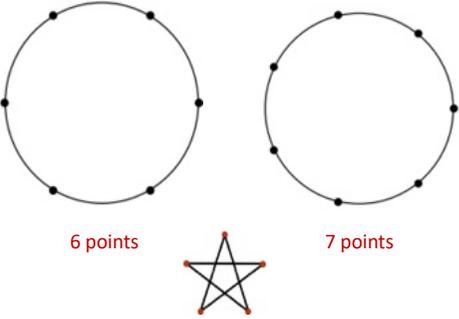
Draw 6 dots. Join the dots to make a star pattern.

Can you do it without taking your pencil off the paper?

Try 7 dots.

Comment on the CHAT

<https://aiminghigh.aimssec.ac.za/path-to-the-stars/>



6 points 7 points

What about lessons for young learners involving these activities?

7



Suggestions for Path to the Stars Lessons for Ages 4 - 7



1. Do a few minutes of Clap Counting or Skip Counting daily starting with counting in 2's up to 10. Progress to other multiples and counting up to 20 and higher.
2. Make paper hats and write numbers on them so everyone has their own number.
3. The whole class stands in order in a circle and skip counts. One person in the centre points to the numbers on the hats as everyone counts.
4. To introduce odds and evens alternate learners in the circle sit down. The class should count again so it's obvious who has even and who has odd numbers on their hats.
5. The evens and odds should stand in 2 lines and count.
6. Make a circle with 8 learners and make a star by passing the string to every third learner as shown.
7. Make a circle of 8 learners and make a star by passing string to every alternate learner. Talk about what is the same and what is different between the 2 stars.
8. Pencil and paper activities: join points on circles with straight lines to make stars and talk about multiples.



Join every 2nd point

Join every 3rd point

Resources: Newspaper, string, scissors, worksheet with points on circles.

8

PATH TO THE STARS

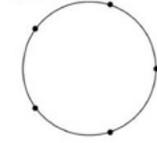
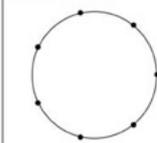
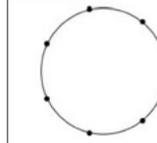
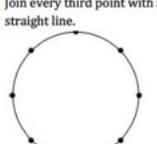
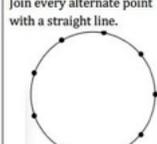
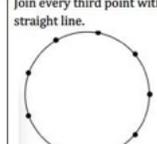
JOIN POINTS TO MAKE STARS AND TALK ABOUT WHAT YOU NOTICE

Choose any one of these circles and follow instructions.

Tell us what you find out on the CHAT.

Note the number of points in each diagram.

Find this on page 5 of the Path to the Stars Inclusion and Home Learning Guide.

Join every alternate point with a straight line. 	Join every alternate point with a straight line. 	Join every third point with a straight line. 
Join every third point with a straight line. 	Join every alternate point with a straight line. 	Join every third point with a straight line. 

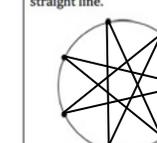
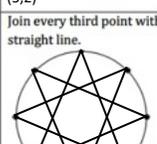
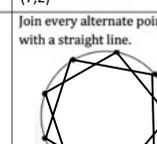
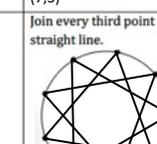
9

JOIN POINTS TO MAKE STARS AND TALK ABOUT WHAT YOU NOTICE

Spot the Odd One Out!

Can you explain the labelling (5, 2), (7, 2) etc

What rule would give a diameter?

Join every alternate point with a straight line.  <p>(5,2)</p>	Join every alternate point with a straight line.  <p>(7,2)</p>	Join every third point with a straight line.  <p>(7,3)</p>
Join every third point with a straight line.  <p>(8,3)</p>	Join every alternate point with a straight line.  <p>(9,2)</p>	Join every third point with a straight line.  <p>(9,3)</p>

10

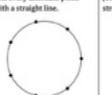
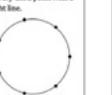
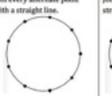
Suggestions for Path to the Stars Lessons for Ages 7 - 9

- Do a few minutes of Clap Counting or Skip Counting daily. Start with counting in 2's then progress to multiples of 9 and counting to 100.
- Make paper hats and write numbers on them so everyone has their own number (e.g. from 1 to 34 if there are 34 learners in the class.)
- The class should stand in a circle and clap count going round the circle several times so in a class of 34 person 1 is also 35, 69 and 103.
- Make a circle with 8 learners and make a star by passing the string to every third learner.


- Give each learner a worksheet with points on circles like the one shown so they can make stars by joining points with straight lines. Talk about the stars and why they are different. Spot the odd one out.

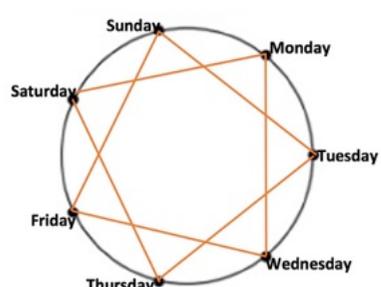
Resources: Newspaper, string, scissors, worksheet with points on circles.

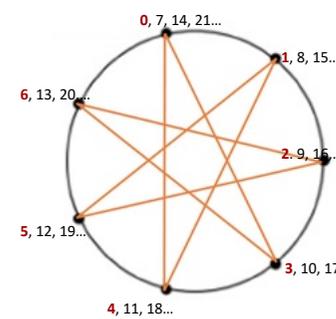
JOIN POINTS TO MAKE STARS AND TALK ABOUT WHAT YOU NOTICE

Join every alternate point with a straight line. 	Join every alternate point with a straight line. 	Join every third point with a straight line. 
Join every third point with a straight line. 	Join every alternate point with a straight line. 	Join every third point with a straight line. 

11

STARS * MULTIPLES * CYCLES * CLOCK ARITHMETIC





12



STARS * MULTIPLES * CYCLES * CLOCK ARITHMETIC



Laying the foundations

Where do we meet equivalence in mathematics?

13



FRACTIONS

$$0.5 = \left\{ \frac{1}{2}, \frac{2}{4}, \frac{3}{6} \dots \frac{2021}{4042} = \dots \right\}$$

You already know about equivalence classes of fractions

$$0.2727 \dots = \left\{ \frac{3}{11}, \frac{6}{22}, \frac{9}{33} \dots \frac{81}{297} = \dots \right\}$$

14



MULTIPLE PATTERNS

<https://aiminghigh.aimssec.ac.za/multiple-patterns>



Now we will look at multiples and factors in a different way

Example:
Multiples of 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

<https://aiminghigh.aimssec.ac.za/multiple-patterns/>

15



MULTIPLE PATTERNS

BREAK
WE'LL START AGAIN ON THE HOUR
<https://aiminghigh.aimssec.ac.za/multiple-patterns/>

Choose one grid.
Shade all the squares with multiples of the given number.

If this is done in groups each person could choose a different set of multiples so the group can compare their different patterns.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 3

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 4

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 7

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 8

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 9

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 10

16



PRIME SIEVE.

<https://aiminghigh.aimssec.ac.za/prime-sieve/>



On the 100 square grid, circle the number 2 and make a line through 4, 6, 8, 10 and the rest of the 2 times table.

Circle the number 3 and make a line through 6, 9, 12, 15 and all the rest of the 3 times table.

Do the same for the 5 and 7 times tables.

What do you notice about the multiples of 8, 9 and 10?

What can you say about the numbers that are not crossed out?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

17



PRIME SIEVE

<https://aiminghigh.aimssec.ac.za/years-6-9-prime-sieve/>



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

In a 100 square shade 1, then not 2, 3, 5 or 7 but all their multiples. You are left with all the prime numbers up to 100. Do it. Then explain why it works.

18

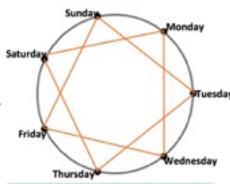


Suggestions for Path to the Stars Lessons for Ages 9 - 11



1. Watch the James Tanton video <https://bit.ly/YouTubeJamesTantonMultiplicationFacts>
2. Play Fizz Buzz and other counting games.
3. Make 7-point stars as 'People Maths' with learners in a circle and string. **Discuss 7-cycles like days of the week** but also with numbers.
4. If learners have not done it before, make a circle of 8 learners and make a star by passing string to every alternate learner.
5. Make another circle with 8 learners and make a star by passing the string to every third learner. Talk about what is the same and what is different.
7. Give each learner a worksheet with points on circles so they can make stars by joining points by straight lines. Talk about the stars and why they are different. Spot the odd one out. (See slides 10 and 11)
8. Colour multiple patterns of 2, 3, 4,... 10 on separate 100 squares (see slides 16 and 17.)
9. Follow instructions for the prime sieve on slide 18. Use the Learning Pack for PRIME SIEVE on the AIMING HIGH website <https://aiminghigh.aimssec.ac.za/prime-sieve/>

Resources: Worksheet with points on circles and 100 squares.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

19





AIMS

African Institute for
Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE
6 MELROSE RD, MUIZENBERG 7975
CAPE TOWN, SOUTH AFRICA
T: +27 (0) 21 787 9326



AN AIMING HIGH LEARNING PACK IS A WEBPAGE CONTAINING

A learning activity with links to:

- a. PDF of the worksheet
- b. Templates and instructions for making resources
- c. Videos
- d. Notes for Teachers with
 - solutions
 - curriculum links and learning objectives
 - diagnostic quizzes
 - suggestions for teaching
 - key questions to guide learning
 - follow up ideas and links
- e. Inclusion Guides for School and Home Learning with
 - a starter activity for a mixed-age group to do together
 - a collection of learning activities to suit all ages from 4 to 18+
 - solutions and everything from the Notes for Teachers

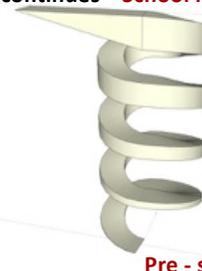
<https://aiminghigh.aimssec.ac.za/>

20

A new era for mathematics education

All AIMSSEC resources are available free under a Creative Commons License on the AIMING HIGH website and on the AIMSSEC App for use offline.

The spiral continues School leaving year



AIMSSEC resources for lessons in school with activities for Differentiation and Inclusion

Inclusion and HOME-LEARNING Guides

- * Mixed Age Groups
- * Common starter & MATHEMATICAL THEME
- * Age-appropriate learning activities for all

Pre - school

21

PATH TO THE STARS

SCISSORS PAPER ROCK VARIATIONS



SPR WIDE SPLITS
Step to side each time you lose.

SPR - played in China in 200 AD
The Wide-Splits photo was taken in Tiananmen Square

WIDE SPLITS
Step to side each time you lose.

FORWARD SPLITS
Each starts one foot behind other, toe to heel. Winner puts front foot behind back. Loser steps forward to put front foot toe to toe with opponent.

Kitsune-ken (狐拳) Japanese Scissors-Paper-Rock

22

ROCK PAPER SCISSORS GAME WITH 7 ELEMENTS

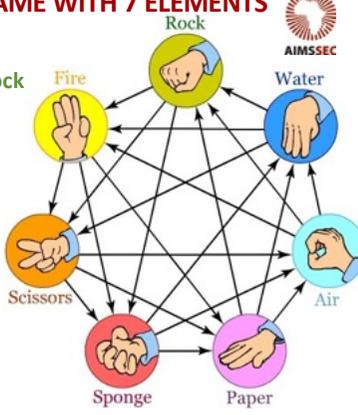
3, 4, 5, 6 & 7 CYCLES
e.g. Rock > Scissors > Sponge > Water > Rock

WHOLE CLASS GAME DIRECTED BY TEACHER

Each player chooses an element and has an equal chance of winning or losing at each turn.

Lines join each point to every other point. Arrows show which element wins.

How many cycles can you find?
What lengths are the cycles?



<https://aiminghigh.aimssec.ac.za/years-scissors-paper-rock/>

23

ROCK PAPER SCISSORS GAME WITH 7 ELEMENTS

3, 4, 5, 6 & 7 CYCLES
e.g. Rock > Scissors > Sponge > Water > Rock

Why play this game?
Cycles are a fundamental and important idea in Mathematics.

<https://aiminghigh.aimssec.ac.za/years-scissors-paper-rock/>

24

Suggestions for Path to the Stars Lessons for Ages 11 - 16

1. Make 7-point stars as 'People Maths' with learners in a circle and string. Discuss 7-cycles like days of the week and with numbers (see slide 7). Discuss 12 and 24 cycles as on the clock and months of the year, and other cycles such as seasons and phases of the moon.
2. Discuss equivalence classes and compare with equivalent fractions (slide 15).
3. Talk about the application of cycles and loops in computer science.
4. Play ROCK, PAPER, SCISSORS and the version with 7 elements (see slide 25). Identify 3, 4, 5, 6 & 7 CYCLES for this version of the game. Talk about directed and undirected networks (see arrows in the diagram).
5. If learners have not done it before, make a circle of 8 learners and make a star by passing string to every alternate learner. Then make another circle with 8 learners and make a star by passing the string to every third learner. Talk about what is the same and what is different and multiples of 2 and 3.
6. Colour multiple patterns of 2, 3, 4,... 9 on separate 100 squares (see slides 18.)
7. Follow instructions for the prime sieve on slide 20. Use the Learning Pack for PRIME SIEVE on the AIMING HIGH website. <https://aiminghigh.aimssec.ac.za/prime-sieve/>

Resources: String, scissors, worksheet with points on circles and 100 squares.

25

STARS * MULTIPLES * CLOCK ARITHMETIC

MODULO 5 **MODULO 7** **MODULO 8**

Just as you cycle around the week, and Saturday comes every 7 days, the numbers in each list are EQUIVALENT in that modular system. Today is Saturday 24th April and 3rd, 10th and 17th were also Saturdays!

26

Poncelet Circles

(8,3) & (8,5) (9,2) & (9,7)

(6,1) & (6,5) (9,4) & (9,5)

(5,2) & (5,3)

This one never returns to the starting point.

Poncelet's Porism

The points generated by drawing tangents repeatedly form a dynamical system which may have cycles. If the path constructed is closed for one starting point it is closed for all starting points.

27

Suggestions for Path to the Stars Lessons for Ages 16 - 18

1. Make 5-point, 7-point and 8-point circles by joining point on a circle. Discuss equivalence of numbers that label the same point on the circle and cycles that come back to the starting point. Compare with 7-cycles like days of the week and 12 and 24 cycles as on the clock and months of the year, and other cycles such as seasons and phases of the moon.
2. Compare with equivalent fractions (slide 15).
3. Discuss periodic functions and principal values of angles in solutions of trigonometric equations.
4. Talk about directed and undirected networks using arrows in network diagrams.
5. Compare with Poncelet Circles formed by drawing tangents to an inner circle, not necessarily concentric with the outer circle (see slide 29).
6. Do the Handshakes and Mystic Rose activities and make connections between the networks and concepts involved. <https://aiminghigh.aimssec.ac.za/handshakes/> <https://aiminghigh.aimssec.ac.za/mystic-rose/>

28



AIMS African Institute for
Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE



Free lesson resources: <http://aiminghigh.aimssec.ac.za>
 AIMSSEC Website: <http://aimssec.ac.za>
COLLABORATIVE PROFESSIONAL DEVELOPMENT <https://aiminghigh.aimssec.ac.za/category/cpd>
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS
<https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/>
PATH TO THE STARS AND RELATED RESOURCES
 Path to the Stars <https://aiminghigh.aimssec.ac.za/path-to-the-stars/>
 Multiple Patterns <https://aiminghigh.aimssec.ac.za/multiple-patterns/>
 Prime sieve <https://aiminghigh.aimssec.ac.za/prime-sieve/>
 Handshakes or Bows <https://aiminghigh.aimssec.ac.za/handshakes/>
 Mystic Rose <https://aiminghigh.aimssec.ac.za/mystic-rose/>

29



AIMS African Institute for
Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE



Free lesson resources: <http://aiminghigh.aimssec.ac.za>
 Collaborative Professional Development <https://aiminghigh.aimssec.ac.za/category/cpd>
 AIMSSEC APP: <https://aimssec.app> (to download the resources and use resources offline)
 AIMSSEC Website: <http://aimssec.ac.za>
 Toni Beardon LAB11@cam.ac.uk
 Caroline Ainslie caroline@bubblymaths.co.uk
**Enquire about signing up for an AIMSSEC course
 as a self-funding student** admin@aimssec.ac.za



Thanks for participating.

30