Global Teacher Empowerment Network GTEN

LOTTERIES – TO BET OR NOT TO BET

Saturday 12 March 2022 15:00 – 17:00 London

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

MATHS TOYS

Global Teacher Empowerment Network (GTEN)

PROGRAMME: LOTTERIES – TO BET OR NOT TO BET- Lucky Numbers and Mathsland Lottery

Learning Spiral

IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:
Probability
Listing all possible events (sample space)
Calculating the chance of winning
Expected profit
How lotteries are run and the very small chance of winning.
The Monty Hall Problem

UPPER SECONDARY
LOWER SECONDARY
UPPER PRIMARY
LOWER PRIMARY
EARLY YEARS
STARTER ACTIVITY

11. Summary
10. Discussion on Gambling
9. Monty Hall Version of Lucky Numbers
8. Monty Hall Problem – win a car
7. Using a tree diagram method without drawing the tree
6. The 6 Numbers from 49 Game – Mathsland Lottery
5. Learning Objectives
4. Tree diagrams
3. Listing all the possibilities
2. The 3 Numbers from 6 Game – Lucky Numbers
1. The 2 Numbers from 6 Game

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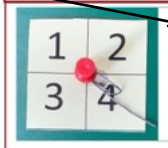
4 LUCKY NUMBERS GAMES

https://aiminghigh.aimssec.ac.za/lucky-numbers/

The first game for young learners. They will intuitively understand that their chance of winning is 1 in 4.



1 IN 4 GAME

Pick a card, spin, Win a point if your number comes up.

1 IN 6 GAME

Pick a card, spin, Win a point if your number comes up.

The second game for young learners. They will intuitively understand that their chance of winning is 1 in 6.

Make a set cards

2 IN 6 GAME & 3 IN 6 GAME

The 1 in 4 game (with 4 cards) and the 1 in 6 game are for younger players.

Move on to the 2 in 6 game (2 spins) and then move on again to the 3 in 6 game (3 spins).

What is your chance of winning?

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A SIMPLE EXAMPLE – 1 LUCKY NUMBER IN 4

A GAME FOR VERY YOUNG PLAYERS

Each player takes a number card from the bag and puts it back.
Spin the spinner to choose the winning number.
If it lands exactly on a line, spin again.
Record 1 point for the winner or winners.
The match ends when someone reaches a total score of 10 points.

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A SIMPLE LOTTERY EXAMPLE – LUCKY NUMBERS

In the Lucky Numbers Game six balls are numbered 1 to 6. Three balls are chosen at the same time, at random, from the six numbers, in no special order.

When you play this game you get a ticket with 3 numbers written on it.
You win a prize if your 3 numbers match the 3 numbers on the chosen balls.
What is your chance of winning a prize?
If you find this problem difficult try the simpler case where 2 balls are chosen from 6 and you get a ticket with 2 numbers.

What method would you use to show that the probability of winning the **2 in 6 Lucky Numbers Game** is 1/15?

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AN EVEN SIMPLER EXAMPLE – 2 LUCKY NUMBERS IN 6

Note that the order in which the numbers occur is not relevant so (1, 2) and (2, 1) are the same.
The possible choices of 2 numbers are:

(1 , 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 3) (2, 4) (2, 5) (2, 6)
(3, 4) (3, 5) (3, 6)
(4, 5) (4, 6)
(5, 6)

One of these pairs of numbers must be the winning combination so there is a 1 in 15 chance of winning.

Think of all possible outcomes for 3 numbers.
How would you list them systematically so you know that you have found all the possibilities?

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A SIMPLE EXAMPLE – 2 LUCKY NUMBERS IN 6

Use the tree diagram to find the probability of getting both lucky numbers. Put answer on chat.

Simple case – choose 2 balls from 6.
You win a prize for getting both lucky balls

Using the top branches of the tree diagram, the probability is 2/6 multiplied by 1/5 giving 1/15.

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A SIMPLE EXAMPLE – 3 LUCKY NUMBERS IN 6

ALL POSSIBLE OUTCOMES for 3 numbers chosen from 6:

(1,2,3) (1,2,4) (1,2,5) (1,2,6)
(1,3,4) (1,3,5) (1,3,6)
(1,4,5) (1,4,6)
(1,5,6)
(2,3,4) (2,3,5) (2,3,6)
(2,4,5) (2,4,6)
(2,5,6)
(3,4,5) (3,4,6)
(3,5,6)
(4,5,6)

One of these triples of numbers must be the winning combination so there is a 1 in 20 chance of winning.

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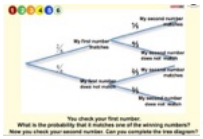
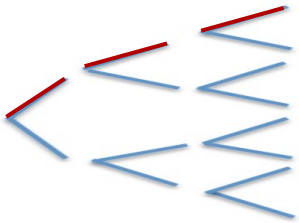
A SIMPLE EXAMPLE – LUCKY NUMBERS





In the **3 Numbers in 6 Game** we don't need to draw the complete tree diagram but just use the branches at the top where all three chosen numbers are winners.

What is the probability of winning the 3 Numbers in 6 Game?

3/6 x 2/5 x 1/4 = 1/20

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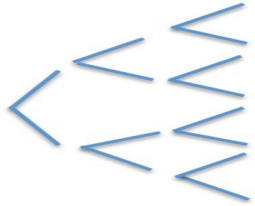
LUCKY NUMBERS



EXPECTED PROFIT:
If 100 people pay R10 to play the 3-number game and the prize is R150, would the organisers expect to make a profit? If so why?

If 100 people pay R10 to play the 3 balls game,
the expectation is that there would be 1/20 x 100 = 5 winners of a R150 prize.
The pay-out would be R750.
The organisers would collect R1000 and so make a profit of R250.
If the tickets cost more than R7.50
then the organisers make a profit on average.

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A SIMPLE EXAMPLE – LUCKY NUMBERS

Learning objectives:

This activity has distinct learning objectives:

(1) to make a **systematically ordered list of all possible sets of numbers** that can be chosen in a game and use it to work out the probability of winning;

(2) to use **tree diagrams** to work out the probability of winning.

Different lessons perhaps for different objectives for different age groups.

For older learners the teacher can combine both learning objectives.

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A SIMPLE EXAMPLE – LUCKY NUMBERS

Why do this activity?

The class can play the game for fun and then collect data from the whole class to find an **experimental estimate of the probability** of winning.

Then they can work on calculating the true probability, either by **systematically listing all the possibilities** (learning objective 1) or using a **tree diagram** (learning objective 2).

(3) This problem offers a simple case of a lottery type game to help learners to develop an understanding that they can then build on to work out the probability of winning in the National Lottery, and to discover how little chance there is of winning, so it's not worth losing money on gambling.

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MATHOPIA LOTTERY

In the Mathopia Lottery, 49 balls are numbered 1 to 49 and 6 balls are chosen at random without replacing any of the balls so that 6 different winning numbers are chosen.

Each lottery ticket has 6 numbers and you win a top prize if your 6 numbers match the 6 numbers chosen that week.

Is buying lottery tickets a waste of money?

What is your chance of winning the top prize?

The probability of all my six numbers winning the jackpot is 1 in 14 million.

If I buy 10 tickets a week it could still be thousands of years (on average) before I'd win a prize. **Let's see why the chance of winning is so low.**

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REMEMBER THE LUCKY NUMBERS GAME
3 WINNING NUMBERS IN 6

All possible outcomes for 3 numbers chosen from 6:

(1,2,3) (1,2,4) (1,2,5) (1,2,6)
(1,3,4) (1,3,5) (1,3,6)
(1,4,5) (1,4,6)
(1,5,6)
(2,3,4) (2,3,5) (2,3,6)
(2,4,5) (2,4,6)
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(3,4,5) (3,4,6)
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One of these triples of numbers must be the winning combination so there is a 1 in 20 chance of winning.

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MATHOPIA LOTTERY

In the Mathopia Lottery, 49 balls are numbered 1 to 49 and 6 balls are chosen at random without replacing any of the balls so that 6 different winning numbers are chosen.
What's my chance of winning the jackpot if I buy a ticket?

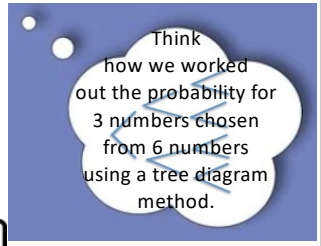
Let's see how the chance of winning is calculated.

Remember that the probability of winning

the 3 Numbers in 6 Game is:

3/6 x 2/5 x 1/4 = 1/20 that is 1 in 20.

How would you calculate the probability of winning the 6 Numbers in 49 Game?



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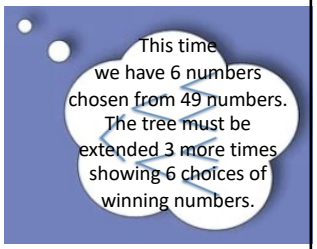
MATHOPIA LOTTERY

Remember that for the **3 balls in 6 Game** the chance of winning the jackpot is 1 in 20.
For **the 6 balls in 49 Game** we must imagine a tree diagram with 6 branchings.

The probability that my 1st number wins is 6/49.
There will then be 5 winning numbers left out of 48.
The probability that both my 1st and 2nd numbers win is 6/49 x 5/48.

What is the probability of winning the top prize for 6 balls chosen from 49?



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MATHOPIA LOTTERY

The probability of winning the jackpot is:

6/49 x 5/48 x 4/47 x 3/46 x 2/45 x 1/44

$= 7.15 \times 10^{-8}$

= 1 in 14 million

= 1 in 14,000,000

If you buy a ticket once a week then you would expect to win once in about 270 thousand years.

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NATIONAL LOTTERIES – MOST HAVE 6 LUCKY BALLS

THE NATIONAL LOTTERY

The UK lottery has 59 balls.
South Africa's National Lottery has 52.
Spain has 49 balls.

The jackpot is the big prize for getting 6 balls.
With more than 49 balls there is an even smaller chance of winning the lottery than for the Mathopia Lottery.
There are much smaller prizes for matching 5 balls or 4 balls.

The big question is:

Are there better ways of spending your money?

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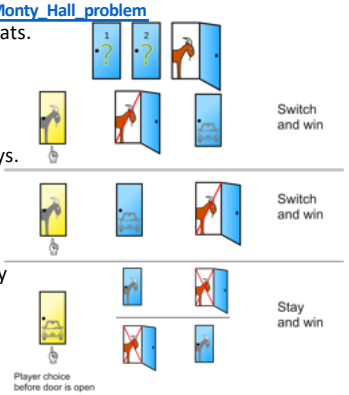
MONTY HALL PROBLEM

https://en.wikipedia.org/wiki/Monty_Hall_problem

There are 3 boxes, one contains a car, two contain goats.

You choose Door 1.
One of the other doors is opened to reveal a goat.

You have the option to change your choice of doors.
Should you change? We'll look at this in different ways.



At the start the probability that Box 1 contained the car was 1/3. That doesn't change.

Given the extra information, we know the probability that the unopened box contains the car is 2/3.

So you should change your choice?

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MONTY HALL PROBLEM

https://en.wikipedia.org/wiki/Monty_Hall_problem https://youtu.be/7u6kFlWZOWg

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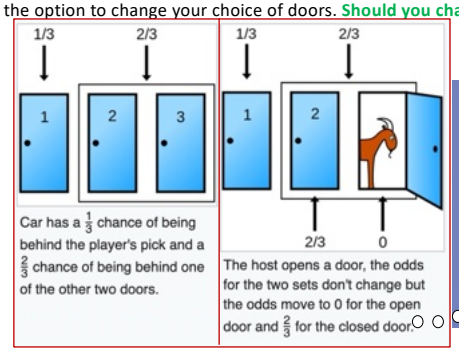
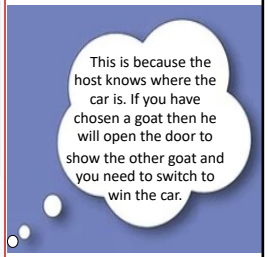
You choose Door 1. One of the other doors is opened to reveal a goat.
You have the option to change your choice of doors. **Should you change?**





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LUCKY NUMBERS – MONTY HALL

This version of Lucky Numbers is similar to the Monty Hall Problem but the probabilities are different.

1. The host picks 3 winning numbers and keeps them secret.
2. You choose 3 numbers from 1 to 6 and show them to the host.

3. The host then chooses one of the 3 numbers that you did not choose and declares that to be a losing number.
4. You then have the option to change your choice of numbers.

Should you change your choice?

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MONTY HALL VERSION OF LUCKY NUMBERS

With 5 numbers A, B, C, D, E there are 10 combinations
(A,B,C) (A,B,D) (A,B,E) (A,C,D) (A,C,E) (A,D,E) (B,C,D) (B,C,E) (B,D,E) (C,D,E)

Initially your chance of winning is $\frac{1}{20}$ = 5%

and your chance of losing is $\frac{19}{20}$.

You get more information.
You are told one of the other numbers is a loser.
It is taken out of play.
Now there are only 5 numbers in play.
Should you change your numbers?

The chance of one choice of 3 numbers chosen from 5 being the winning combination is now $\frac{1}{10}$

So your chance of winning by changing your choice of 3 numbers is

$$\frac{19}{20} \times \frac{1}{10} = 9.5\%$$

By changing, your chance of winning goes up from 5% to 9.5%

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WHAT IS GAMBLING? WHY STUDY PROBABILITY?

Gambling is an activity where:

- two or more parties place at risk something of value (the stakes)
- in the hope of winning something of greater value (the prize)
- where the result depends on the outcome of events unknown to the participants at the time of the bet.

According to the North American Foundation for Gambling Addiction, gambling worldwide has greatly increased over recent years due to internet technologies, with a corresponding increase in problem gambling and gambling addiction.

Commercial gambling operations typically depend on mathematical facts which ensure that the company cannot lose in the long run, these companies are not in fact gambling because they are not taking risks.

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Cardano

Huygens

Fermat

Pascal

HISTORY OF THE STUDY OF GAMBLING ODDS AND PROBABILITY

https://en.wikipedia.org/wiki/Probability_theory

The modern mathematical theory of probability originated in attempts by Gerolamo Cardano, in the sixteenth century, to analyze gambling and games of chance.

Pierre de Fermat and Blaise Pascal explored these ideas further in the seventeenth century.

Christiaan Huygens published a book on gambling and the theory of probability in 1657

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HISTORY OF THE STUDY OF GAMBLING ODDS AND PROBABILITY

Pierre Laplace
1749 - 1827

In the 19th century, Pierre Laplace published what is now considered to be the complete basic theory of probability.

https://en.wikipedia.org/wiki/Probability_theory

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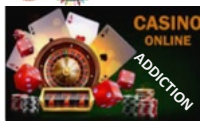
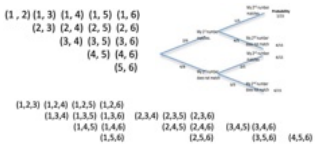
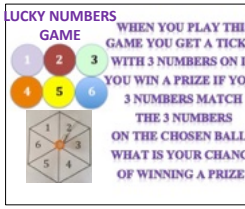
TO BET OR NOT TO BET SUMMARY

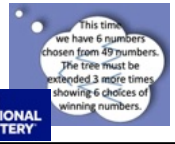




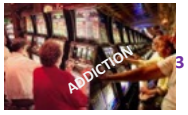


(A,B,C) (A,B,D) (A,B,E) (A,C,D) (A,C,E) (A,D,E) (B,C,D) (B,C,E) (B,D,E) (C,D,E)

In Mathopia Lottery with 6 chosen from 49 balls the chance of winning is **1 in 14 million**



THE NATIONAL LOTTERY



In the Monty Hall version of the Lucky Numbers Game, by changing your choice of 3 numbers, your chance of winning goes up from 5% to 9.5%

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AIMSSEC Website: http://aimssec.ac.za
AIMING HIGH Free lesson resources: http://aiminghigh.aimssec.ac.za
AIMSSEC APP: https://aimssec.app **to download the resources for use offline**

LUCKY NUMBERS
https://aiminghigh.aimssec.ac.za/lucky-numbers/

MATHSLAND LOTTERY
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MONTY HALL PROBLEM
https://en.wikipedia.org/wiki/Monty_Hall_problem/

NUMBERPHILE EXPLANATION OF MONTY HALL PROBLEM
https://youtu.be/7u6kFlWZOWg

COLLABORATIVE PROFESSIONAL DEVELOPMENT https://aiminghigh.aimssec.ac.za/category/cpd
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS
https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/

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Global Teacher Empowerment Network (GTEN)

For teachers in primary and secondary schools, colleges and universities

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Enquire about signing up for an AIMSSEC course as a self-funding student admin@aimssec.ac.za

Thanks for coming to this workshop.

Use the AIMSSEC ideas on AIMING HIGH and add comments.

Share what you have learned with other teachers.

Try to help all your learners to have a

'YES I CAN'

attitude to mathematics.

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