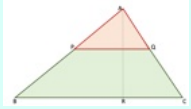
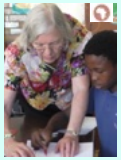
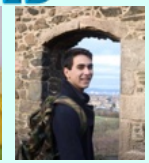
Global Teacher Empowerment Network GTEN

Saturday 22 May 2021 16.00 – 18.00 London Time

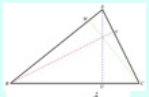
TRI-FOLD

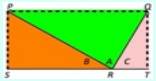


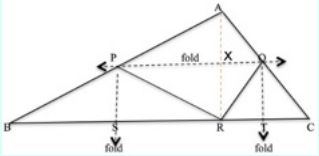


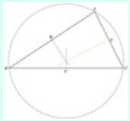


Toni Beardon Caroline Ainslie Jonathan Shipp









1

TRI-FOLD

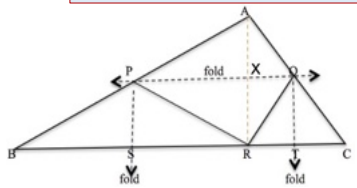
COMMON THEME FROM AGE 4 TO 18 – PAPER FOLDING

DURING THIS WORKSHOP YOU WILL:

- ✓ Bring your cut-out triangles (any size and angles you wish)
- ✓ Make a few simple folds
- ✓ Work through practical activities that will help your students to discover geometrical properties by themselves
- ✓ Explore properties of reflections and enlargements
- ✓ Investigate proofs of geometry theorems

You need to have ready:

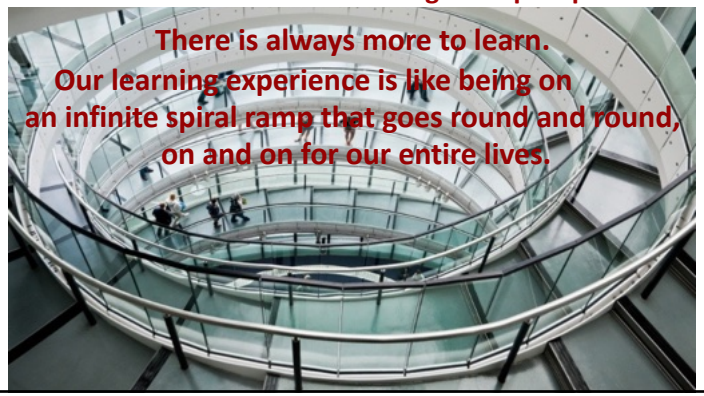
- 5 different paper triangles, any shape (pre-cut),
- 1 triangle cut out of cardboard,
- Paper, Pencil, Ruler, Eraser,
- Compasses,
- 3 Colouring pens or pencils



2

TRI-FOLD

Build skills and understanding on a spiral path



3

WHAT AIMSSEC OFFERS YOU FOR SPIRAL LEARNING

All AIMSSEC resources are available free under a Creative Commons License on the AIMING HIGH website and on the AIMSSEC App for use offline.

The spiral continues School leaving year

For **lessons in school** AIMSSEC resources provide activities for Differentiation and Inclusion.

For **home-learning** AIMSSEC resources provide for
* Mixed Age Groups
* A common starter & MATHEMATICAL THEME
* Age-appropriate learning activities for all.

Pre - school

4

SPIRAL LEARNING AND TRI-FOLD

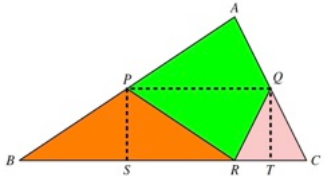
In Primary School we lay the foundations for later learning.

We plant the seeds, and we begin to cultivate,

insights, visualization and understanding of mathematics.

Practical experiences underpin later abstract thinking.

In Secondary School we build understanding and skills for lifelong learning.



A picture is worth a thousand words.

5

TRI-FOLD * EARLY YEARS * FREE PLAY

One photo shows some triangles cut from the page of a magazine.
The other two photos show the same triangles with
one fold and with two folds.

Ages 4 - 6: Give the children some paper triangles. Ask them to:
Make **one** fold anywhere in the triangle.
Ask: What happens? What shapes can you make?

Ages 6 – 8: Also ask them to make **two** folds anywhere in the triangle.
Ask: What happens? What shapes can you make?
Play with your folded paper triangles.
Talk about what you notice about them.
Perhaps colour them and stick them in your book or on a poster.
Ask: Can you make a fold giving 2 triangles? What about 3 triangles?
What other shapes can you make?
What do you notice about your shapes?

6

TRI-FOLD * EARLY YEARS

Grown-ups, teachers and parents: This should be free play with some talk about the shapes and very little formal teaching.

Learning about the different types of triangle, and different quadrilaterals, comes much later when the children are older.

Talk with the children about the shapes they make. Ask questions to encourage them to talk about quadrilaterals they have made and their properties. Give them the names if any of these **special quadrilaterals** like squares, rectangles, kites, rhombuses, parallelograms and trapezia (trapezoids if you are American).

Ask: "What's the same and what's different?" about the shapes.

7

TRI-FOLD FREE PLAY
Creature made from 2 triangles

8

TRI-FOLD

Here the same activity can be directed to making named shapes.

Taking the **special quadrilaterals** one by one, show the children pictures and ask them to make squares, rectangles, kites, rhombuses, parallelograms and trapezia (trapezoids if you are American).

For example, can they make a square by folding a triangle?
How many folds?
What is the shape of the bit of the triangle that sticks out?
(*Answer, in this picture it's a right angled triangle.)*
Is there more than one way to make a square?

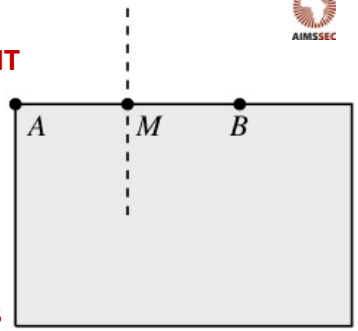
Ask questions to encourage them to talk about these quadrilaterals and their properties. Ask: "What's the same and what's different?" about the shapes.

9

TRI-FOLD

HOW TO HALVE A LINE SEGMENT

To find the midpoint M of a line segment AB fold A onto B making a fold-line (shown as a dashed line) and mark mid-point M.



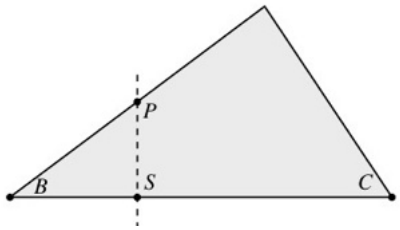
The same folding method works for halving any line segment.
You can prick a hole at points A and B to help you align them when folding (not necessary for points on an edge of the paper).

10

TRI-FOLD

HOW TO DRAW A PERPENDICULAR FROM A POINT TO A LINE

To drop a perpendicular PS from a point P onto a line BC, take point B, make a fold through P, keep B on the line BC, and mark the fold line.



What do you notice about the angles ∠PSB and ∠PSC?
Explain your ideas.

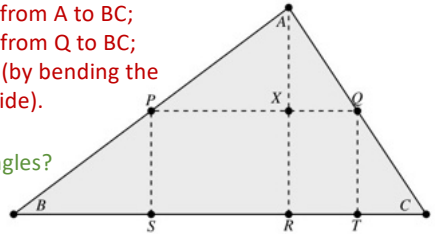
11

TRI-FOLD TRIANGLE INVESTIGATION

Label your triangle ABC then make folds to:
(1) find and mark the midpoint P of AB;
(2) find and mark the midpoint Q of AC;
(3) fold along *PQ*. What happens to the point *A*?
(4) construct the perpendicular PS from P to BC;
(5) construct the perpendicular AR from A to BC;
(6) construct the perpendicular QT from Q to BC;
(7) check that lengths PS = XR = QT (by bending the paper until the line segments coincide).

You will have 5 minutes to do the investigation. Write comments, questions and answers in the chat line.

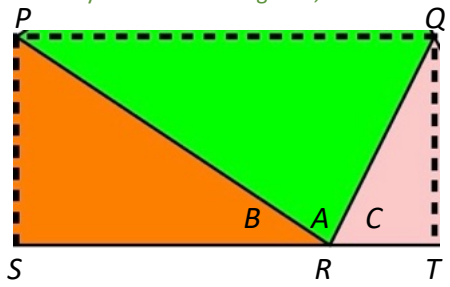
Find all the right angles.
How do you know they are right angles?
What shape is PSTQ?
How do you know?



12

TRI-FOLD
FOLDING AND ANGLES

Make 3 folds on *PQ*, *PS* and *QT* as in the diagram.
What happens?
What does this tell you about the angles *A*, *B* and *C*?



This proves that the angles of a triangle add up to 180°

13

TRI-FOLD
FOLDING TO COMPARE LENGTHS

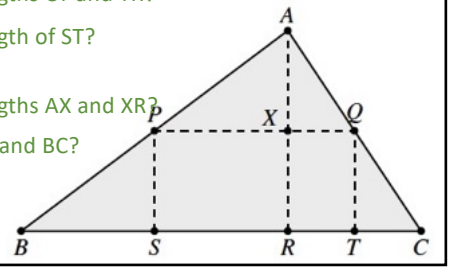
Fold along PS.
What does this tell you about the lengths BS and SR?

Fold along QT.
What does this tell you about the lengths CT and TR?

What does this tell you about the length of ST?

Fold along PQ.
What does this tell you about the lengths AX and XR?

What can you say about the lines PQ and BC?



14

TRI-FOLD FOLDING & THE AREA OF A TRIANGLE

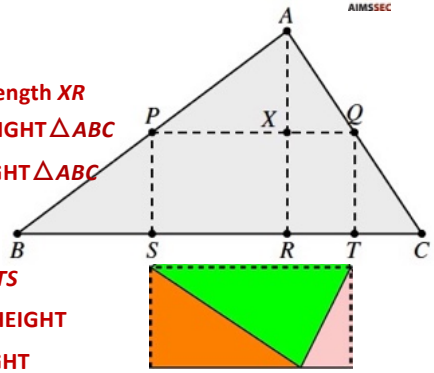
Length $ST = \frac{1}{2}$ BASE $\triangle ABC$

Length $XR = \frac{1}{2}$ HEIGHT $\triangle ABC$

Area rectangle $PQTS$ = Length ST × Length XR

Area rectangle $PQTS = \frac{1}{2}$ BASE $\times \frac{1}{2}$ HEIGHT $\triangle ABC$

Area rectangle $PQTS = \frac{1}{4}$ BASE × HEIGHT $\triangle ABC$



Area $\triangle ABC$ = 2 × Area rectangle $PQTS$

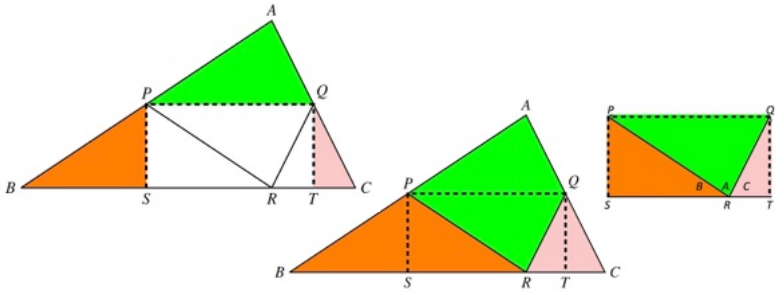
$= 2 \times \frac{1}{4}$ BASE × HEIGHT

$= \frac{1}{2}$ BASE × HEIGHT

15

TRI-FOLD
FOLDING, REFLECTION AND CONGRUENT TRIANGLES

Colour triangle *APQ* and its mirror image. Where is the mirror line?
Colour triangle *PBS* and its mirror image. Where is the mirror line?
Colour triangle *QTC* and its mirror image. Where is the mirror line?



16

TRI-FOLD
FOLDING, ENLARGEMENTS AND SIMILAR TRIANGLES

Look at the triangle *APQ.*

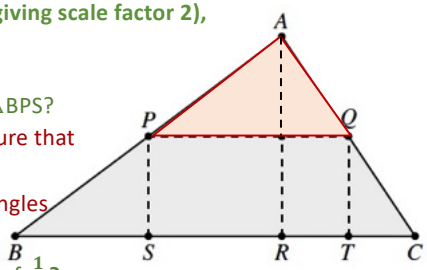
If △APQ is enlarged from centre *A* so that the image points are **twice the distance from A (giving scale factor 2),** what would the image be?

Can you see an enlargement of △BPS?

Find another triangle in the picture that has an enlarged image?

Describe the pairs of similar triangles that you can see in the diagram.

What happens for a scale factor of $\frac{1}{2}$?



17

TRI-FOLD – TO PROVE THE RATIO THEOREM

1. Draw △ABC.
2. Fold the baseline BC back onto itself so that the fold line goes through point A. Mark the point R at the base of the fold. AR is called an altitude of the triangle.
3. Make a fold by placing point A anywhere on AR. Mark the fold-line PQ with P and Q on the edges of △*ABC.*
4. What do you notice about the lines AR and PQ?

(*Answer: By construction, AR and PQ are perpendicular.*)

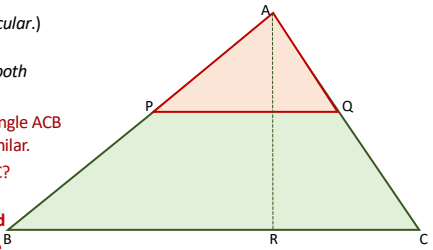
5. What do you notice about the lines PQ and BC?

(*Answer: By construction, PQ and BC are parallel, both perpendicular to AR.*)

6. Notice angle APQ = angle ABC and angle AQP=angle ACB (corresponding angles) so △*ABC* and △*APQ* are similar.
7. What else do you notice about △APQ and △ABC?

(*Answer: △ABC is an enlargement of △APQ).*

8. **We have shown that *PQ* and *BC* are parallel and △*APQ* and △*ABC* are similar, so it follows that the three ratios *AP:AB, AQ:AC* and *PQ:BC* are equal (each ratio is the SCALE FACTOR of the enlargement).**



18

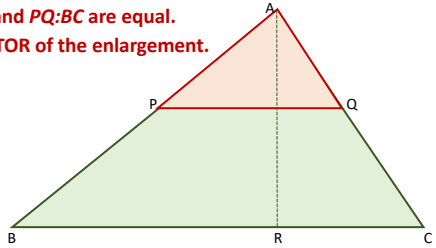
TRI-FOLD – PROOF OF THE RATIO THEOREM SUMMARY

By construction, AR and PQ are perpendicular.

By construction, PQ and BC are parallel, both perpendicular to AR.

△ABC is an enlargement of △APQ .

We have shown that *PQ* and *BC* are parallel and △*APQ* and △*ABC* are similar, so it follows that the three ratios *AP:AB, AQ:AC* and *PQ:BC* are equal. Each ratio is the SCALE FACTOR of the enlargement.



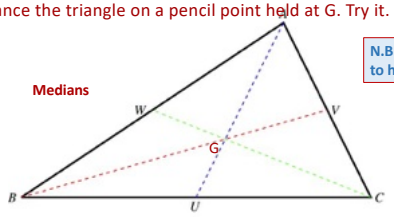
19

TRI-FOLD TO FIND THE MEDIANS OF A TRIANGLE

THE NEXT 4 DRAWING AND CONSTRUCTION ACTIVITIES CAN BE DONE AFTER THIS WORKSHOP. LET'S DISCUSS THEM IN THE GTEN TEACHERS' WHATSAPP GROUP.

1. Draw triangle ABC (which should be cut out of cardboard).
2. Fold to find U the mid-point of BC.
3. Fold along AU and draw the median AU.
4. Repeat to find V and W and the medians BV and CW.
5. What do you notice about the medians that you have found by folding?
6. You can balance the triangle on a pencil point held at G. Try it.

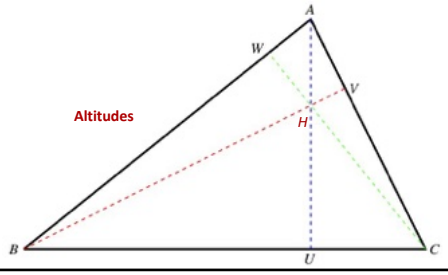
N.B. You can prick holes through points to help you align them when folding.



20

TRI-FOLD TO FIND THE ALTITUDES OF A TRIANGLE

1. Draw triangle ABC (which can be cut out).
2. Fold to find AU the perpendicular from A to BC (one of the altitudes of triangle ABC).
3. Repeat to find the altitudes BV and CW.
4. What do you notice about the altitudes that you have found by folding?

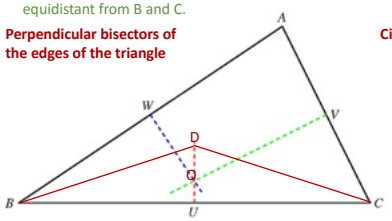


21

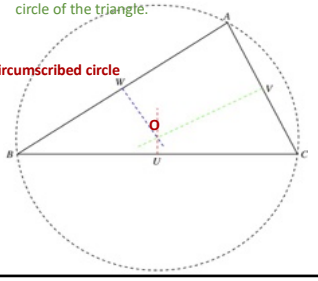
TRI-FOLD TO FIND THE PERPENDICULAR BISECTORS OF THE EDGES OF THE TRIANGLE

1. Draw triangle ABC. (Don't cut this one out)
2. Fold to find U the mid-point of BC and mark this fold line.
3. Repeat to find V and W the midpoints of AC and AB.
4. What do you notice about the fold lines?
5. Label any point D on the on the red fold line through U, the perpendicular bisector.
6. Explain why all points on the perpendicular bisector are equidistant from B and C.
7. Label the intersection of the perpendicular bisectors as O
8. With your compass point at O, draw a circle centre O through A.
9. What do you notice about the circle?
10. Explain why this construction gives the circumscribed circle of the triangle.

Perpendicular bisectors of the edges of the triangle



Circumscribed circle

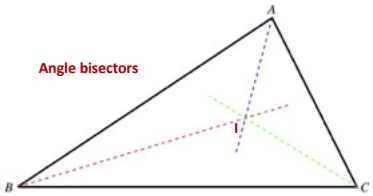


22

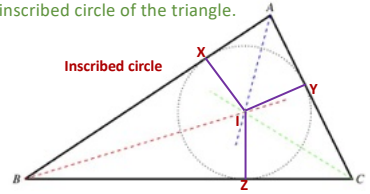
TRI-FOLD TO FIND ANGLE BISECTORS

1. Draw triangle ABC (which can be cut out).
2. Fold through A so that AB goes onto AC.
3. Repeat for B and C.
4. What do you notice about the angles?
5. What do you notice about the fold lines?
6. Label the intersection of the angle bisectors as point I
7. With your compass point at I, draw a circle centre I that just touches the edge AB. Label this point X.
8. What do you notice about the circle?
9. Explain why this construction gives the inscribed circle of the triangle.

Angle bisectors



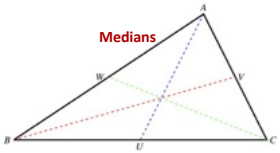
Inscribed circle



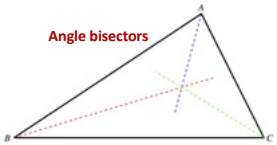
23

TRI-FOLD
FOLDING TO FIND CONCURRENT LINES IN TRIANGLES

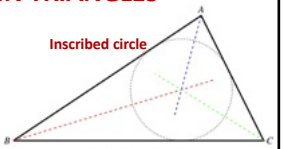
Medians



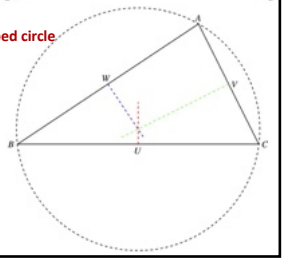
Angle bisectors



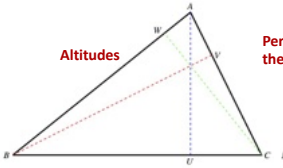
Inscribed circle



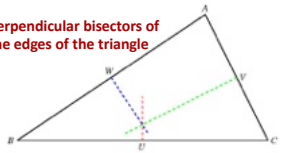
Circumscribed circle



Altitudes



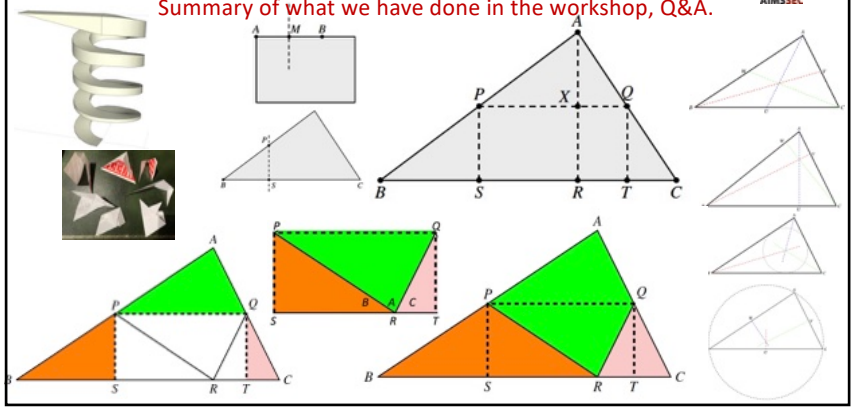
Perpendicular bisectors of the edges of the triangle



24

TRI-FOLD

Summary of what we have done in the workshop, Q&A.



25

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

Global Teacher Empowerment Network (GTEN)

For teachers in primary and secondary schools, colleges and universities.

TRI-FOLD AND OTHER RESOURCES

TRI-FOLD https://aiminghigh.aimssec.ac.za/tri-fold/

ANGLE SUM https://aiminghigh.aimssec.ac.za/angle-sum/

Free lesson resources: http://aiminghigh.aimssec.ac.za

AIMSSEC APP: https://aimssec.app (to download the resources and use resources offline)

AIMSSEC Website: http://aimssec.ac.za

COLLABORATIVE PROFESSIONAL DEVELOPMENT https://aiminghigh.aimssec.ac.za/category/cpd

MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS

https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/

26

LET'S PLAY MATHEMATICALLY AND LEARN

Order from AMAZON or TARQUIN https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically

Play Mathematically

- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

27

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

Thanks for coming to this workshop.
Use the AIMSSEC ideas on AIMING HIGH and add comments.
Share what you have learned with other teachers.
Try to help all your learners to have a 'Yes I Can' attitude to mathematics.

28