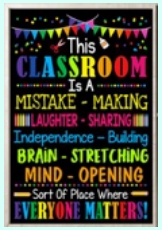
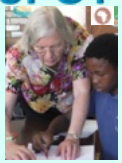
Global Teacher Empowerment Network GTEN

Saturday 6 November 2021 16.00 – 18.00 London Time

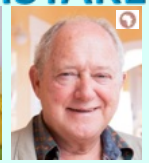
SPOT THE MISTAKE



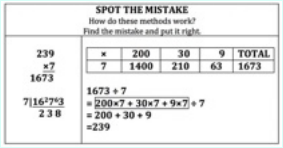


Toni Beardon

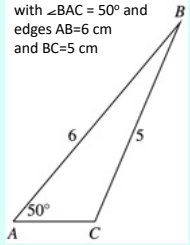
Caroline Ainslie



Barrie Barnard



Not the only triangle ABC with ∠BAC = 50° and edges AB=6 cm and BC=5 cm



INVALID ARGUMENT

When it rains we stay in doors.
We are staying indoors so it must be raining.

1

AIMS | African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

MATHS TOYS

Global Teacher Empowerment Network (GTEN)

PROGRAMME FOR SPOT THE MISTAKE WORKSHOP 6 November 2021

IMPROVE KNOWLEDGE AND UNDERSTANDING OF:

methods of + - × ÷ calculations;
inverse operations;
logical reasoning;
percentages;
conditions for congruency;
(Mis)representation of data.

Learning Spiral

UPPER SECONDARY	10. Modelling and Problem solving in using and applying mathematics
	9. Misrepresentation of data
	8. Representation of sales figures
	7. Percentages Paradox
LOWER SECONDARY	6. Mistakes in using Mathematical Notation
	5. Cooling graph
	4. Geometrical construction misconception
UPPER PRIMARY	3. Percentages Misconceptions
	2. Spot the mistakes in - × ÷ calculations
LOWER PRIMARY	3. Spot the Mistakes Lessons
EARLY YEARS	2. Early years reasoning
STARTER ACTIVITY	1. Logical Reasoning

2

SPOT THE MISTAKE – LEARN FROM MISTAKES

Do the activities.
Questions are in green
This icon suggest discussion of the activity.
Please participate even if you make guesses.
That way you'll get more benefit
out of the workshop.

3

SPOT THE MISTAKE
LEARN FROM MISTAKES

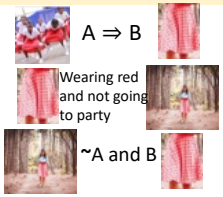
4

STARTER ACTIVITY – LOGICAL REASONING

Think about two statements A and B that can either be true or false.

If A is false then not-A is true and vice versa.

Consider 4 arguments and try to decide if they are VALID (correct) arguments or (INVALID) (mistaken) arguments.



ARE THESE ARGUMENTS VALID OR INVALID?

1. If Layla went to a party then she wore a red skirt.
Layla wore a red skirt therefore she went to a party.
If A then B **A ⇒B**
B therefore A **B ⇒A**
INVALID

2. If she is my sister then she is younger than I am.
She is my sister, therefore she is younger than I am.
If A then B **A ⇒B**
A therefore B **A ⇒B**
VALID

3. If she is my sister then she is younger than I am.
She is not younger than I am, therefore she is not my sister.
If A then B **A ⇒B**
not-B therefore not-A **~B ⇒~A**
VALID

4. If Layla went to a party then she wore a red skirt.
Layla did not go to a party therefore she did not wear a red skirt.
If A then B **A ⇒B**
not-A therefore not-B **~A ⇒~B**
INVALID

5

SPOT THE MISTAKE – LEARN FROM MISTAKES

Statement A: $x = 5$
Statement B: $x^2 = 25$

It is true that A ⇒ B
$x = 5 \Rightarrow x^2 = 25$.

Is it true that B ⇒ A
$x^2 = 25 \Rightarrow x = 5$?

No because $(-5)^2 = 25$ so x could be – 5.

Statement A: The number is 0.
Statement B: We add this number to itself and we multiply it by itself and we get the same answer.

A ⇒ B
because 0 + 0 = 0 × 0 = 0

Is it true that B ⇒ A?
No because the number could be 2 as 2 + 2 = 2 × 2 = 4.

Just because something is true we can't assume that the converse is true.

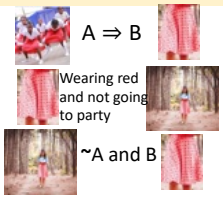
6

STARTER ACTIVITY – LOGICAL REASONING

Think about two statements A and B that can either be true or false.

If A is false then not-A is true and vice versa.

Consider 4 arguments and try to decide if they are VALID (correct) arguments or (INVALID) (mistaken) arguments.



DO THESE ARGUMENTS SEEM VALID OR INVALID?

1. If Layla went to a party then she wore a red skirt.
Layla wore a red skirt therefore she went to a party.
If A then B **A ⇒B**
B therefore A **B ⇒A**
INVALID

2. If she is my sister then she is younger than I am.
She is my sister, therefore she is younger than I am.
If A then B **A ⇒B**
A therefore B **A ⇒B**
VALID

3. If she is my sister then she is younger than I am.
She is not younger than I am, therefore she is not my sister.
If A then B **A ⇒B**
not-B therefore not A **~B ⇒~A**
VALID

4. If Layla went to a party then she wore a red skirt.
Layla did not go to a party therefore she did not wear a red skirt.
If A then B **A ⇒B**
not-A therefore not B **~A ⇒~B**
INVALID

7

SPOT THE MISTAKE – EARLY YEARS REASONING

Many young children will recognize and understand that the following reasoning is wrong:

When it rains we stay in doors.
We are staying indoors so it must be raining. INVALID

They may not recognize that the following arguments are also wrong.

What do you think?

When Mummy comes home she puts me to bed.
I don't need to go to bed until Mummy comes home. INVALID

Last time we went to Granny's house we had chocolate cake.
We are going to Granny's house so we'll have chocolate cake. INVALID

I did not like the taste of cheese when I was 2 years old.
I won't like the taste of cheese now I am 5 years old. INVALID

8

SPOT THE MISTAKES LESSONS – LOWER PRIMARY

Create an atmosphere in your class that everyone believes that mistakes are good learning experiences.

Make a note (mental or written) of all the mistakes in mathematics work that are made by your class.

Frequently write out a few examples of incorrect working on the board and ask the class to work in pairs to spot and explain the mistakes.

The ask pairs of learners to show the mistakes to the class and to explain them.

9

UPPER PRIMARY

Spot the Mistakes is a useful teaching strategy.

It can be used for diagnostic assessment.

Learners have to investigate and compare different methods for themselves.

SPOT THE MISTAKES PUZZLE 1

https://aiminghigh.aimssec.ac.za/spot-the-mistake/

SPOT THE MISTAKE

How do these methods work?
Find the mistake and put it right.

$$\begin{array}{r} 79 \\ \times\ 8 \\ \hline 642 \end{array}$$

$$8\,\overline{)\,63^{7}2}\quad 79$$

×	70	9	TOTAL
8	560	72	632

$632 \div 8$ "How many 80's & 8's"?
$= (80\times7 + 8\times9) \div 8$
$= (560 + 72) \div 8 = 79$

What happens when you multiply by 8 and then divide your answer by 8?

10

UPPER PRIMARY

Learners engage with different methods, find out how and why the methods work, and learn that + and - are inverse operations, and × and ÷ are inverse operations, like putting on and taking off socks and shoes.

$5 \times 2 + 3 = 13$
$(13 \div 2) - 3 \neq 5$ X
$(13 - 3) \div 2 = 5$ ✅

SPOT THE MISTAKES PUZZLE 2

https://aiminghigh.aimssec.ac.za/spot-the-mistake/

SPOT THE MISTAKE

How do these methods work?
Find the mistake and put it right.

$$\begin{array}{r} 239 \\ \times 7 \\ \hline 1673 \end{array}$$

$$7\,\overline{)\,16^{2}7^{6}3}\quad 238$$

×	200	30	9	TOTAL
7	1400	210	63	1673

$1673 \div 7$
$= (700\times2 + 70\times3 + 7\times9) \div 7$
$= (1400 + 210 + 63) \div 7$
$= 239$

11

UPPER PRIMARY

Explore different methods of doing multiplication calculations.

SPOT THE MISTAKES PUZZLE 3

https://aiminghigh.aimssec.ac.za/years-4-7-spot-the-mistake/

SPOT THE MISTAKE

How do these methods work?
Find the mistake and put it right.

$$\begin{array}{r} 29 \\ \times 51 \\ \hline 145 \\ 290 \\ \hline 435 \end{array}$$

×	20	9
50	1000	450
1	20	9

29×51
$= 1000 + 450 + 20 + 9$
$= 1479$

29×51
$= 30 \times 51 - 51$
$= 1530 - 51$
$= 1479$

29×51
$= 29 \times 50 + 29$
$= 145 \times 100 + 29$
$= 1450 + 29$
$= 1479$

12

UPPER PRIMARY

Doing quick accurate calculations is for calculators and computers.

Understanding and estimating calculations is for humans.

SPOT THE MISTAKES PUZZLE 4

https://aiminghigh.aimssec.ac.za/years-4-7-spot-the-mistake/

SPOT THE MISTAKE

How do these methods work?
Find the mistake and put it right.

```
 129
×34
 486
3870
4356
```

×	100	20	9	TOTAL
30	3000	600	270	3870
4	400	80	36	516

129 × 34
= 3870 + 516
= 4386

13

LOWER SECONDARY PERCENTAGES MISCONCEPTIONS

SHOULD YOU BELIEVE THINGS ARE WHAT THEY SEEM?

The government cut funding to education by 10% one year and increased it by 11% the following year.

Should the voters have been satisfied that education was now better funded?

Suppose the funding was 1 million. It was cut to 900 000.
Then 900 000 was increased by 11%.

What would the funding be after the increase?

PRICES REDUCED IN A SALE BY 50%

In High Season some prices in a store were increased by 40% so an item costing 100 was increased to 140. In an end of season sale those prices were reduced to 70 and marked as a 50% reduction.

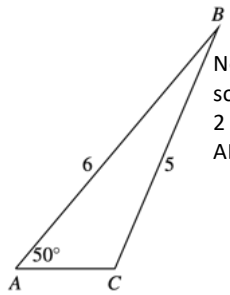
Was this really a 50% reduction or only a 30% reduction?

14

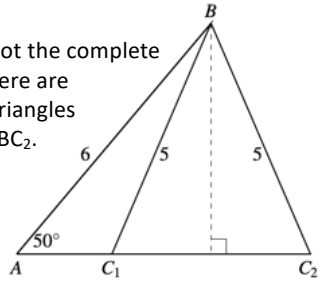
LOWER SECONDARY SPOT THE MISTAKE IN GEOMETRY

Draw triangle ABC with ∠BAC = 50° and edges AB=6 cm and BC=5 cm.
Are all the angles acute?

Is the answer no as shown by the diagram on the left?



No, this is not the complete solution, there are 2 possible triangles ABC_1 and ABC_2.



15

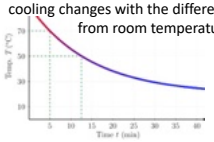
LOWER SECONDARY MISTAKES IN REPRESENTING DATA

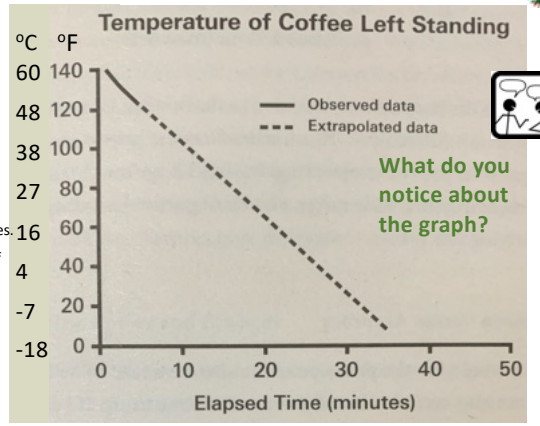
My cup of coffee started at 63°C.
I observed the temperature decreasing over time like this:

Elapsed time (mins)	Temp °C (to nearest degree)
0	63
1	60
2	57
3	54

The coffee loses 3°C every minute.
I used this information to draw a graph to find the temperature after 30 minutes.

It's a mistake to assume that the rate of cooling remains constant. The rate of cooling changes with the difference from room temperature.





What do you notice about the graph?

16

LOWER SECONDARY MISTAKES IN USING MATHEMATICAL NOTATION

Mzu worked out 4^{3^2} as 4096.

Najwa worked out 4^{3^2} as 262144.

Who was right?

Both did their calculations correctly.

Mzu worked out $4^3 = 64$ and $64^2 = 4096$.

Najwa worked out $3^2 = 9$ and $4^9 = 262144$.

The expression 4^{3^2} is not uniquely defined.

The mathematical notation should have used brackets.

4^{3^2} could be read as $(4^3)^2$ or as $4^{(3^2)}$.

Mzu calculated $(4^3)^2$ and Najwa calculated $4^{(3^2)}$

17

UPPER SECONDARY PERCENTAGES PARADOX

You and a friend each try to score goals in basketball games. On Saturday you got 7 goals out of 8 attempts, but your friend got 2 goals out of 2 attempts, so your rating was 7/8 or 87.5% and hers was 2/2 or 100%.

In the match on Sunday you scored 1 goal from 2 attempts and she scored 5 out of 8 attempts so again her rating of 5/8 or 62.5% was better than yours of ½ or 50%. Your friend scored at higher rates than you on each of two days.

Does that mean your friend has a higher rating when the two days are combined?

Not necessarily, work it out!

SIMPSON'S PARADOX

You scored 8 goals out of 10, a rating of 80%.
She scored 7 goals out of 10, a rating of 70%, so for the whole weekend your rating was better!

THIS SHOWS YOU HAVE TO BE CAUTIOUS WHEN INTERPRETING DATA AND DRAWING CONCLUSIONS.

Day	You	Your friend
Saturday	$\frac{7}{8} = 87.5\%$	$\frac{2}{2} = \mathbf{100\%}$
Sunday	$\frac{1}{2} = 50\%$	$\frac{5}{8} = \mathbf{62.5\%}$
Total	$\frac{8}{10} = \mathbf{80\%}$	$\frac{7}{10} = 70\%$

18

UPPER SECONDARY PERCENTAGES PARADOX

Drug name	AntiCynicismia	AntiMisantropia
Group A	436 out of 545 people were cured, or **80%** success rate	9 out of 10 people were cured, or **90%** success rate
Group B	245 out of 350 people were cured, or **70%** success rate	16 out of 20 people were cured, or **80%** success rate
Group C	48 out of 80 people were cured, or **60%** success rate	21 out of 30 people were cured, or **70%** success rate
Group D	10 out of 20 people were cured, or **50%** success rate	180 out of 300 people were cured, or **60%** success rate
Group E	2 out of 5 people were cured, or **40%** success rate	320 out of 640 people were cured, or **50%** success rate
TOTALS	741 cures out of 1000 74.1%	546 cures out of 1000 54.6%

These are the results for a clinical trial of two new drugs *AntiCynicismia* and *AntiMisantropia*

Which drug is best?

There is no mistake here. It is another example of Simpson's Paradox.

People responsible for interpreting data need to be aware that averaging percentages, when the percentages relate to different totals, often gives this type of result.

19

UPPER SECONDARY - SIMPSON'S PARADOX

Suppose we see the graph as a distance time graph.
The gradients of the lines show the average speeds for different parts of the journeys.

OR_1 shows a journey of 90 kilometres in 1 hour at an average speed of 90 km.p.h.

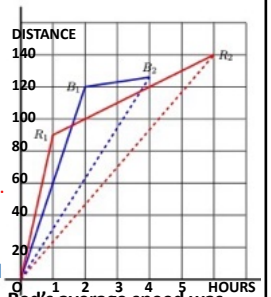
R_1R_2 shows Red's journey for the next 5 hours when he travelled for 50 km and stopped to visit a friend (an average of 10 km.p.h).

OB_1 shows a journey of 120 kilometres in 2 hours at an average speed of 60 km.p.h., slower than the first leg of Red's journey.

B_1B_2 shows Blue's journey for the next 2 hours when he travelled 6 km (an average of 3 km.p.h) and stopped for lunch.

OR_2 shows Red's average speed for the whole journey of 140 km in 6 hours an average of 23.3 km.p.h. (to 1 decimal place)

OB_2 shows Blue's average speed of 126 km in 4 hours an average of 31.5 km.p.h. **Note: 23.3 < 31.5**



Red's average speed was HIGHER than Blue's on both legs of the journey but Red's overall average speed was LOWER than Blue's.

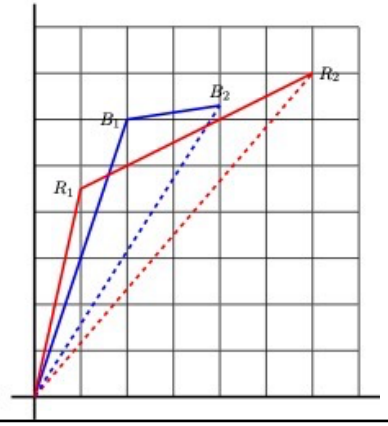
20

UPPER SECONDARY - SIMPSON'S PARADOX

The paradox occurs in many contexts when groups of data show one particular trend, but this trend is reversed when the groups are combined together.

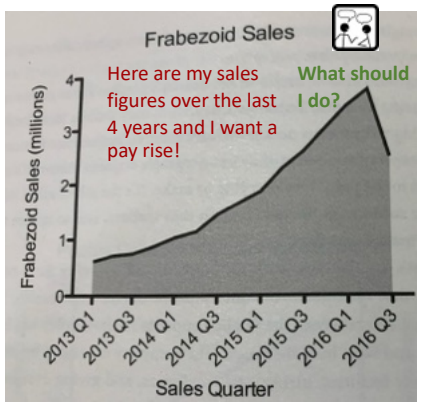
Simpson's paradox can be viewed for example in terms of percentages, ratios, gradients of line segments or speeds.

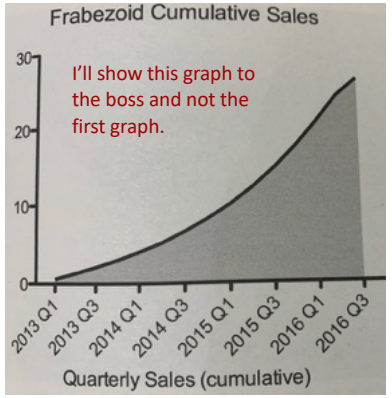
Understanding and identifying the paradox is important for interpreting data correctly.



21

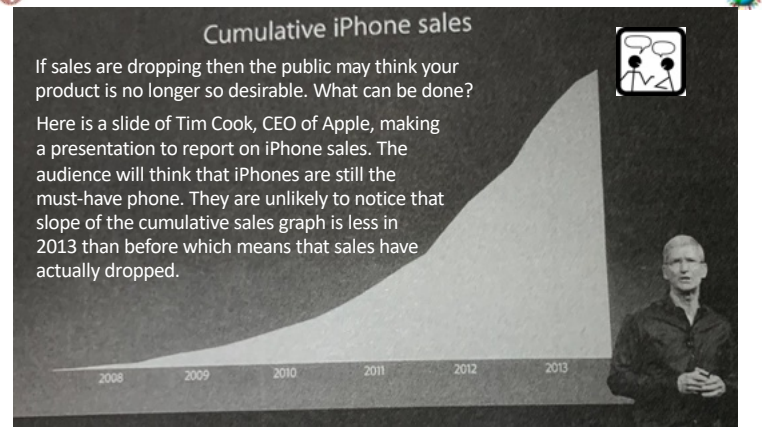
UPPER SECONDARY SPOT THE MISTAKES IN REPRESENTING DATA





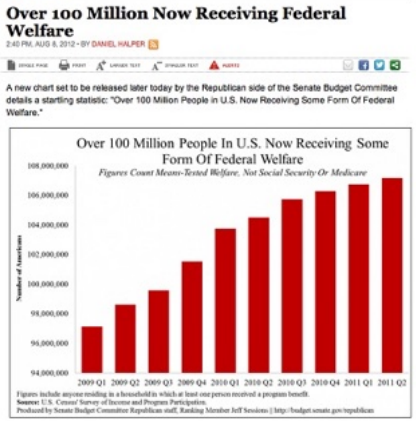
22

UPPER SECONDARY SPOT THE MISTAKES IN REPRESENTING DATA



23

UPPER SECONDARY SPOT THE MISTAKES IN REPRESENTING DATA



Misleading graphs are sometimes **deliberately** misleading and sometimes it's a case of people **not understanding the data** behind the graph they create.

The figures in this example include anyone living in a household where at least one person received benefits.

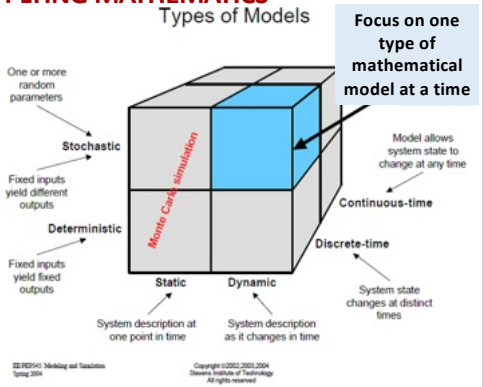
Classic types of **misleading** graphs include cases where the vertical scale is too big or too small, or it skips numbers or, as in this example, it doesn't start at zero making the increase look much bigger than it really is.

24

MATHEMATICAL MODELLING AND PROBLEM SOLVING IN USING AND APPLYING MATHEMATICS

Reading comprehension is important; many mistakes arise from misinterpretation of written information.

Mistakes also arise from building a mathematical model that does not exactly fit the specifications because of a failure to understand a context fully.



25

Global Teacher Empowerment Network (GTEN)

NEW SKILLS NEW HOPES NEW HORIZONS
for teachers and learners worldwide

SPOT THE MISTAKE RESOURCES

SPOT THE MISTAKES PUZZLE https://aiminghigh.aimssec.ac.za/spot-the-mistakes/

A FIELD GUIDE TO LIES and STATISTICS by Daniel Levitin
Penguin Viking ISBN: 978-0-241-24000-7

SIMPSON'S PARADOX https://brilliant.org/wiki/simpsons-paradox/

To apply to join the GTEN Teachers WhatsApp Group and to get information about GTEN write to Layla Thompson <jjthompson@aimssec.ac.za>

26

AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

AIMSSEC GTEN YouTube Channel
https://www.youtube.com/c/MathsToys/videos

AIMSSEC Website: http://aimssec.ac.za
AIMING HIGH Free lesson resources: http://aiminghigh.aimssec.ac.za

COLLABORATIVE PROFESSIONAL DEVELOPMENT
https://aiminghigh.aimssec.ac.za/category/cpd
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOPS
https://aiminghigh.aimssec.ac.za/manage-your-own-professional-development-workshops/

Toni Beardon LAB11@cam.ac.uk
Caroline Ainslie caroline@bubblymaths.co.uk

27

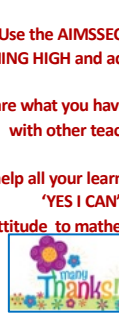
AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

Thanks for coming to this workshop.

Use the AIMSSEC ideas
on AIMING HIGH and add comments.

Share what you have learned
with other teachers.

Try to help all your learners to have a
'YES I CAN'
attitude to mathematics.



28