


DO-TALK-RECORD

the basis of a good lesson




DO Break up the broccoli
TALK What do you notice?
RECORD Make notes.

DO the activities pretending that you only know what your learners know.

TALK to other teachers in your group, or comment on the chat. How might learners discover new ideas by doing the activity?



RECORD for yourself as a teacher and also in a way that would help learners.



DO Count the black triangles.
TALK What do you notice?
RECORD Describe the pattern and other properties of this shape.

You will have time to do these activities later in this workshop


5



WHAT IS A FRACTAL?

A fractal is a shape, or curve, that is made up of infinitely many of copies of itself.

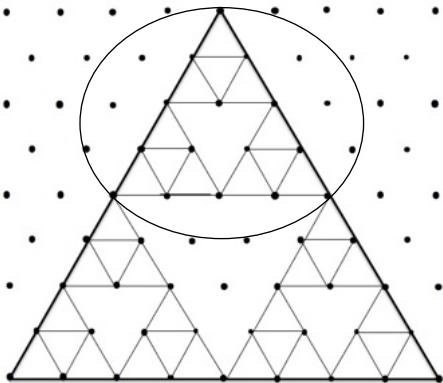
The copies may get smaller and smaller so that the shape does not grow to be unmanageably large.



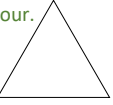
6

STARTER ACTIVITY FRACTAL TRIANGLE

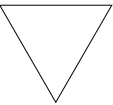


Colour the **small** triangles that are this way up all the same colour.



Start with the triangles in the ring.


Leave the triangles like this uncoloured



7

FRACTALS IN NATURE

- Rivers
- Lightning
- Coastlines
- Trees
- Root systems
- Ferns
- Broccoli
- Lungs
- Vascular system ...



8

FRACTALS IN NATURE

A fern drawn by a computer code and a photo – which is which?

Bacteria

Veins in a leaf

Fractals are used by the film industry to produce images . Randomness is introduced to make the images look more real.

9

FRACTALS

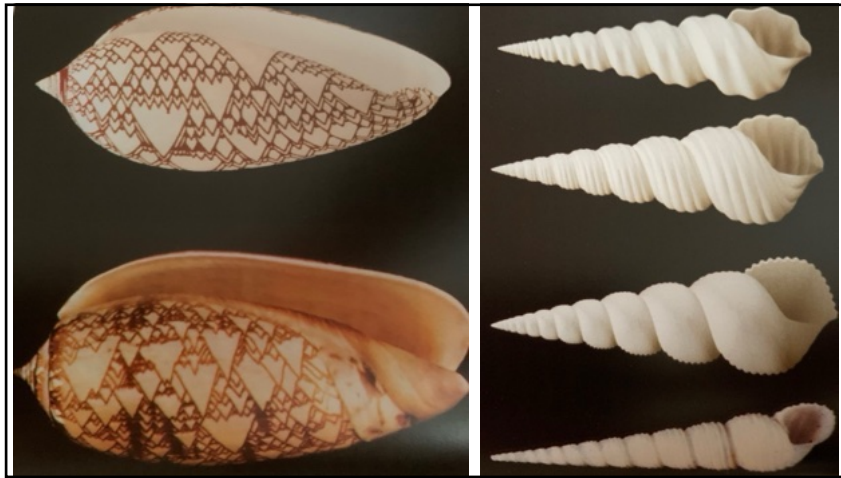
10

FRACTALS

11

Patterns on shells. Some are fractal-like.

12



13

SIERPINSKI TRIANGLES

The Sierpinski triangles contain copies of themselves at smaller scales. The first few stages are shown in the picture. To form the fractal the process is repeated infinitely often.

Stage 0 Stage 1 Stage 2 Stage 3 Stage 4 ...

IN YOUR IMAGINATION
 Repeat the process of joining the midpoints of the edges of the black triangles and removing the inverted triangles at each stage leaving a white space.

Alternatively: Imagine a triangle, reduce it so that the lengths of the edges are halved, imagine two copies and arrange the three triangles with two side by side and one on top as shown. Repeat this process on and on forever to form a fractal.

14

Sierpinski Triangle Stage 5

27 stage 2 triangles like this make the stage 5 Sierpinski triangle.

Stage 6 takes 81 stage 2s
 Stage 7 takes 243 stage 2s and so on for ever.

Imagination can repeatedly shrink the Sierpinski triangles making infinitely many smaller and smaller copies inside each of the triangles in the picture.

In the limiting case, with infinitely many iterations, you get the Sierpinski Fractal.

15

Sierpinski Triangle Stage 6

3 stage 5 triangles like this make the stage 6 Sierpinski triangle.

16

SIERPINSKI TRIANGLE POSTER CLASS ACTIVITY

PREPARE
Depending on the age of the learners, give each learner a stage 2 or stage 3 Sierpinski triangle to colour in.
For very young children, give them each a triangle that is already coloured in.

TO MAKE THE POSTER
Each learner can stick their triangle onto the backing sheet.
Use 9, 27, 81 or 243 triangles or more to make your poster.
Notes for Teachers give templates and advice.

Give each learner a template to colour in.

This orange shaded Sierpinski Triangle is a Stage 4.

How many Stage 3 triangles make stage 4?

How many make Stages 5, 6 and 7?

17

SIERPINSKI TRIANGLE POSTER

PREPARE
Depending on the age of the learners, give each learner a stage 2 or stage 3 Sierpinski triangle to colour in.
For very young children, give them each a triangle that is already coloured in.

TO MAKE THE POSTER
Each learner can stick their triangle onto the backing sheet.
Use 9, 27, 81 or 243 triangles or more to make your poster.
Notes for Teachers give templates and advice.

Give each learner a template to colour in.

The number of stage 3 triangles needed for each stage is given here.

You need a backing sheet at least 120 centimetres wide and 105 high for stage 7 and we recommend 140 by 125 cm or more.

18

QUESTIONS FOR MATHEMATICS LESSONS

Imagine this pattern growing inwards at smaller and smaller scales. The fractal is formed by repeating this process infinitely often.

This Fractal is called the Sierpinski Gasket.

- How many triangles are shaded at each stage?
- What fraction of the area of the gasket is shaded at each stage?
- What fraction is unshaded?
- What scale factors give the similarity (enlargement) at each stage?

In answering these questions what patterns do you notice?

Put answers in table - Worksheet #2

19

SIERPINSKI TRIANGLE

Stage	Number of shaded triangles (black)	Edge length of shaded triangles. Linear scale factor	Fraction of area of one shaded triangle. Area scale factor	Total fraction of area shaded	Fraction unshaded
Stage 0					
Stage 1					
Stage 2					
Stage 3					
Stage 4					
Stage 5					

20

SIERPINSKI TRIANGLE

Stage	Number of shaded triangles	Edge length of shaded triangles. Linear scale factor	Fraction of area of one shaded triangle. Area scale factor	Total fraction of area shaded	Fraction unshaded
Stage 0	1	1	1	1	0
Stage 1	3	1/2	1/4	3/4 = 75%	1/4 = 25%
Stage 2	9	1/4	1/16	9/16 = 56%	7/16 = 44%
Stage 3	27	1/8	1/64	27/64 = 42%	37/64 = 58%
Stage 4	81	1/16	1/256	81/256 = 32%	175/256 = 68%
Stage 5	243	1/32	1/1024	243/1024 = 24%	781/1024 = 76%

Download the pdf of this presentation from <https://aiminghigh.aimssec.ac.za/30-minute-fractals-lesson/> and see follow up slide 49 for more on area.

21

SIERPINSKI TRIANGLE LEARNING OBJECTIVES

DO-TALK-RECORD

By making the Sierpinski triangles, describing what they see, and noticing and talking about the geometrical properties, young learners develop visualization and language skills.

Older learners can investigate number patterns and the numbers of triangles in the sequences of patterns, spot similar triangles and work out scale factors, investigate areas and even sum infinite geometric series to find the total area covered by the fractal.

22

REPTILE FRACTAL

Make an enlargement of this shape by putting the 4 pieces together edge to edge.

The L shape can be enlarged again and again infinitely often.
It can also be repeatedly reduced infinitely often to form a fractal.

23

REPTILE FRACTAL

The Sphinx, another example of a reptile that can be repeatedly filled infinitely often with smaller and smaller copies of itself, and also enlarged similarly to form a fractal.

24

SQUAREFLAKE

1. Start with a square (stage 0)

2. Replace each edge with a zig-zag

3. Repeat step 2 infinitely often.

What happens to the area inside the curve?
How does the perimeter change?

Squareflakes stage 0 (red) and stage 1 black

Squareflake stage 2

25

SQUAREFLAKE TESSELLATION

Can you fit multiple congruent copies of the Stage 1 squareflake together like pieces of a jigsaw to tessellate the plane?

END OF PART 1

26