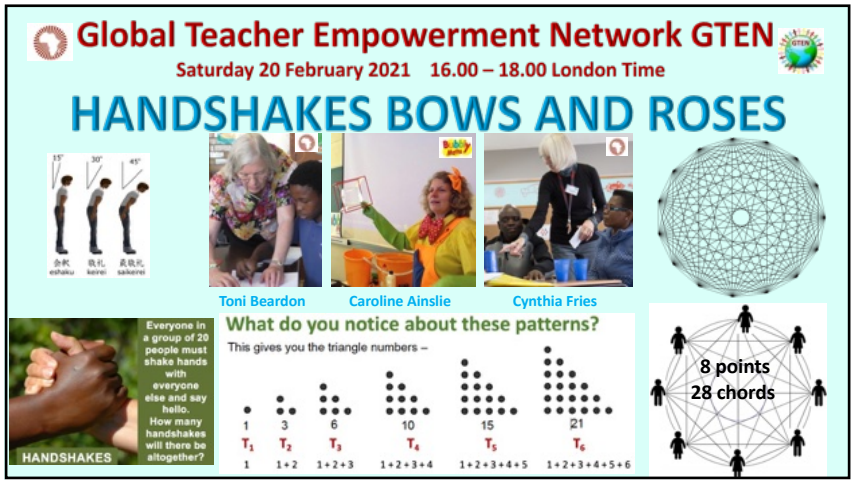
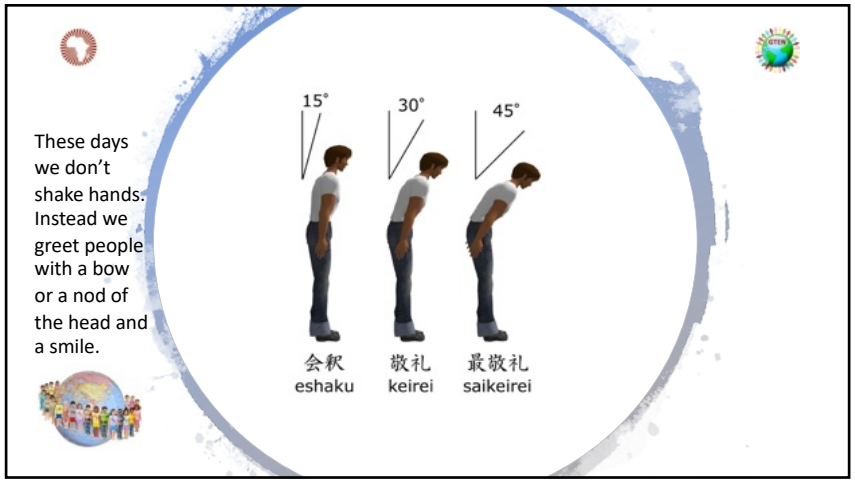

Global Teacher Empowerment Network GTEN
Saturday 20 February 2021 16.00 – 18.00 London Time
HANDSHAKES BOWS AND ROSES
Toni Beardon
Caroline Ainslie
Cynthia Fries
Everyone in a group of 20 people must shake hands with everyone else and say hello. How many handshakes will there be altogether?
HANDSHAKES
What do you notice about these patterns?
This gives you the triangle numbers –
1 3 6 10 15 21
T1 T2 T3 T4 T5 T6
1 1+2 1+2+3 1+2+3+4 1+2+3+4+5 1+2+3+4+5+6
8 points
28 chords

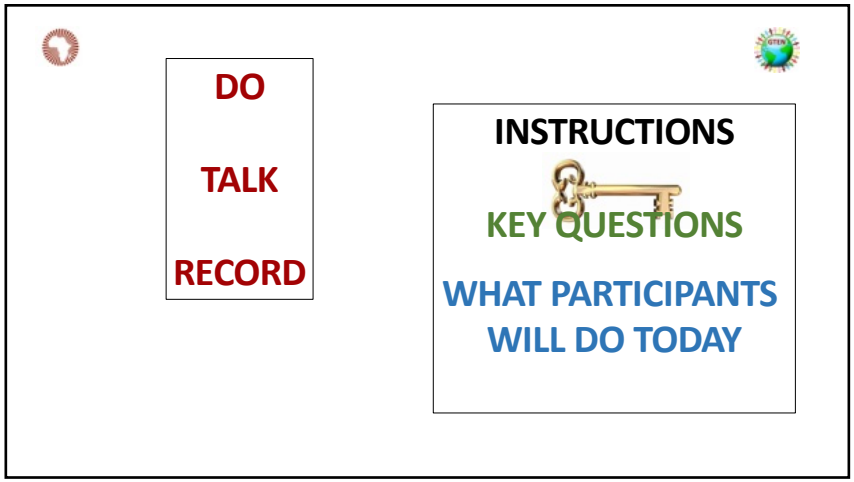
1


GTEN workshop Handshakes, Bows & Roses
https://aiminghigh.aimssec.ac.za/handshakes
Saturday 20 February 2021 16.00-18.00 London time
HANDSHAKES
Everyone in a group of 20 people must shake hands with everyone else and say hello. How many handshakes will there be altogether?
The first hour is about teaching primary and pre-school maths, the second hour about secondary maths, all on the same theme.

2

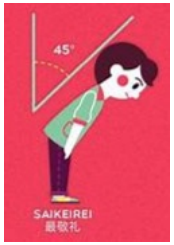

These days we don't shake hands. Instead we greet people with a bow or a nod of the head and a smile.
15°
30°
45°
会釈 eshaku
敬礼 keirei
最敬礼 saikeirei

3


DO
TALK
RECORD
INSTRUCTIONS
KEY QUESTIONS
WHAT PARTICIPANTS WILL DO TODAY

4

PRE-SCHOOL



Get the children to talk about greetings (handshakes, touching elbows, hugging, bowing etc). Practise the Japanese bow.

Encourage them to discuss the difference between saying hello to one person at a time and saying hello to everyone, either verbally or with a greeting like a handshake.

Introduce the idea of everyone in the room greeting everyone else.

Ask how we could find out how many greetings that would be.

Try this out, either with people or get a few soft toys and enact (play) the scene. First use 2 soft toys, then 3, then 4.

Only continue as long as the children enjoy it.

Perhaps come back to the same 'game' on other occasions.

DO Bowing	**TALK** about what they are doing	**RECORD** Draw a picture

5

LEARNING OBJECTIVES FOR PRE-SCHOOL CHILDREN

THEME: NUMBER AND SHAPE PATTERNS

LEARNING OBJECTIVES:

- simple counting;
- applying maths (counting) to ordinary life (greetings).

GENERIC COMPETENCES:

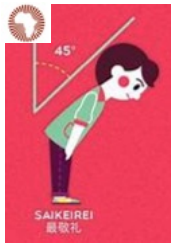
- connecting actions and ideas;
- developing communication skills.

6

LOWER PRIMARY AGES 5/6 TO 8

7

LOWER PRIMARY



Get the children to talk about greetings (handshakes, bowing, elbow touching, hugging, etc). They should try out the Saikeirei and talk about bowing at different angles.

Introduce the idea of everyone in the room greeting everyone else. Ask **"How will you find out the number of greetings?"** Make sure everyone understands that this means counting all the one-to-one greetings, that is looking directly at one other person and bowing to them.

DO: See the next slide then try this out as PEOPLE MATHS, first with 2 people, then 3, then 4, then 5.

The learners should talk about the problem in groups. They should see if they can think of any ideas about how to work out the number of greetings for 6 people without actually counting them.

Groups who find the answer for 6 people, should then try to find the answer for 20.

8

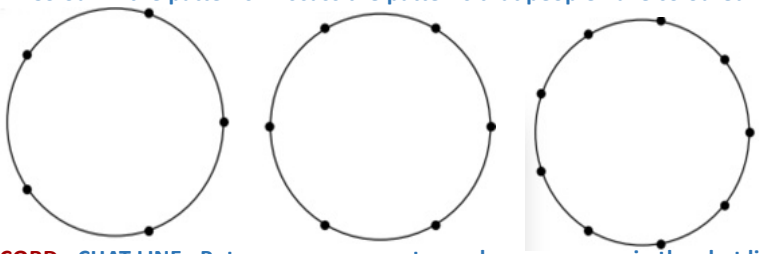
JOIN THE DOTS

DO: Join each dot to all the dots on the same circle.

TALK about: What patterns do you see?

What is the same and what is different about the patterns?

Colour in the patterns. Discuss the patterns that people have coloured.



RECORD - CHAT LINE: Put your name, country and your answers in the chat line.

9

LEARNING OBJECTIVES FOR LOWER PRIMARY LEARNERS

THEME: NUMBER AND SHAPE PATTERNS

LEARNING OBJECTIVES Learners have the opportunity to:

- investigate and extend number and geometry patterns;
- develop visualization skills;
- describe observed relationships or rules in learners' own words.

GENERIC COMPETENCES: Learners have the opportunity to:

- describe logical connections between their actions and ideas;
- think creatively and logically;
- develop communication skills.

10

UPPER PRIMARY AGES 8 TO 11

11

12

13

UPPER PRIMARY - HANDSHAKES

DO & TALK *Either* try it if you can make up groups of 5 people.

RECORD *Or* draw 5 dots around a circle and draw chords.

Join each dot to all the other dots.

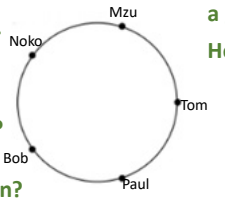
Imagine each chord represents a greeting. How many chords?

How many greetings?

Four children all greet each other one-to-one.

How many greetings is that?

What about a group of 5 children?



How many greetings would there be if everyone in your class greeted everyone else?

14

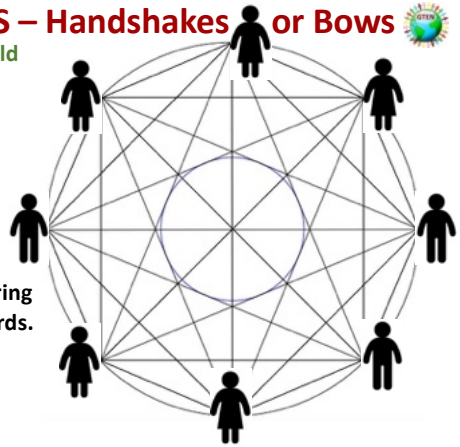
PEOPLE MATHS – Handshakes or Bows

How many handshakes would there be with 8 people?

DO Stand in a circle. Suppose each chord represents one handshake.

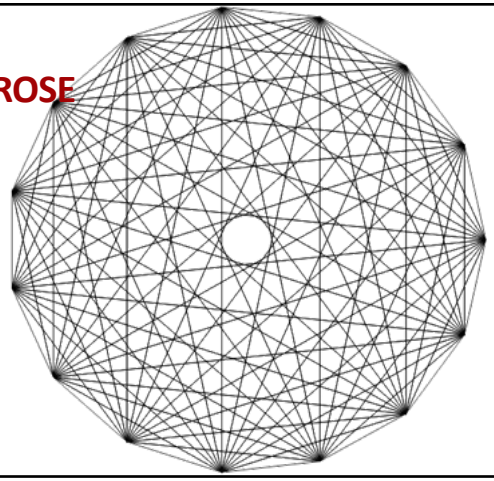
With a few people and a ball of string you can try this and count the chords.

TALK With 8 people how many handshakes?



15

MYSTIC ROSE



What do you see?

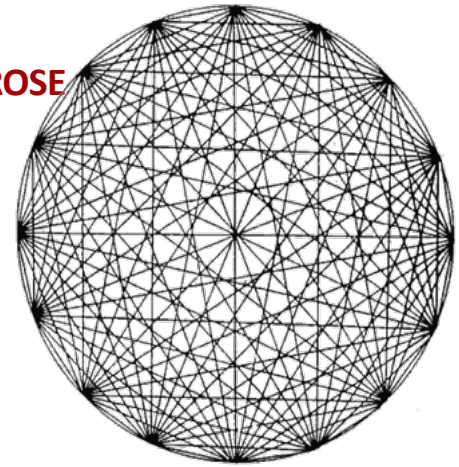
Describe this design.

How would you draw this design?

How many points on the circumference of the circle?

16

MYSTIC ROSE



What do you see now?

Describe this design.

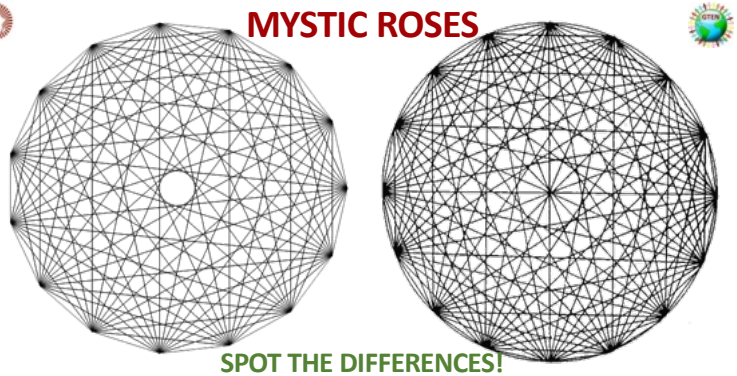
How would you draw this design?

How many points on the circumference of the circle?

16

17

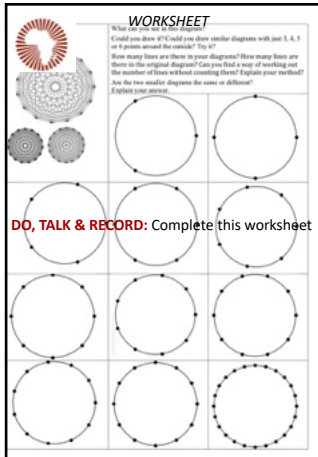
MYSTIC ROSES



SPOT THE DIFFERENCES!

How many points on the circumferences of these circles?

18



DO, TALK & RECORD: Complete this worksheet

KEY QUESTIONS

Ask these questions to guide learning

- Suppose you only have 3 points on the circle and every point is joined to every other point, then how many lines would there be?
- What about the number of lines for 4 points, or 5 points, or 6 points …?
- How many points are there around that circle?
- Can you count the number of straight lines (chords) in that diagram?
- If you had 100 points you would not want to count the lines, but could you work out how many lines there should be?

19

HANDSHAKES, BOWS & ROSES

Number of people	Number of greetings or chords
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	

KEY QUESTIONS Ask these questions to guide learning

If you know the number of people, how do you work out the number of greetings?

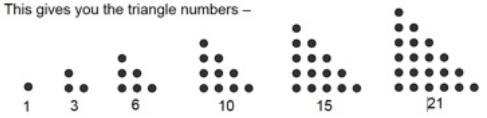
Fill in the rest of the table. Can you see a pattern?

Can you explain the pattern in the sequence of numbers?

How many greetings would there be for 20 people?
What about 100 people?

Can you explain how you worked out the number of greetings?

This gives you the triangle numbers –



20

HANDSHAKES, BOWS & ROSES – 2 methods

METHOD 1

20 people each greet 19 others giving 20 × 19 = 380 greetings.

Each greeting is counted twice so the number of greetings is 380/2 = 190

METHOD 2

The 2nd person to enter the room greets 1 person.
The 3rd person to enter the room greets 2 people.
The 4th greets 3 people,
The 5th greets 4 people… and so on.

To get the number of greetings for 20 people you add
1 + 2 + 3 + 4 +… + 19 = 190

In a class of 11 year olds typically some groups will use Method 1 and other groups Method 2.

If your class finds patterns not answers then it is often better not to tell them the answer but rather to leave the investigation unfinished and come back to it at a later time.

21

UPPER PRIMARY

THEME: NUMBER AND SHAPE PATTERNS

LEARNING OBJECTIVES: Learners have the opportunity to:

- develop visualization skills;
- investigate and extend number and geometry patterns looking for relationships and the rules defining the patterns;
- describe observed relationships or rules in learners' own words;
- solve problems involving whole numbers and number patterns.

GENERIC COMPETENCES: Learners have the opportunity to:

- think creatively and logically;
- develop communication skills.

22

SECONDARY 11 - 16

23

LOWER & UPPER SECONDARY Handshakes or Bows

DO

TALK

RECORD

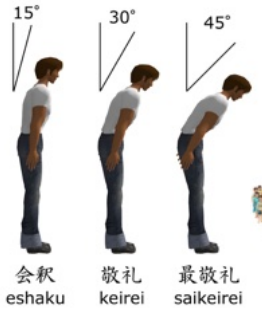
INSTRUCTIONS

KEY QUESTIONS

WHAT PARTICIPANTS WILL DO TODAY

24

LOWER & UPPER SECONDARY Handshakes or Bows



25

LOWER & UPPER SECONDARY Handshakes or Bows

Introduce the idea of everyone in the room greeting everyone else. Make sure everyone understands that this means counting all the one-to-one greetings making eye contact first.

This is like a Football League where every team plays every other team once.

Ask "How would you find the number of greetings?

TALK: The learners should talk about the problem in groups. They should see if they can think of any ideas about how to work out the number of greetings for 6 people without actually counting them.

If a group finds the answer for 6 people, then they should try to find the answer for 20.

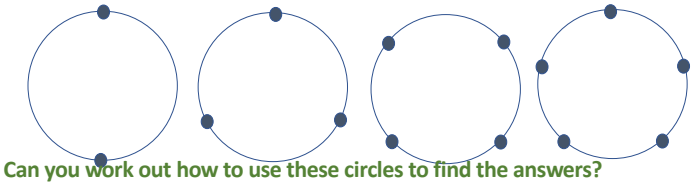
DO: See the next 2 slides then try this out as PEOPLE MATHS, first with 2 people, then 3, then 4, then 5.

26

LOWER & UPPER SECONDARY Handshakes or Bows

TALK: How many greetings would there be with 2 people?

What about 3 people? Or 4? Or 5?



Can you work out how to use these circles to find the answers?

DO: Try this out as PEOPLE MATHS, first with 2 people, then 3, then 4, then 5 or find the answers using these circles. If you have done this before then colour your patterns. What does your colouring show?

Put your name, your country and your 4 answers on the chatline.

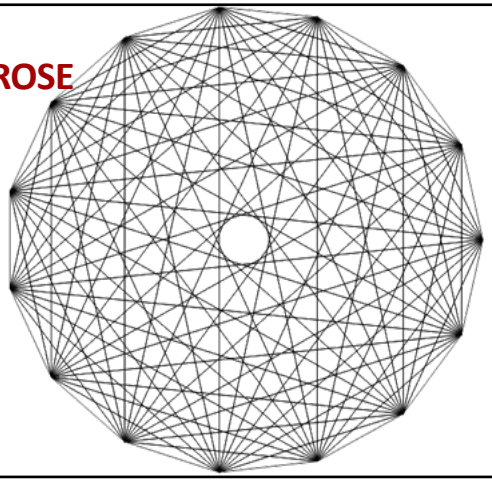
27

KEY QUESTIONS-Ask these questions to guide learning

- **How can you be sure everyone has shaken hands with everyone else once and only once?**
- **How many handshakes would there be between 3 people? Or between 4 people?**
- **Can you use the same reasoning to find the answer for the whole class?**
- **Can you explain how you found your answer?**
- **Could you find the number of handshakes for the whole school without actually counting them?**
- **What is special about the numbers of handshakes in different sized classes?**

28

MYSTIC ROSE



What do you see?

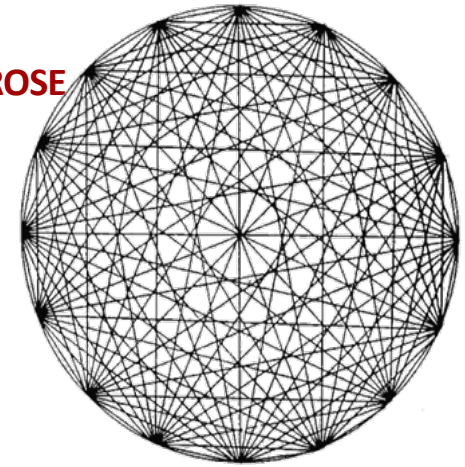
Describe this design.

How would you draw this design?

How many points on the circumference of the circle?

29

MYSTIC ROSE



What do you see now?

Describe this design.

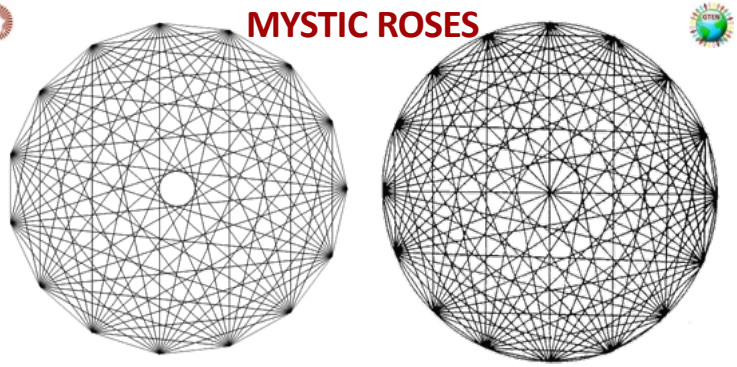
How would you draw this design?

How many points on the circumference of the circle?

16

30

MYSTIC ROSES



SPOT THE DIFFERENCES!

How many points on the circumferences of these circles?

31

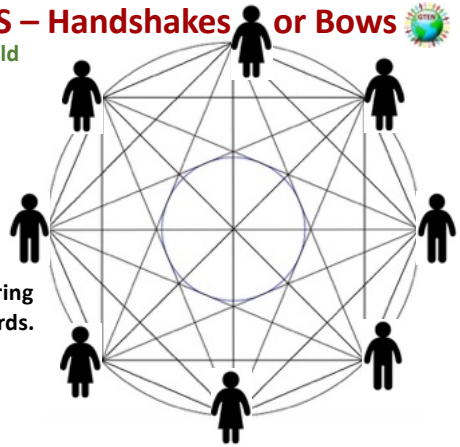
PEOPLE MATHS – Handshakes or Bows

How many handshakes would there be with 8 people?

DO: Stand in a circle. Suppose each chord represents one handshake.

With a few people and a ball of string you can try this and count the chords.

TALK: With 8 people how many handshakes?

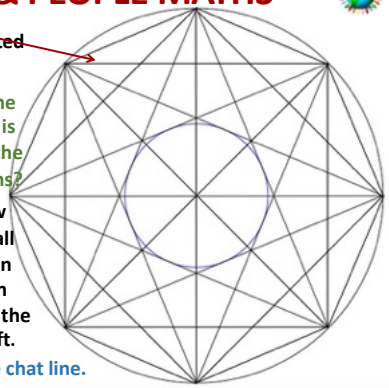


32

MYSTIC ROSES & PEOPLE MATHS

This is not a Mystic Rose
These are (8, 3)
Poncelet Circles

This is an 8-pointed Mystic Rose.



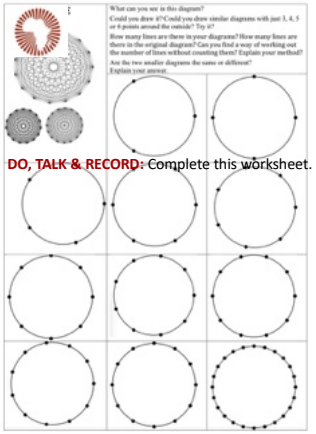
TALK: What is the same and what is different about the two constructions?

DO: With a few people and a ball of string you can make your own Mystic Roses like the group on the left.

DO & RECORD: Answer this question on the chat line.

Can you make connection between the mystic rose and handshake problem?

33



DO, TALK & RECORD: Complete this worksheet.

KEY QUESTIONS
Ask these questions to guide learning

- Suppose you only have 3 points on the circle and every point is joined to every other point, then how many lines would there be?
- What about the number of lines for 4 points, or 5 points, or 6 points …
- How many points are there around that circle?
- Can you count the number of straight lines (chords) your diagram?
- If you had 100 points you would not want to count the lines but could you work out how many lines there would be?

34

MYSTIC ROSES

Number of points	Number of chords
2	1
3	3
4	6
5	10
6	15
7	21
8	
9	
10	

Sequences

How many red balls would be in the fifth picture?

Term in the sequence: 1 2 3 4 5

Number of balls: 1 3 6 10 ?

TALK: Discuss the patterns and connections.

35

LOWER SECONDARY 11 - 16

THEME: NUMBER AND SHAPE PATTERNS

LEARNING OBJECTIVES: Learners have the opportunity to:

- develop visualization skills;
- investigate and extend number and geometry patterns looking for relationships between numbers;
- describe and justify the general rules for the observed relationships in the learner's own words or in algebraic language;
- solve problems involving whole numbers and number patterns.

GENERIC COMPETENCES: Learners have the opportunity to:

- think creatively and reason logically;
- be creative and innovative;
- apply knowledge and skills;
- solve and interpret problems;
- develop communication skills.

36

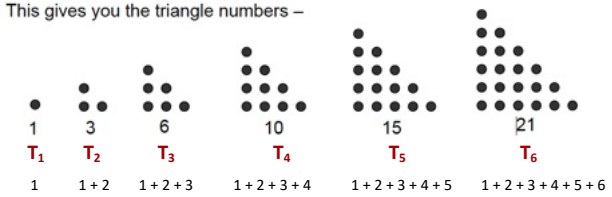
UPPER SECONDARY 16+

37

UPPER SECONDARY 16+ If the students have not met it before then introduce the problem as for Lower Secondary

What do you notice about these patterns?

This gives you the triangle numbers –



How many dots in T_{100}?

38

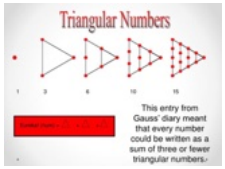
Clever Carl

Numbers	1	2	3	4	5	…	99	100
Reversed	100	99	98	97	96	…	2	1
Totals	**101**	**101**	**101**	**101**	**101**		**101**	**101**

$$T_{100} = \frac{1}{2}(100 \times 101) = 5050$$

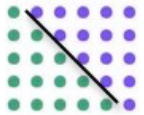
DO: Give the students the table with many gaps to fill in. Then ask them what they notice about it. Tell them the story and discuss the method.

Answer on the chatline: "What Key Questions would you ask your students?"



39

DO Think: WHAT DO YOU NOTICE IN THIS PICTURE?



Two T_5 triangle numbers here make a rectangle

$2T_5 = 5 \times 6 = 30$

so $T_5 = 15$

Similarly $2T_{100} = 100 \times 101$

so $T_{100} = 5050$

$T_7 = 1 + 2 + 3 + 4 + 5 + 6 + 7$

$T_7 = 7 + 6 + 5 + 4 + 3 + 2 + 1$

$2T_7 = 8 + 8 + 8 + 8 + 8 + 8 + 8$ so $T_7 = 56/2$

40

Generalisation for all arithmetic series

PROOF OF THE FORMULA $S_n = \frac{1}{2} n\,[2a + (n\text{-}1)d]$

for the sum of n terms S_n of an arithmetic series with first term a and common difference d.

$S_n = \quad a \quad + \quad (a + d) \quad + \quad (a + 2d) + \ldots \quad + (a + (n\text{-}1)d)$

Writing the terms in the reverse order:

$S_n = \quad (a + (n\text{-}1)d) + (a + (n\text{-}2)d) \quad + \quad \ldots \quad + \quad a$

Adding pairs of terms:

$2S_n = \quad n\,[2a + (n\text{-}1)d]$

Dividing by 2:

$S_n = \frac{1}{2} n\,[2a + (n\text{-}1)d]$

41

WHAT DO YOU NOTICE ABOUT THIS PICTURE?

$T_6 + T_7$

$7 \times 7 = 49$

42

MYSTIC ROSES, PEOPLE MATHS AND HANDSHAKES

TALK: Suppose each chord in the picture represents a handshake.

Is everybody shaking hands with everyone else?

DO: Then 8 people each shake hand with 7 other people

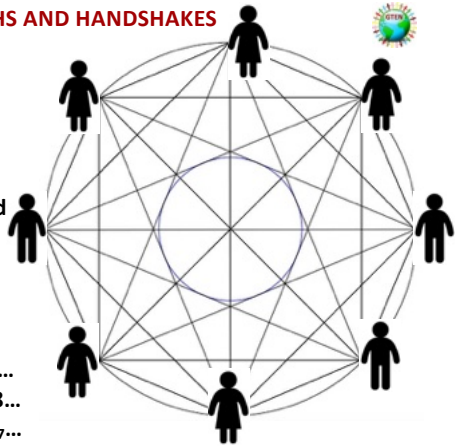
How many chords are there in this Mystic Rose?

RECORD: $\frac{1}{2}(8 \times 7) = 28$

People: 2, 3, 4, 5, 6, 7, 8…

Handshakes: 1, 3, 6, 10, 15, 21, 28…

Triangle nos: $T_1, T_2, T_3, T_4, T_5, T_6, T_7$…



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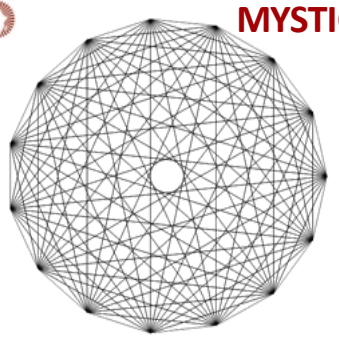
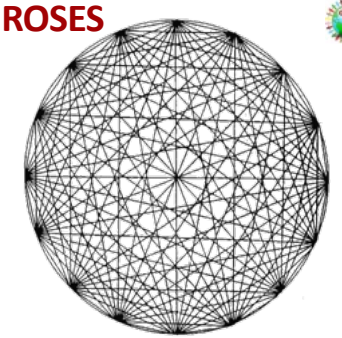
MYSTIC ROSES



15 people shake hands with 14 other people making $\frac{1}{2}(15 \times 14) = 105$ handshakes

$T_{15} = 105$



16 people shake hands with 15 other people making $\frac{1}{2}(16 \times 15) = 120$ handshakes

$T_{16} = 120$

44

UPPER SECONDARY 16+

THEME: NUMBER AND SHAPE PATTERNS

LEARNING OBJECTIVES: Learners have the opportunity to:

- investigate number patterns leading to arithmetic sequences and summing series;
- understand the sequence, algebraic representation and summing n terms of series;
- develop visualization skills;
- solve problems involving numbers and geometry patterns.

GENERIC COMPETENCES: Learners have the opportunity to:

- think creatively and reason logically;
- apply knowledge and skills;
- solve and interpret problems;
- develop communication skills.

45

LET'S PLAY MATHEMATICALLY AND LEARN

Order from AMAZON or TARQUIN https://www.tarquingroup.com/products/aiming-

Play Mathematically

- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

Thanks for coming to this workshop.

Use the AIMSSEC ideas on AIMING HIGH and add comments.

Share what you have learned with other teachers.

Try to help all your learners to have a

'YES I CAN'

attitude to mathematics.

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Enquire about signing up for an AIMSSEC course as a self-funding student admin@aimssec.ac.za

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