

Global Teacher Empowerment Network GTEN

LOTTERIES – TO BET OR NOT TO BET


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AIMS

African Institute for
Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE

Global Teacher Empowerment Network (GTEN)

PROGRAMME: LOTTERIES – TO BET OR NOT TO BET- Lucky Numbers and Mathsland Lottery

Learning Spiral

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:
 Probability
 Listing all possible events (sample space)
 Calculating the chance of winning
 Expected profit
 How lotteries are run and the very small chance of winning.
 The Monty Hall Problem

UPPER SECONDARY

LOWER SECONDARY

UPPER PRIMARY

LOWER PRIMARY

EARLY YEARS

STARTER ACTIVITY

11. Summary
10. Discussion on Gambling
9. Monty Hall Version of Lucky Numbers
8. Monty Hall Problem – win a car
7. Using a tree diagram method without drawing the tree
6. The 6 Numbers from 49 Game – Mathsland Lottery
5. Learning Objectives
4. Tree diagrams
3. Listing all the possibilities
2. The 3 Numbers from 6 Game – Lucky Numbers
1. The 2 Numbers from 6 Game

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4 LUCKY NUMBERS GAMES

<https://aiminghigh.aimssec.ac.za/lucky-numbers/>

The first game for young learners. They will intuitively understand that their chance of winning is 1 in 4.

1 IN 4 GAME

Pick a card, spin, Win a point if your number comes up.

1 IN 6 GAME

Pick a card, spin, Win a point if your number comes up.

The second game for young learners. They will intuitively understand that their chance of winning is 1 in 6.

2 IN 6 GAME & 3 IN 6 GAME

The 1 in 4 game (with 4 cards) and the 1 in 6 game are for younger players.

Move on to the 2 in 6 game (2 spins) and then move on again to the 3 in 6 game (3 spins).

What is your chance of winning?

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A SIMPLE EXAMPLE – 1 LUCKY NUMBER IN 4

1
2
3
4


A GAME FOR VERY YOUNG PLAYERS

Each player takes a number card from the bag and puts it back.

Spin the spinner to choose the winning number. If it lands exactly on a line, spin again.


Record 1 point for the winner or winners.

The match ends when someone reaches a total score of 10 points.



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A SIMPLE LOTTERY EXAMPLE – LUCKY NUMBERS



In the Lucky Numbers Game six balls are numbered 1 to 6. Three balls are chosen at the same time, at random, from the six numbers, in no special order.


When you play this game you get a ticket with 3 numbers written on it.
 You win a prize if your 3 numbers match the 3 numbers on the chosen balls.
 What is your chance of winning a prize?
 If you find this problem difficult try the simpler case where 2 balls are chosen from 6 and you get a ticket with 2 numbers.

What method would you use to show that the probability of winning the 2 in 6 Lucky Numbers Game is 1/15?



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AN EVEN SIMPLER EXAMPLE – 2 LUCKY NUMBERS IN 6



Note that the order in which the numbers occur is not relevant so (1, 2) and (2, 1) are the same.
 The possible choices of 2 numbers are:
 (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
 (2, 3) (2, 4) (2, 5) (2, 6)
 (3, 4) (3, 5) (3, 6)
 (4, 5) (4, 6)
 (5, 6)

One of these pairs of numbers must be the winning combination so there is a 1 in 15 chance of winning.

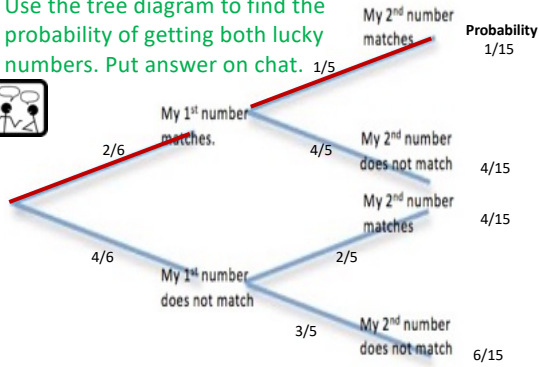
Think of all possible outcomes for 3 numbers. How would you list them systematically so you know that you have found all the possibilities?



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A SIMPLE EXAMPLE – 2 LUCKY NUMBERS IN 6

Use the tree diagram to find the probability of getting both lucky numbers. Put answer on chat. 1/5



Simple case – choose 2 balls from 6.
 You win a prize for getting both lucky balls

Using the top branches of the tree diagram, the probability is 2/6 multiplied by 1/5 giving 1/15.

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A SIMPLE EXAMPLE – 3 LUCKY NUMBERS IN 6

ALL POSSIBLE OUTCOMES for 3 numbers chosen from 6:

(1,2,3) (1,2,4) (1,2,5) (1,2,6)
 (1,3,4) (1,3,5) (1,3,6)
 (1,4,5) (1,4,6)
 (1,5,6)
 (2,3,4) (2,3,5) (2,3,6)
 (2,4,5) (2,4,6)
 (2,5,6)
 (3,4,5) (3,4,6)
 (3,5,6)
 (4,5,6)

One of these triples of numbers must be the winning combination so there is a 1 in 20 chance of winning.

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A SIMPLE EXAMPLE – LUCKY NUMBERS

In the **3 Numbers in 6 Game** we don't need to draw the complete tree diagram but just use the branches at the top where all three chosen numbers are winners.

What is the probability of winning the 3 Numbers in 6 Game?

$$3/6 \times 2/5 \times 1/4 = 1/20$$

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LUCKY NUMBERS

EXPECTED PROFIT:
 If 100 people pay R10 to play the 3-number game and the prize is R150, would the organisers expect to make a profit? If so why?

If 100 people pay R10 to play the 3 balls game, the expectation is that there would be $1/20 \times 100 = 5$ winners of a R150 prize.
 The pay-out would be R750.
 The organisers would collect R1000 and so make a profit of R250.
 If the tickets cost more than R7.50 then the organisers make a profit on average.

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A SIMPLE EXAMPLE – LUCKY NUMBERS

Learning objectives:
 This activity has distinct learning objectives:

- (1) to make a **systematically ordered list of all possible sets of numbers** that can be chosen in a game and use it to work out the probability of winning;
- (2) to use **tree diagrams** to work out the probability of winning.

Different lessons perhaps for different objectives for different age groups.
 For older learners the teacher can combine both learning objectives.

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A SIMPLE EXAMPLE – LUCKY NUMBERS

Why do this activity?
 The class can play the game for fun and then collect data from the whole class to find an **experimental estimate of the probability** of winning.
 Then they can work on calculating the true probability, either by **systematically listing all the possibilities** (learning objective 1) or using a **tree diagram** (learning objective 2).
 (3) This problem offers a simple case of a lottery type game to help learners to develop an understanding that they can then build on to work out the probability of winning in the National Lottery, and to discover how little chance there is of winning, so it's not worth losing money on gambling.

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MATHOPIA LOTTERY






In the Mathopia Lottery, 49 balls are numbered 1 to 49 and 6 balls are chosen at random without replacing any of the balls so that 6 different winning numbers are chosen.

Each lottery ticket has 6 numbers and you win a top prize if your 6 numbers match the 6 numbers chosen that week.


Is buying lottery tickets a waste of money?
What is your chance of winning the top prize?




The probability of all my six numbers winning the jackpot is 1 in 14 million.

If I buy 10 tickets a week it could still be thousands of years (on average) before I'd win a prize. **Let's see why the chance of winning is so low.**

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REMEMBER THE LUCKY NUMBERS GAME 3 WINNING NUMBERS IN 6




All possible outcomes for 3 numbers chosen from 6:


(1,2,3) (1,2,4) (1,2,5) (1,2,6)
 (1,3,4) (1,3,5) (1,3,6)
 (1,4,5) (1,4,6)
 (1,5,6)
 (2,3,4) (2,3,5) (2,3,6)
 (2,4,5) (2,4,6)
 (2,5,6)
 (3,4,5) (3,4,6)
 (3,5,6)
 (4,5,6)


One of these triples of numbers must be the winning combination so there is a 1 in 20 chance of winning.

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MATHOPIA LOTTERY





In the Mathopia Lottery, 49 balls are numbered 1 to 49 and 6 balls are chosen at random without replacing any of the balls so that 6 different winning numbers are chosen. What's my chance of winning the jackpot if I buy a ticket?


Let's see how the chance of winning is calculated.

Remember that the probability of winning the 3 Numbers in 6 Game is:


$3/6 \times 2/5 \times 1/4 = 1/20$ that is 1 in 20.

How would you calculate the probability of winning the 6 Numbers in 49 Game?


Think how we worked out the probability for 3 numbers chosen from 6 numbers using a tree diagram method.




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MATHOPIA LOTTERY





Remember that for the 3 balls in 6 Game the chance of winning the jackpot is 1 in 20. For the 6 balls in 49 Game we must imagine a tree diagram with 6 branchings.


The probability that my 1st number wins is 6/49. There will then be 5 winning numbers left out of 48. The probability that both my 1st and 2nd numbers win is 6/49 x 5/48.

What is the probability of winning the top prize for 6 balls chosen from 49?

This time we have 6 numbers chosen from 49 numbers. The tree must be extended 3 more times showing 6 choices of winning numbers.

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MATHOPIA LOTTERY



The probability of winning the jackpot is:

$$6/49 \times 5/48 \times 4/47 \times 3/46 \times 2/45 \times 1/44$$

$$= 7.15 \times 10^{-8}$$

$$= 1 \text{ in } 14 \text{ million}$$

$$= 1 \text{ in } 14,000,000$$

If you buy a ticket once a week then you would expect to win once in about 270 thousand years.

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NATIONAL LOTTERIES – MOST HAVE 6 LUCKY BALLS



The UK lottery has 59 balls.
South Africa's National Lottery has 52.
Spain has 49 balls.

The jackpot is the big prize for getting 6 balls.
With more than 49 balls there is an even smaller chance of winning the lottery than for the Mathopia Lottery.

There are much smaller prizes for matching 5 balls or 4 balls.

The big question is:

Are there better ways of spending your money?



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MONTY HALL PROBLEM

https://en.wikipedia.org/wiki/Monty_Hall_problem

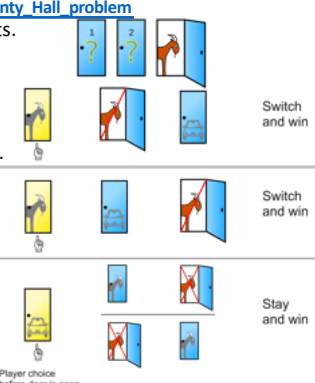
There are 3 boxes, one contains a car, two contain goats.

You choose Door 1.

One of the other doors is opened to reveal a goat.

You have the option to change your choice of doors.

Should you change? We'll look at this in different ways.



At the start the probability that Box 1 contained the car was 1/3. That doesn't change.

Given the extra information, we know the probability that the unopened box contains the car is 2/3.

So you should change your choice?

Player choice before door is open

Switch and win

Stay and win

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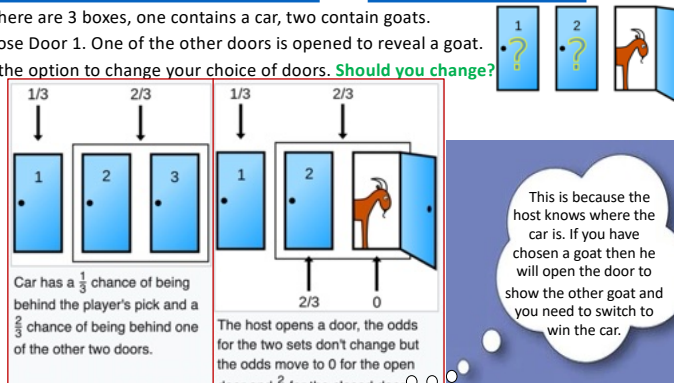
MONTY HALL PROBLEM

https://en.wikipedia.org/wiki/Monty_Hall_problem <https://youtu.be/7u6kFIWZOWg>

There are 3 boxes, one contains a car, two contain goats.

You choose Door 1. One of the other doors is opened to reveal a goat.

You have the option to change your choice of doors. **Should you change?**



Car has a $\frac{1}{3}$ chance of being behind the player's pick and a $\frac{2}{3}$ chance of being behind one of the other two doors.


The host opens a door, the odds for the two sets don't change but the odds move to 0 for the open door and $\frac{2}{3}$ for the closed door.

This is because the host knows where the car is. If you have chosen a goat then he will open the door to show the other goat and you need to switch to win the car.


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HISTORY OF THE STUDY OF GAMBLING ODDS AND PROBABILITY

https://en.wikipedia.org/wiki/Probability_theory




Cardano




Huygens

The modern mathematical theory of probability originated in attempts by Gerolamo Cardano, in the sixteenth century, to analyze gambling and games of chance.



Fermat



Pascal


Pierre de Fermat and Blaise Pascal explored these ideas further in the seventeenth century.

Christiaan Huygens published a book on gambling and the theory of probability in 1657

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HISTORY OF THE STUDY OF GAMBLING ODDS AND PROBABILITY

Pierre Laplace
1749 - 1827




In the 19th century, Pierre Laplace published what is now considered to be the complete basic theory of probability.

https://en.wikipedia.org/wiki/Probability_theory

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TO BET OR NOT TO BET SUMMARY



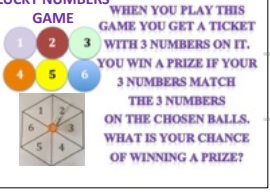
CASINO ONLINE ADDICTION

(1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 3) (2, 4) (2, 5) (2, 6)
(3, 4) (3, 5) (3, 6)
(4, 5) (4, 6)
(5, 6)

(1,2,3) (1,2,4) (1,2,5) (1,2,6)
(1,3,4) (1,3,5) (1,3,6) (2,3,4) (2,3,5) (2,3,6)
(1,4,5) (1,4,6) (2,4,5) (2,4,6) (3,4,5) (3,4,6)
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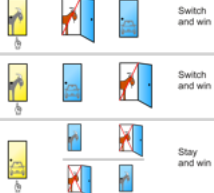
LUCKY NUMBERS GAME

WHEN YOU PLAY THIS GAME YOU GET A TICKET WITH 3 NUMBERS ON IT. YOU WIN A PRIZE IF YOUR 3 NUMBERS MATCH THE 3 NUMBERS ON THE CHOSEN BALLS. WHAT IS YOUR CHANCE OF WINNING A PRIZE?



Player choice before door is open

(A,B,C) (A,B,D) (A,B,E) (A,C,D) (A,C,E) (A,D,E)
(B,C,D) (B,C,E) (B,D,E)
(C,D,E)



Switch and win

Switch and win

Stay and win


In Mathopia Lottery with 6 chosen from 49 balls the chance of winning is **1 in 14 million**

THE NATIONAL LOTTERY

This time we have 6 numbers chosen from 49 numbers. The title must be exceeded 3 more times showing 6 choices of winning numbers.


In the Monty Hall version of the Lucky Numbers Game, by changing your choice of 3 numbers, your chance of winning goes up from 5% to 9.5%

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AIMSSEC Website: <http://aimssec.ac.za>

AIMING HIGH Free lesson resources: <http://aiminghigh.aimssec.ac.za>

AIMSSEC APP: <https://aimssec.app> to download the resources for use offline

LUCKY NUMBERS
<https://aiminghigh.aimssec.ac.za/lucky-numbers/>

MATHSLAND LOTTERY
<https://aiminghigh.aimssec.ac.za/mathsland-lottery/>

MONTY HALL PROBLEM
https://en.wikipedia.org/wiki/Monty_Hall_problem/

NUMBERPHILE EXPLANATION OF MONTY HALL PROBLEM
<https://youtu.be/7u6kFIWZOWg>

COLLABORATIVE PROFESSIONAL DEVELOPMENT <https://aiminghigh.aimssec.ac.za/category/cpd>

MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS
<https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/>

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Global Teacher Empowerment Network (GTEN)
For teachers in primary and secondary schools, colleges and universities




AIMSSEC GTEN YouTube Channel
<https://www.youtube.com/c/MathsToys/videos>




AIMSSEC FACEBOOK <https://www.facebook.com/aimsseca/>

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LET'S PLAY MATHEMATICALLY AND LEARN

Order from AMAZON or TARQUIN <https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically>



Play Mathematically

- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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Thanks for coming to this workshop.

Use the AIMSSEC ideas on AIMING HIGH and add comments.

Share what you have learned with other teachers.

Try to help all your learners to have a **'YES I CAN'** attitude to mathematics.



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Enquire about signing up for an AIMSSEC course as a self-funding student admin@aimssec.ac.za

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