

DIGIT DETECTIVE - Getting there.... Start them young

A
 $\triangle + 9 = 10$
 $\triangle - \triangle = \square$

B
 $5 + \triangle = 9$
 $7 - \triangle = \square$

Rules

- Find a number between 0 and 9 that goes into each shape.
- On each card the same number must go into the same shape.
- The answer to the sum must be correct.

Thinking algebraically

C
 $\square + \triangle + \triangle - 22$ $\square + \triangle - 13$
 What is \triangle ?
 What is \square ?
 Explain how you found your answers

Explain how you found your answer for Card C.

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DIGIT DETECTIVE - Getting there.... Start them young

Put 10 counters into 2 boxes in different ways. You need 110 counters altogether; they can be stones, dried peas or beans.

One has been done for you. $4 + 6 = 10$ is an example of a **NUMBER BOND**.

To solve this puzzle, put a number in place of A and another number in place of B to make this addition sum correct.

Can A be any other number? Explain why or why not.

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TIMES FIVE & TIMES NINE - Getting started

Count out some stones or dried peas and arrange them in sets of 9.

9	$1 \times 9 = 10 - 1 = 9$
18	$2 \times 9 = 20 - 2 = 18$
27	$3 \times 9 = 30 - 3 = 27$
36	$4 \times 9 = 40 - 4 = 36$
45	$5 \times 9 = 50 - 5 = 45$
54	$6 \times 9 = 60 - 6 = 54$
63	$7 \times 9 = 70 - 7 = 63$
72	$8 \times 9 = 80 - 8 = 72$
81	$9 \times 9 = 90 - 9 = 81$

Here are 14 handprints (or parts of handprints)
 $14 \times 5 = 70$

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You can always count on a helping hand!

Use your digits... Use your eyes ... Use what you know...

9 x table on your fingers

- Hold your hands in front of you with your fingers spread out.
- For 9×4 bend your 4th finger down (like the picture).
- You have 3 fingers in front of the bent finger and 6 after the bent finger. Thus the answer must be 36!
- The technique works for the 9 times tables up to 10.

[9 Nine Multiplication Times Table Hand Trick - YouTube](#)

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You can always count on a helping hand!
Use your digits... *Do 'hands' only work with 9's?*

Times Tables Using Your Hands - 3 Times Table

Learning multiplication tables is in your own hands - 6, 7, 8, 9 & 10:

- <https://www.youtube.com/watch?v=ovNRpka4JXU>
- <https://www.yestuition.co.uk/learning-times-tables-using-fingers/>
- [Always Remember Your Times Table \(6 to 10\) Finger Multiplication - Why It Works - YouTube](#)

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You can always count on a helping hand!
Use your digits... Use your eyes... Use what you know...

Some children's work

Team 3	Pattern	Detectives
$9 \times 1 = 09$	0 9	$0 + 9 = 9$
$9 \times 2 = 18$	1 8	$1 + 8 = 9$
$9 \times 3 = 27$	2 7	$2 + 7 = 9$
$9 \times 4 = 36$	3 6	$3 + 6 = 9$
$9 \times 5 = 45$	4 5	$4 + 5 = 9$
$9 \times 6 = 54$	5 4	$5 + 4 = 9$
$9 \times 7 = 63$	6 3	$6 + 3 = 9$
$9 \times 8 = 72$	7 2	$7 + 2 = 9$
$9 \times 9 = 81$	8 1	$8 + 1 = 9$
$9 \times 10 = 90$	9 0	$9 + 0 = 9$

We are	finding	Patterns
1 0 9 ↓	09	08 9
1 8	10 8	1 9 8
2 7	11 7	2 0 7
3 6	12 6	3 1 6
4 5	13 5	4 2 5
5 4	14 4	5 3 4
6 3	15 3	6 4 3
7 2	16 2	7 5 2
8 1	17 1	8 6 1
9 0	18 0	9 7 0

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DIGIT DETECTIVE - Getting there.... Start them young

1. Put these numbers in the boxes to make this addition sum correct:

2 2 3 4 5 6

+

Put different numbers in place of the letters to make these addition sums correct.

2. $\begin{matrix} B & B \\ + & A \\ \hline A & C & C \end{matrix}$

3. $\begin{matrix} P & Q \\ + & Q & R \\ \hline Q & R & Q \end{matrix}$

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DIGIT DETECTIVE - Getting there.... Start them young

1. Put these numbers in the boxes to make this addition sum correct:

2 2 3 4 5 6

+

2. $\begin{matrix} B & B \\ + & A \\ \hline A & C & C \end{matrix}$

A must be 1 so BB must be 99
Similarly, Q must be 1

2. $\begin{matrix} 9 & 9 \\ + & 1 \\ \hline 1 & 0 & 0 \end{matrix}$

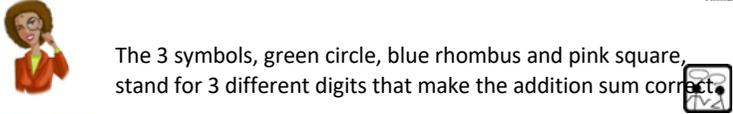
3. $\begin{matrix} P & Q \\ + & Q & R \\ \hline Q & R & Q \end{matrix}$

3. $\begin{matrix} 9 & 1 \\ + & 1 & 0 \\ \hline 1 & 0 & 1 \end{matrix}$

Children can solve these puzzles when they can count and write numbers up to 100

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FIND THE NUMBERS PUZZLE



The 3 symbols, green circle, blue rhombus and pink square, stand for 3 different digits that make the addition sum correct.

Replace the coloured shapes by digits.

You need to be a digit detective and find the clues that will help you to solve this puzzle.

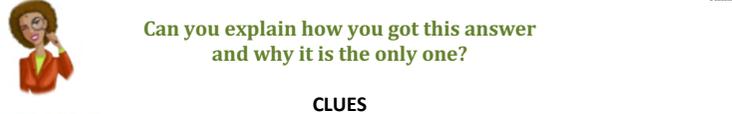
What digit do you think has to go in the hundreds place? Why?

Looking at the pink squares in the units column what can you say about the circle and rhombus?

<https://aiminghigh.aimssec.ac.za/find-the-numbers/>

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FIND THE NUMBERS PUZZLE



Can you explain how you got this answer and why it is the only one?

CLUES

The digit in the hundreds place must be 1.

The squares in the units column are the same digit so the circle and rhombus must add up to 10.

From the tens column we see that the rhombus must be one more than the square.

$$\begin{array}{r} 11 \\ 99 \\ +88 \\ \hline 198 \end{array}$$

<https://aiminghigh.aimssec.ac.za/find-the-numbers/>

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MORE CRYPTARITHM PUZZLES – 3 DIGIT NUMBERS

7 possible solutions

$$\begin{array}{r} \text{TWO} \\ + \text{TWO} \\ \hline \text{FOUR} \end{array}$$

16 solutions

$$\begin{array}{r} \text{ONE} \\ + \text{ONE} \\ \hline \text{TWO} \end{array}$$

How many solutions can you find?

What clues could you give your learners here?

To help the learners the teacher might give them a clue (a partial solution) and ask them to find the rest of the digits or give them one solution and ask them to find others.

<https://nrich.maths.org/twoandtwo>

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MORE CRYPTARITHM PUZZLES – 3 DIGIT NUMBERS

7 possible solutions

	938	928	
	+938	+928	
	1876	1856	
	867	846	
	+867	+846	
	1734	1692	
	836	765	734
	+836	+765	+734
	1672	1530	1468

One of the solutions is

$$\begin{array}{r} 432 \\ + 432 \\ \hline 864 \end{array}$$

To help the learners the teacher might give them one solution and ask them to find other solutions.

Make up some more cryptarithm puzzles like this and find all the solutions.

<https://nrich.maths.org/twoandtwo>

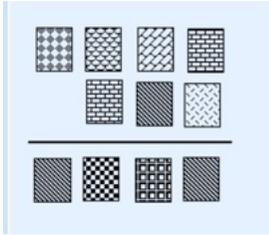
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CRYPTARITHM PUZZLES 4 DIGIT NUMBERS - Getting started:

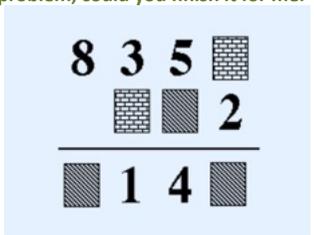
This is one for you to work on later

I have been working on this puzzle that a learner challenged me to solve.
Each type of sticker stands for a different number.
Wherever that same sticker occurs it is the same number.

This is the problem:



So far, I have partially solved the problem, could you finish it for me.



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DIGIT DETECTIVE - Getting there.... Start them young

Here is your key or code book:

F	O	O	T
+B	A	L	L
G	A	M	E

A = 5, B = 6, E = 7, F = 1, G = 8,
L = 3, M = 2, O = 9, T = 4.

What does this reveal?

1	9	9	4
+6	5	3	3
8	5	2	7

Replace B, A, L, L here for another solution:

2004
BALL +
5981

This is one for you to work on later.
This is one of 224 solutions to
FOOT + BALL = GAME
when you replace each letter in by a
different digit to make the sum correct.
How many solutions can you find?

<https://ajimhigh.aimssec.ac.za/football-challenge/>

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DIGIT DETECTIVE

What is the difference between a digit and a number?

We will start with simple calculations so learners will find it easy to explain their ways of thinking about the problems.

12 + ? = 19 1, 2, and 9 are digits here. Twelve (12) and nineteen (19) are numbers.

The question marks ? are missing digits between 0 and 9.

How will you find the missing digit?

Where did you start? Why?

Let's share our methods with each other. Explain how you found your answer.

- Counting on from 12 to 19
- Just knew the answer
- Subtracting 12 from 19. Why?
- Because 2 + 7 = 9

Now what about these examples? ? + 8 = ?4

??? + 150 = 257

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DIGIT DETECTIVE

You are a detective and you have to discover what digits the question marks stand for. They can be any digit from 0 to 9 and they can be the same digit or different digits.

Each ? is a missing digit from 0 to 9 either the same or different digits.

? + 8 = 2?

2? + 4 = 1?

33 + ? = 4?

What do you notice?

How will you find the missing digits?

Explain why this happens?

What clues did you use to help you work out your answers?

How many solutions are there to the calculations?

<https://ajimhigh.aimssec.ac.za/digit-detective/>

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DIGIT DETECTIVE SOLUTIONS



Each ? is a missing digit from 0 to 9 either the same or different digits.

$7 + 8 = 2?$
 $2? + 4 = 1?$
 $33 + ? = 4?$

What do you notice?

$? + 8 = 2?$ is impossible.
 There are no solutions because the largest number you can get from $? + 8 = 9 + 8 = 17$ which is less than 20.

$2? + 4 = 1?$ is impossible.
 There are no solutions because $2?$ has to be at least 20 and by adding 4 you get a number greater than 20.

$33 + ? = 4?$ has 3 solutions.
 You do not get a solution if you add any digit less than 7 to 33 because you get an answer less than 40.

$33 + 7 = 40$
 $34 + 8 = 41$
 $35 + 9 = 42$

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DIGIT DETECTIVE




Each ? is a missing digit from 0 to 9 either the same or different digits.

$7 + 8 = 2?$
 $2? + 4 = 1?$
 $33 + ? = 4?$

What do you notice?

The objective is not trying to teach facts or methods but trying to help learners to think mathematically and to find explanations.

Ask questions like:

How could you check you found all the solutions?
 Why is there more than one solution?
 Why is it you could *not* find the digits in $? + 8 = 2?$
 What about $2? + 4 = 1?$.
 How many solutions did you find to $33 + ? = 4?$



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DIGIT DETECTIVE



MISSING DIGITS

(a) $3?6 + 5? = 383$
 (b) $8?3 - 7? = 815$



What clues did you use? Explain how you solved the puzzles.

LARGEST AND SMALLEST

In the next two puzzles each of the digits must be different.

(c) Find the largest possible total for $?? + ?? =$
 Explain why it is the largest.
 Find the smallest possible total and explain?

MULTIPLICATION AND DIVISION

(d) What about $?? \times ??$
 (e) Now find the missing digit in this division: $816 \div ? = 272$

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DIGIT DETECTIVE – SOME SOLUTIONS



(a) $3?6 + 5? = 383$ has the solution
 $326 + 57 = 383$

The clues are:

- to get the units digit 3 you must add 7 to 6
- then $7 + 6 = 13$ so look for $1 + 5 + ? = 8$ to find the tens digit.

(b) $8?3 - 7? = 815$ has the solution
 $893 - 78 = 815$

The clues are:

- you must find a number to subtract from 13 to get the units digit 5
- then use the inverse operation $815 + 78$ gives the number 893.

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DIGIT DETECTIVE – SOME SOLUTIONS

(c) The largest total for $?? + ?? =$
 is $97 + 86 = 183$
 or $96 + 87 = 183$

To get the largest total you need to make the tens digits as large as possible and then make the units digits as large as possible choosing from the remaining digits.

The smallest is $10 + 23 = 33$ or $13 + 20 = 33$

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DIGIT DETECTIVE – SOLUTIONS TO LARGEST AND SMALLEST

(d) The largest product for $?? \times ??$
 is $96 \times 87 = 8352$.

(Remember you can't repeat a digit)

This is bigger than $97 \times 86 = 8342$

x	90	7	
80	7200	560	7760
6	540	42	582
	7740	602	8342

x	90	6	
80	7200	480	7680
7	630	42	672
	7830	522	8352

To get the largest product we have to multiply the largest units digit by the largest tens digit as shown by the grid method of multiplication.

(e) The missing digit is 3 in this division:
 $816 \div ? = 272$
 $816 \div 3 = 272$

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DIGIT DETECTIVE Why do these activities?

LEARNING OBJECTIVES

- To develop a deeper understanding of place value and inverse operations by challenging learners to act as detectives and to solve number puzzles.
- To get a feel for the size of numbers leading to the understanding of maximum and minimum values.
- To develop mathematical thinking and **communication skills**. Learners share ideas about their intuitive methods in simple addition and subtraction problems and progress to problems including multiplication and division.
- To develop **number sense, logical thinking, problem solving and application of knowledge and skills** by asking for reasons for what they do to solve the puzzles.
- To practise (a) calculations in what seems to be a game;
 (b) **handling problems with no solutions and with multiple solutions;**
 (c) **flexible thinking, creativity and innovative**
 (d) **research, searching for, analysing, and interpreting information.**

Generic skills are in red

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MAGIC OF 37

Complete the number sentences by putting a number or \times or $=$ in the box:

$37 \times 3 \square 111$
 $37 \times \square = 222$
 $37 \square 9 = 333$
 $37 \times \square = 444$
 $\square \times 15 = 555$
 $37 \square 18 = 666$
 $\square \times 21 = 777$
 $37 \times 24 \square 888$
 $\square \times 27 = 999$

First complete the number sentences and then explain why the pattern occurs.

Carry on these multiplications up to 37×54 and then explore the pattern that you get.

Talk about what you notice about this pattern.

Then write down everything that you notice.

<https://ajimhigh.aimssec.ac.za/magic-of-37/magic-of-37/>

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BEAUTIFUL NUMBERS AND MULTIPLES OF 9

? × 9 + 6 = 888 Replace the ?s to complete these calculations.
 ? × 9 + 1 = 88 888 888 Don't use a calculator

Solutions **98 × 9 + 6 = 888**
9 876 543 × 9 + 1 = 88 888 888

Now complete these calculations with a calculator if you wish
 ? × 9 + 5 = 8 888
 ? × 9 + 0 = 888 888 888

Solutions **987 × 9 + 5 = 8 888**
98 765 432 × 9 + 0 = 888 888 888

What patterns do you see?
<https://aiminghigh.aimssec.ac.za/beautiful-numbers/>

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BEAUTIFUL NUMBERS

How would you describe this pattern of calculations?
 How do you find the missing digits?
 Can you explain how this pattern arises?

? × 9 + 7 = 88	9 × 9 + 7 = 88
? × 9 + 6 = 888	98 × 9 + 6 = 888
? × 9 + 5 = 8888	987 × 9 + 5 = 8888
? × 9 + 4 = 88888	9876 × 9 + 4 = 88888
? × 9 + 3 = 888888	98765 × 9 + 3 = 888888
? × 9 + 2 = 8888888	987654 × 9 + 2 = 8888888
? × 9 + 1 = 88888888	9876543 × 9 + 1 = 88888888
? × 9 + 0 = 888888888	98765432 × 9 + 0 = 888888888

<https://aiminghigh.aimssec.ac.za/beautiful-numbers/>

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BEAUTIFUL NUMBERS

How would you describe this pattern of calculations?
 Can you explain how this pattern arises?

? × 9 + 7 = 88	9 × 9 + 7 = 88
? × 9 + 6 = 888	98 × 9 + 6 = 888
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? × 9 + 3 = 888888	98765 × 9 + 3 = 888888
? × 9 + 2 = 8888888	987654 × 9 + 2 = 8888888
? × 9 + 1 = 88888888	9876543 × 9 + 1 = 88888888
? × 9 + 0 = 888888888	98765432 × 9 + 0 = 888888888

Example – the bottom line:
98 765 432 × 9 = 88 888 888
 is made up as follows:

90 000 000 × 9 =	810 000 000
+ 8 000 000 × 9 =	72 000 000
+ 700 000 × 9 =	6 300 000
+ 60 000 × 9 =	540 000
+ 5 000 × 9 =	45 000
+ 400 × 9 =	3 600
+ 30 × 9 =	270
+ 2 × 9 =	18
adding these subtotals:	
= 98 765 432 × 9 =	88 888 888

<https://aiminghigh.aimssec.ac.za/beautiful-numbers/>

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TIMES NINE

Complete the calculations in the picture.
 What do you notice?

To explain how the patterns of numbers arise multiply by (10 – 1) instead of multiplying by 9.

<https://aiminghigh.aimssec.ac.za/times-nine/>

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TIMES NINE

Complete the calculations in the picture.
What do you notice?
Multiply by (10 - 1) instead of multiplying by 9 and explain how the patterns of numbers arise.

Examples

$12\ 345 \times (10 - 1) + 6$
 $= 123\ 450 + 6 - 12\ 345$
 $= 123\ 456 - 12\ 345$
 $= 111\ 111$

$12\ 345\ 678 \times (10 - 1) + 9$
 $= 123\ 456\ 780 + 9 - 12\ 345\ 678$
 $= 123\ 456\ 789 - 12\ 345\ 678$
 $= 111\ 111\ 111$

<https://aiminghigh.aimssec.ac.za/times-nine/>

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TIMES NINE

The picture shows the pyramids in the background because this number pattern, and others like it, were known to the ancient Egyptians.

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THINK OF TWO WHOLE NUMBERS

Here is an alternative version of the “Think of a Number” trick.
Think of two whole numbers under 10.
Take one of them and add 1.
Multiply by 5.
Add 1 again.
Double your answer.
Subtract 1.
Add your second number.
Add 2.
Double again.
Subtract 8.
Halve this number and tell me your answer.
I can work out both your numbers very quickly. How?
Choose some different pairs of numbers and repeat the process.
Can you find a good explanation of how the trick works?

<https://aiminghigh.aimssec.ac.za/think-of-two-numbers/>

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THINK OF TWO WHOLE NUMBERS

Algebraically:
a and *b*
a + 1
 $5a + 5$
 $5a + 6$
 $10a + 12$
 $10a + 11$
 $10a + b + 11$
 $10a + b + 13$
 $20a + 2b + 26$
 $20a + 2b + 18$
 $10a + b + 9$
I subtract 9.
I know the number has $10a + b$

6 and 3 Think of two whole numbers under 10.
6 + 1 = 7 Take one of them and add 1.
35 Multiply by 5.
36 Add 1 again.
72 Double your answer.
71 Subtract 1.
74 Add your second number.
76 Add 2.
152 Double again.
144 Subtract 8.
72 Halve this number and tell me your answer.
 I can work out both your numbers very quickly. How?
 Choose some different pairs of numbers and repeat the process.
 Can you find a good explanation of how the trick works?
<https://aiminghigh.aimssec.ac.za/think-of-two-numbers/>

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FOOTBALL GAME

Thinking algebraically

$$\square + \triangle = 13$$

$$\square + \triangle + \triangle = 22$$

$$\square = ?$$

$$\triangle = ?$$

Think of 2 numbers a & b and $10a + b$

Find the largest possible values for $?? + ?? = ??$ and $?? \times ?? = ???$

Find the smallest possible values. Explain.

What do you notice?

Each 7 is a missing digit from 0 to 9 either the same or different digits.

376 + 57 = 383
873 - 7? = 815

7 + 8 = 27
27 + 4 = 17
33 + 7 = 47

9 x 9 + 7 = 88
98 x 9 + 6 = 888
987 x 9 + 5 = 8888
9876 x 9 + 4 = 88888
98765 x 9 + 3 = 888888
987654 x 9 + 2 = 8888888
9876543 x 9 + 1 = 88888888
98765432 x 9 + 0 = 888888888

816 ÷ ? = 272

8?? ÷ 3 = 272

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AIMSSEC African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

DIGIT DETECTIVE <https://aiminghigh.aimssec.ac.za/digit-detective/>

THINK OF TWO WHOLE NUMBERS <https://aiminghigh.aimssec.ac.za/think-of-two-numbers/>

BEAUTIFUL NUMBERS <https://aiminghigh.aimssec.ac.za/beautiful-numbers/>

MAGIC OF 37 <https://aiminghigh.aimssec.ac.za/magic-of-37/magic-of-37/>

TIMES NINE <https://aiminghigh.aimssec.ac.za/times-nine/>

CRYPTARITHM – FIND TNUMBERS <https://aiminghigh.aimssec.ac.za/find-the-numbers/>

TWO AND TWO <https://nrich.maths.org/twoandtwo>

FOOT BALL CHALLENGE <https://aiminghigh.aimssec.ac.za/football-challenge/>

EGYPTIAN MATHEMATICS NUMBER HEIROGLYPHS <https://discoveringegypt.com/egyptian-hieroglyphic-writing/egyptian-mathematics-numbers-hieroglyphs/>

BABYLONIAN SEXAGESIMAL NUMBER SYSTEM , OLD BABYLONIAN MATHEMATICS AND PLIMPTON 322 <https://www.youtube.com/watch?v=J5Ug3Cr8RUE>

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

GTEN

On the AIMING HIGH website there are freely downloadable worksheets, Inclusion and Home Learning Guides with activities for learners of all ages and attainment levels, Notes for Teachers with solutions, Key Questions and Diagnostic Quizzes for formative assessment.

AIMSSEC GTEN YouTube Channel
<https://www.youtube.com/c/MathsToys/videos>

MATHS TOYS

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AIMSSEC FACEBOOK <https://www.facebook.com/aimssecsa/>

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GTEN

Thanks for coming to this workshop.

Use the AIMSSEC ideas on AIMING HIGH and add comments.

Share what you have learned with other teachers.

Try to help all your learners to have a **'YES I CAN'** attitude to mathematics.

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Enquire about signing up for an AIMSSEC course as a self-funding student admin@aimssec.ac.za

Thank You!

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