

**Global Teacher Empowerment Network GTEN**

**PAPER STICK MATHS**  
 Saturday 19 June 2021 16.00 – 18.00 London Time

Toni Beardon      Caroline Ainslie      Mary Achieng

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**PAPER STICK MATHS**

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**PAPER STICK MATHS FOR AGES 4 TO 18+**

To do the activities you will need 18 paper sticks all the same length (about 10 cm)

**Wholesome Rectangles** The theme for all ages is 'making maths with unit lengths'  
<https://aiminghigh.aimssec.ac.za/wholesome-rectangles/>

**Tets and Octs Puzzles**  
<https://aiminghigh.aimssec.ac.za/tets-and-octs-puzzles/>

**Metre Measures**  
<https://aiminghigh.aimssec.ac.za/metre-measures/>

**8 Paper Sticks videos on AIMSSEC YouTube site:**  
<https://www.youtube.com/c/MathsToys>

**THE LEARNING SPIRAL**

- Ages 16 – 18+ **EIGHT TRIANGLES, THREE SQUARES AND POLY PUZZLE TRANSFORMATIONS AND SYMMETRIES IN 2D AND 3D**
- Ages 11 – 15 **LOOPY CHALLENGE 2 AREA, PERIMETER AND FACTORS**
- Ages 7 – 11 **METRE MEASURES AND LIFE SKILLS**
- Ages 4 – 7 **LOOPY CHALLENGE 1 PLAYING WITH SHAPES**
- Starter activity for ages 4 – 18+ **FOUR TRIANGLES PUZZLE**

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**PAPER STICK MATHS STARTER FOR EVERYONE**

Use 5 sticks to make two triangles into a rhombus.  
 Use one more stick to make 4 triangles.

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**PAPER STICK MATHS**  
**ACTIVITIES FOR 4 TO 7 YEAR OLDS**

The children should play with a loop of 12 sticks.


What can you make?  
What shapes?

Talk about the shapes by name as they make them.


If they have made a triangle, say "You've made a triangle, how many edges does it have?"

Can you make a different triangle?  
Encourage the children to talk about the different types of quadrilateral and about other polygons as they make them.

**LOOPY CHALLENGE 1**



The children should play with the **FOUR TRIANGLE PUZZLE**. Start with 6 paper sticks. Can you make 2 triangles as in the diagram? Using one extra stick can you make 4 triangles?



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**Paper Metre Sticks**




MATHS TOYS | GTEN | Bubbly Maths | AIMSSEC AIMING HIGH

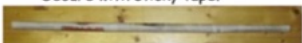
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**PAPER STICK MATHS ACTIVITIES FOR 7 TO 11 YEAR OLDS**  
**METRE MEASURES**

**MAKE A METRE STICK**



Take 8 sheets of newspaper, 4 and 4, fold in 3 as shown. Arrange the paper to make it **EXACTLY ONE METRE** wide. Roll very tightly. Secure with sticky tape.



**Mark your metre stick in 10 cm sections.**  
**Mark one section in centimetres.**


**3 or 4 lessons for ages 7 – 11**  
**Your class can work in pairs, make metre sticks, then:**

- estimate lengths and distances and then take measurements to check their estimates;
- measure classrooms, playground, sports pitches etc.;
- draw accurate scale diagrams;
- measure their heights and draw graphs and work out mean, median and mode.

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**PAPER STICK MATHS**  
**IMPORTANT LIFE-SKILLS**

**METRE MEASURES**  
<https://aimshigh.aimssec.ac.uk/metre-measures/>



Roll up newspaper. Make metre sticks. Your class can measure their heights, or the classroom, playground or football pitch and draw plans.

**ACTIVITY FOR ALL**  
How good are you at judging lengths and distances? Make some guesses and check by measuring. Practice to improve your performance.

- Estimating lengths and distances
- Interpreting scale on maps and plans
- Drawing plans to scale for designing objects to be made.
- Reading & interpreting information from scale diagrams.
- Collecting data for statistical analysis and studies.

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**PAPER STICK MATHS ACTIVITIES FOR 9 TO 16 YEAR OLDS**  
**Loopy Challenge 2**

Use your loop of 12 sticks to find all the different rectangles that are possible with a perimeter of 12 units.

Imagine making rectangles from loops of different lengths.  
 How many can you make with a perimeter of 14 units? What about 16 units or 18 units?  
 If you don't have the sticks, then sketch the shapes.

Young children should have lots of practical experience with the loops of paper sticks.  
 Learners will make progress until they no longer need the loops.

Rectangles with Perimeter of 12 Units

3 x 3 Rectangle (Square)      2 x 4 Rectangle      1 x 5 Rectangle

Find the area of each rectangle.

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**PAPER STICK MATHS LESSONS FOR 11 – 16 YEAR OLDS**  
**LEARNERS WORK IN GROUPS & SHARE IDEAS WITH OTHER GROUPS**

Each group will make rectangles from a loop of a **different** length to the other groups and then share their discoveries with the whole class.

Give each group an even number of sticks. Tell them to make a loop with all but one of the sticks. Can they make a rectangle with an odd number of sticks?

Tell them to add the remaining stick so their loop has an even number of sticks and to make as many **different rectangles as possible**.

Then for each of their rectangles tell them to write down its area and perimeter.

**DIFFERENTIATION:** Low attainers can be given loops for numbers that don't have many factors and so have few possible rectangles. High attainers can be given loops for which it is more of a challenge to find all the factors.

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**PAPER STICK MATHS LESSONS CONTINUED**  
**LESSON ON AREA AND PERIMETER AGES 11 – 16**

This table shows a way to record discoveries.  
 Each cell shows all the rectangles with the same perimeter.

<b>Breadth</b>	1	1	1	2	1	2	1	2	3	1	2	3	4	1	2	3	4	...			
<b>Length</b>	1	2	3	2	4	3	5	4	3	6	5	4	7	6	5	4	8	7	6	5	...
<b>Perimeter</b>	4	6	8	8	10	10	12	12	12	14	14	14	16	16	16	16	18	18	18	18	20
<b>Area</b>	1	2	3	4	4	6	5	8	9	6	10	12	7	12	15	16	8	14	18	20	

The class could share their discoveries and make a poster.

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**PAPER STICK MATHS LESSONS FOR 11 – 16 CONTINUED**

This table shows a way to record discoveries.  
 Each cell shows all the rectangles with the same perimeter.

<b>Breadth</b>	1	1	1	2	1	2	1	2	3	1	2	3	4	1	2	3	4	...			
<b>Length</b>	1	2	3	2	4	3	5	4	3	6	5	4	7	6	5	4	8	7	6	5	...
<b>Perimeter</b>	4	6	8	8	10	10	12	12	12	14	14	14	16	16	16	16	18	18	18	18	20
<b>Area</b>	1	2	3	4	4	6	5	8	9	6	10	12	7	12	15	16	8	14	18	20	...

The class could share their discoveries and make a poster.  
 They could add factor bugs to the class poster!

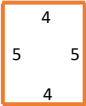
**Think of a whole number.**  
**Draw its factor bug to show pairs of factors.**  
 Antennae show the number. Pairs of legs show other factors.  
 Bugs like 17 and 19 have no legs.  
 12 has 4 legs, 36 has 8 legs.

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**PAPER STICK MATHS LESSONS FOR 11 TO 16 YEAR OLDS CONTINUED**  
**SHAPES WITH A PERIMETER OF 18 UNITS**

What shapes can we make with a perimeter of 18 units using a loop of 18 sticks?  
 How many rectangles? How many triangles? What other shapes?

Hint: To investigate all possible triangles with a perimeter of 18 units break the loop in one place.



If you don't have the sticks, then just sketch the shapes.

What are the perimeters?  
 What are the areas?

Do shapes with the same perimeter have the same area?

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**PAPER STICK MATHS LESSONS FOR 11 TO 16 YEAR OLDS CONTINUED**  
**HOW MANY TRIANGLES?**

What shapes can we make with a loop of 18 sticks?

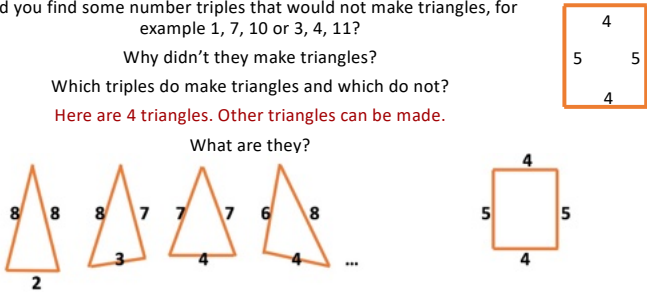
Did you find some number triples that would not make triangles, for example 1, 7, 10 or 3, 4, 11?

Why didn't they make triangles?

Which triples do make triangles and which do not?

Here are 4 triangles. Other triangles can be made.

What are they?




During the break, write the numbers that give the other triangles on the chat.  
 If you have a computer, then you could draw the triangles on the SHARED WHITEBOARD.

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**PAPER STICK MATHS**  
**THE TRIANGLE INEQUALITY**

Try to make a triangle with edges of lengths 10, 1 and 7.  
 Why is it impossible?

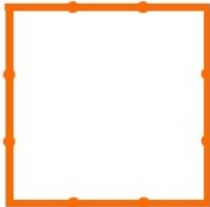


To make a triangle,  
**any two edges need together to be longer than the third edge.**

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**PAPER STICK MATHS LESSON FOR 11 TO 18+**  
**THE EIGHT TRIANGLE PUZZLE**

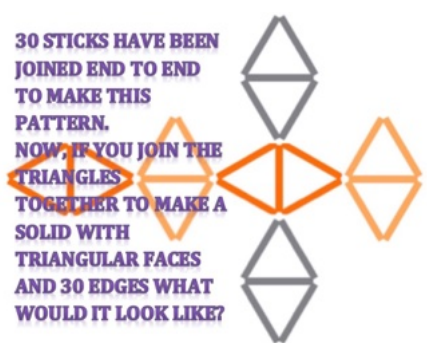

From a loop of 12 sticks make a polyhedron with 8 triangular faces.



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**PAPER STICK MATHS LESSON FOR 11 TO 18+  
THE POLY PUZZLE**

30 STICKS HAVE BEEN JOINED END TO END TO MAKE THIS PATTERN. NOW, IF YOU JOIN THE TRIANGLES TOGETHER TO MAKE A SOLID WITH TRIANGULAR FACES AND 30 EDGES WHAT WOULD IT LOOK LIKE?





The answer to the Poly Puzzle  
30 sticks  
30 edges  
12 vertices  
20 faces.

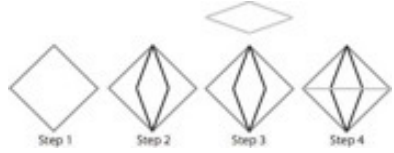
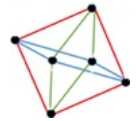
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**PAPER STICK MATHS LESSON FOR 16 TO 18+  
THE THREE SQUARES PUZZLE**

Start with 12 sticks and make with 3 squares.  
Use your 3 squares to make a polyhedron with 8 triangular faces.



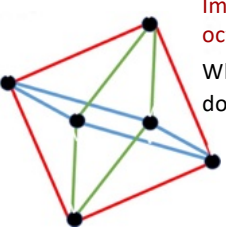
The 3 squares make an OCTAHEDRON.  
What do you notice about it?


12 edges  
6 vertices  
8 faces.

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**PAPER STICK MATHS LESSON FOR 16 TO 18+  
SOME SYMMETRIES**



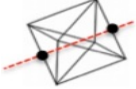
Imagine cutting an octahedron in half.  
What sort of symmetry does that show?



Imagine spinning an octahedron about each of these 3 axes.

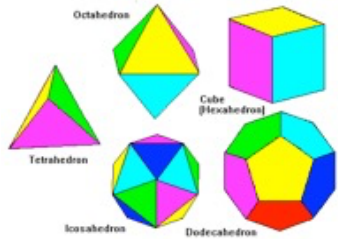



What sort of symmetry does that show?



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**PAPER STICK MATHS LESSON FOR 16 TO 18+  
REGULAR POLYHEDRA**

You can make SKELETONS of these polyhedra and many others from paper sticks.  
To make a display hang them from the ceiling.  
How many strings do you need to use for each one so that, hanging under their own weight, they take a regular shape?

Which polyhedra hold their shape?  
Which structures are rigid?

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**REGULAR POLYHEDRA**

**SEMI REGULAR POLYHEDRA**

**CODES:**  
 Tetrahedron 333      Cube 444  
 Octahedron 3333      Dodecahedron 555  
 Icosahedron 33333

Can you decipher the code?

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**PAPER STICK MATHS LESSON FOR 16 TO 18+  
 APPLICATIONS OF SYMMETRY GROUPS - CRYSTALLOGRAPHY**

The mathematics of *group theory* provides the language needed to describe the symmetries of atoms, molecules and crystals. Group theory explains exactly what symmetries enable a unit cell to pack neatly and repetitively to make a three-dimensional crystal.

See articles by Rachel Thomas in the Plus online magazine:  
<https://plus.maths.org/content/through-looking-glass>  
<https://plus.maths.org/content/shattering-crystal-symmetries>

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**APPLICATIONS OF SYMMETRY GROUPS – PARTICLE PHYSICS**

In 2013, Peter Higgs and Francois Englert were awarded the Nobel prize in Physics. In 1964 they had predicted a new particle using the mathematics of symmetry groups and knowledge of physics. The existence of the particle, called the Higgs Boson, was verified in 2012 at CERN in Switzerland, the European Organisation for Nuclear Research.

**Physicists Just Found 4 New Subatomic Particles That May Test The Laws of Nature**

PATRICK KOPPENBURG, THE CONVERSATION 5 MARCH 2021

This month is a time to celebrate. CERN has just announced the discovery of four brand new particles at the Large Hadron Collider (LHC) in Geneva.

This means that the LHC has now found a total of 59 new particles, in addition to the Nobel prize-winning Higgs boson, since it started colliding protons – particles that make up the atomic nucleus along with neutrons – in 2009.

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**AIMS** African Institute for Mathematical Sciences  
 SCHOOLS ENRICHMENT CENTRE

**Global Teacher Empowerment Network (GTEN)**

Workshops for teachers in primary and secondary schools, colleges and universities.

**Wholesome Rectangles**  
<https://aiminghigh.aimssec.ac.za/wholesome-rectangles/>

**Tets and Octs Puzzles**  
<https://aiminghigh.aimssec.ac.za/tets-and-ocets-puzzles/>

**Icodahedron Puzzle**  
<https://aiminghigh.aimssec.ac.za/icosahedron-puzzle/>

**Metre Measures**  
<https://aiminghigh.aimssec.ac.za/metre-measures/>

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AIMSSEC YouTube Channel  
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**NEW SKILLS  
NEW HOPES  
NEW HORIZONS**  
AIMSSEC for teachers and learners worldwide

**ADDRESSING THE ISSUES**

- Need for in-service training.
- Inequalities in educational opportunities
- Lack of teaching materials to engage learners & develop understanding & skills
- Need for different teaching methods and new skills to thrive in the 21<sup>st</sup> century
- Underqualified teachers in rural and disadvantaged communities.

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**NEW SKILLS  
NEW HOPES  
NEW HORIZONS**  
for educators and students globally

Free lesson resources:  
<http://aiminghigh.aimssec.ac.za>

Collaborative Professional Development  
<https://aiminghigh.aimssec.ac.za/category/cpd>

AIMSSEC Website: <http://aimssec.ac.za>

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Enquire about signing up for an AIMSSEC course as a self-funding student [admin@aimssec.ac.za](mailto:admin@aimssec.ac.za)



**Thanks for participating.**

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