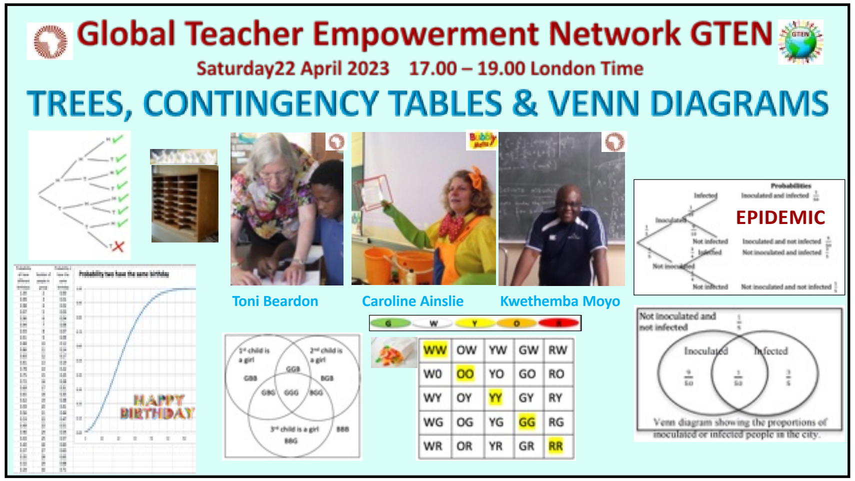
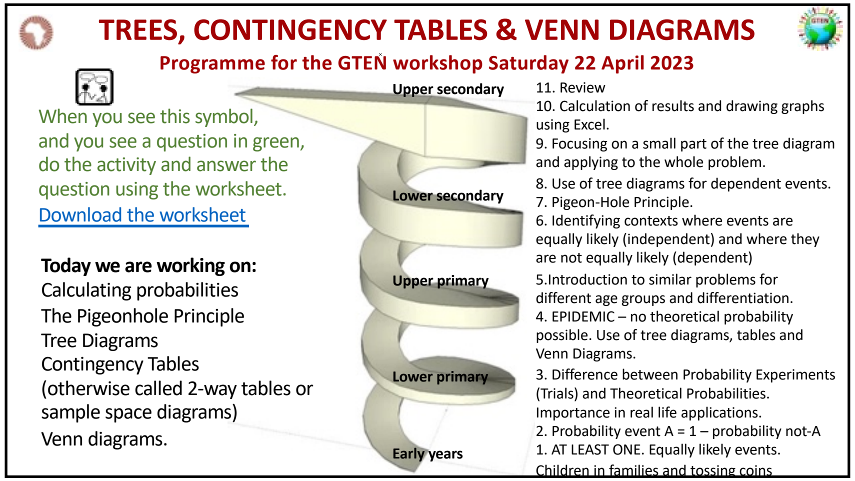
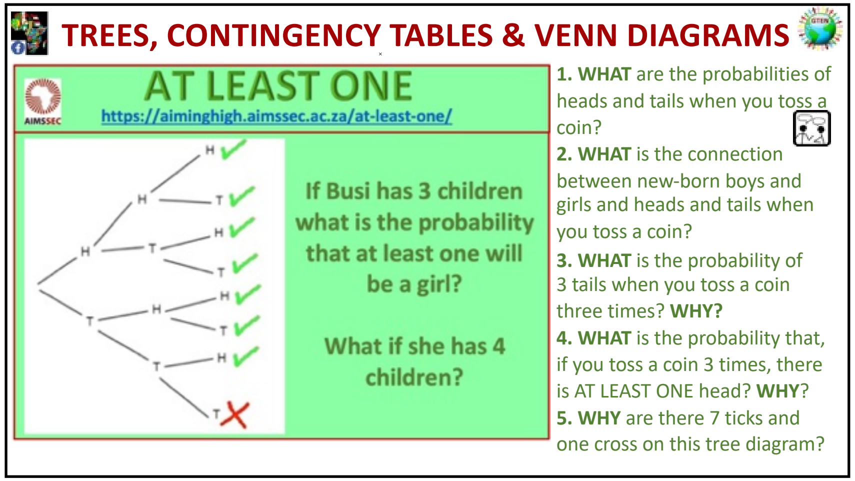

Global Teacher Empowerment Network GTEN
Saturday22 April 2023 17.00 – 19.00 London Time
TREES, CONTINGENCY TABLES & VENN DIAGRAMS
Toni Beardon
Caroline Ainslie
Kwethemba Moyo
EPIDEMIC
HAPPY BIRTHDAY

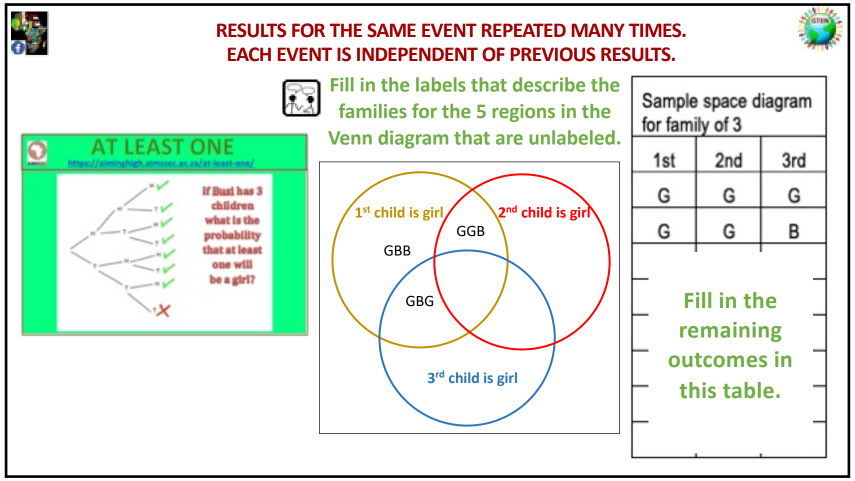
1


TREES, CONTINGENCY TABLES & VENN DIAGRAMS
Programme for the GTEN workshop Saturday 22 April 2023
When you see this symbol, and you see a question in green, do the activity and answer the question using the worksheet.
Download the worksheet
Today we are working on:
Calculating probabilities
The Pigeonhole Principle
Tree Diagrams
Contingency Tables
(otherwise called 2-way tables or sample space diagrams)
Venn diagrams.
Upper secondary
Lower secondary
Upper primary
Lower primary
Early years
11. Review
10. Calculation of results and drawing graphs using Excel.
9. Focusing on a small part of the tree diagram and applying to the whole problem.
8. Use of tree diagrams for dependent events.
7. Pigeon-Hole Principle.
6. Identifying contexts where events are equally likely (independent) and where they are not equally likely (dependent)
5.Introduction to similar problems for different age groups and differentiation.
4. EPIDEMIC – no theoretical probability possible. Use of tree diagrams, tables and Venn Diagrams.
3. Difference between Probability Experiments (Trials) and Theoretical Probabilities. Importance in real life applications.
2. Probability event A = 1 – probability not-A
1. AT LEAST ONE. Equally likely events.
Children in families and tossing coins

2


TREES, CONTINGENCY TABLES & VENN DIAGRAMS
AT LEAST ONE
https://aiminghigh.aimssec.ac.za/at-least-one/
If Busi has 3 children what is the probability that at least one will be a girl?
What if she has 4 children?
1. WHAT are the probabilities of heads and tails when you toss a coin?
2. WHAT is the connection between new-born boys and girls and heads and tails when you toss a coin?
3. WHAT is the probability of 3 tails when you toss a coin three times? WHY?
4. WHAT is the probability that, if you toss a coin 3 times, there is AT LEAST ONE head? WHY?
5. WHY are there 7 ticks and one cross on this tree diagram?

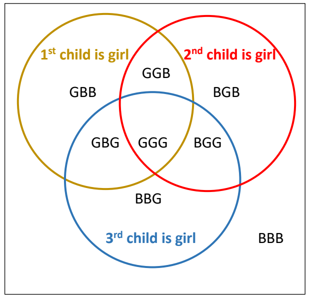
3


RESULTS FOR THE SAME EVENT REPEATED MANY TIMES.
EACH EVENT IS INDEPENDENT OF PREVIOUS RESULTS.
Fill in the labels that describe the families for the 5 regions in the Venn diagram that are unlabeled.
AT LEAST ONE
1st child is girl
2nd child is girl
3rd child is girl
GGB
GBB
GBG
Sample space diagram for family of 3
1st 2nd 3rd
G G G
G G B
Fill in the remaining outcomes in this table.

4

RESULTS FOR THE SAME EVENT REPEATED MANY TIMES.
EACH EVENT IS INDEPENDENT OF PREVIOUS RESULTS.

Fill in the labels that describe the families for the 5 regions in the Venn diagram that are unlabeled.



Sample space diagram for family of 3		
1st	2nd	3rd
G	G	G
G	G	B
G	B	G
B	G	G
G	B	B
B	G	B
B	B	G
B	B	B

5

If Busi has 3 children, the probability that **AT LEAST ONE** will be a girl
= 1 – probability of 3 boys

SAMPLE SPACE

List all the ways families can be made up, in age order Notation: girls G and boys B.

Family size	All possible outcomes for boys and girls in family in age order
1 child	B or G
2 children	BB, BG, GB or GG
3 children	BBB, BGB, GBB, GGB, BBG, BGG, GBG or GGG
4 children	BBBB, BGBB, GBBB, GGBB, BBGB, BGGB, GBGB or GGGB BBBG, BGBG, GBBG, GGBG, BBGG, BGGG, GBGG or GGGG

For 3 children there are 8 possibilities.
The probability of at least one girl ???

For 4 children there are 16 possibilities.
The probability of at least one girl ???

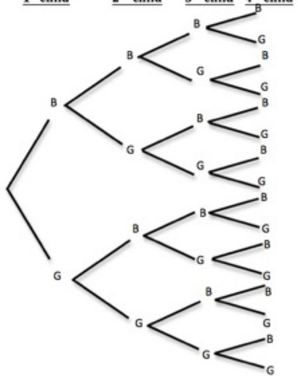
6

If Busi has 3 children, the probability that **AT LEAST ONE** will be a girl
= 1 – probability of 3 boys

For 3 children there are 8 possibilities.
The probability of at least one girl =
$1 - \frac{1}{8} = \frac{7}{8}$

For 4 children there are 16 possibilities.
The probability of at least one girl =
$1 - \frac{1}{16} = \frac{15}{16}$

FAMILIES OF 4 CHILDREN
All probabilities are ½
B represents boy and G represents girl.



7

Diagnostic Assessment

Answer on the chat A, B, C or D whichever you think is the correct answer.

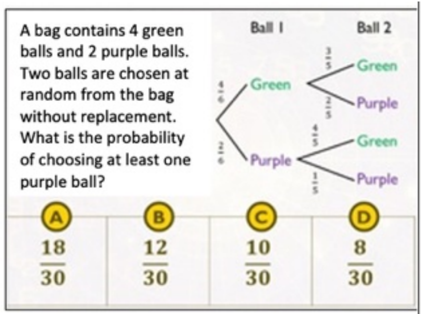
The correct answer is A: $1 - \frac{12}{30} = \frac{18}{30} = \frac{3}{5}$

Possible misconceptions:

B. This is the probability of choosing no purple balls.

C. This is the probability that the 2nd ball chosen is purple. Learners giving this answer may not have understood the question.

D. Learners may have thought like this:
I think D is the right answer is because $\frac{2}{6} \times \frac{4}{5} = \frac{8}{30}$

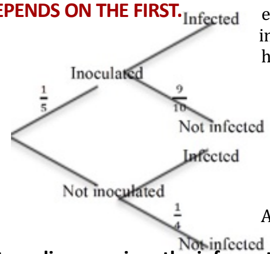


- Notice how the learners respond. Ask a learner who gave answer A to explain why they gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for the answer.
- Do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. https://diagnosticquestions.com

8

EPIDEMIC

RESULTS FOR TWO CONSECUTIVE EVENTS. THE SECOND EVENT DEPENDS ON THE FIRST.



A city is hit by an epidemic. 20% of the inhabitants of the city have been inoculated against a certain disease.

The chance of escaping infection amongst those inoculated is 90%. Amongst the rest it is 25%.

SPACE DIAGRAM (2-way table)	Inoculated	Not inoculated	Totals
Infected			
Not infected			
Totals	20%	80%	100%

The tree diagram gives the information.

Fill in the probabilities on the other 3 branches.

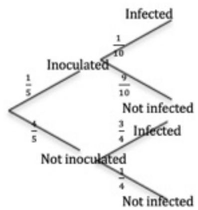
Can you use this information to fill in the contingency (2-way) table?

Compare the 3 diagrams.

Can you work out the probabilities of the 4 outcomes?

9

EPIDEMIC



Probabilities

Inoculated and infected $\frac{1}{50}$

Inoculated and not infected $\frac{9}{50}$

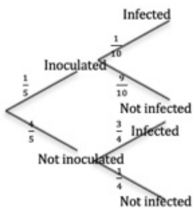
Fill in probabilities for people not inoculated

Do these proportions add up to 1? (i.e.100%)

	Inoculated	Not inoculated	Totals
Infected	10% of 20% = $\frac{1}{10} \times \frac{1}{5} = \frac{1}{50}$ =2%	Fill in probabilities for people not inoculated	2% + 60% = 62%
Not infected	90% of 20% = $\frac{9}{10} \times \frac{1}{5} = \frac{9}{50}$ =18%		18% + 20% =38%
Totals	20%		100%

10

EPIDEMIC



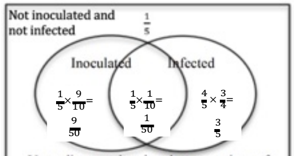
Probabilities

Inoculated and infected $\frac{1}{50}$

Inoculated and not infected $\frac{9}{50}$

Not inoculated and infected $\frac{3}{5}$

Not inoculated and not infected $\frac{1}{5}$



Venn diagram showing the proportions of inoculated or infected people in the city.

	Inoculated	Not inoculated	Totals
Infected	10% of 20% = $\frac{1}{10} \times \frac{1}{5} = \frac{1}{50}$ =2%	75% of 80% = $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$ =60%	2% + 60% = 62%
Not infected	90% of 20% = $\frac{9}{10} \times \frac{1}{5} = \frac{9}{50}$ =18%	25% of 20% = $\frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$ = 20%	18% + 20% =38%
Totals	20%	80%	100%

11

WHY TEACH PROBABILITY?

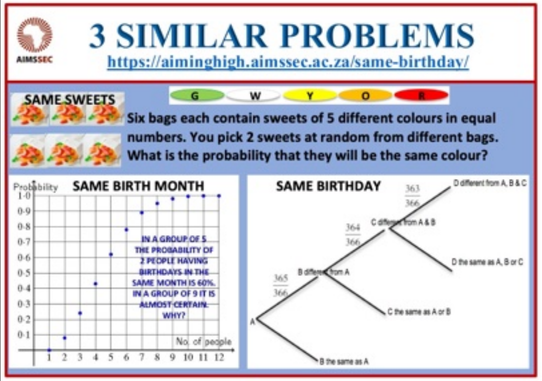
We need to give children the best possible opportunity to thrive when they leave school. We must prepare them for a job market where existing knowledge and skills have limited value unless they can be applied to solve real life problems.

By collecting and analysing data, probability can be applied in many important ways to predict outcomes that help solve today's complex problems and improve the quality of life for all.

For example: clinical trials in medicine, calculation of risk in insurance, sales predictions in commerce, weather forecasting and many more.

12

WE NEED TO KNOW WHETHER SUCCESSIVE EVENTS ARE INDEPENDENT OR NOT.



In the **SAME SWEETS** problem, the probability of choosing a particular colour does not depend on what has been chosen before.

The probabilities of successive events are **independent**.

If you choose two people from a group, the probability of the second person having the **SAME BIRTH MONTH OR BIRTHDAY** as the first **depends** on the birthday of the first person. **WHY?**

13

SAME COLOUR SWEETS

G W Y O R

Six bags each contain green, white, yellow, orange and red sweets with equal numbers of each colour (5 flavours).

You pick sweets from different bags without looking.

What is the probability of drawing a yellow sweet?

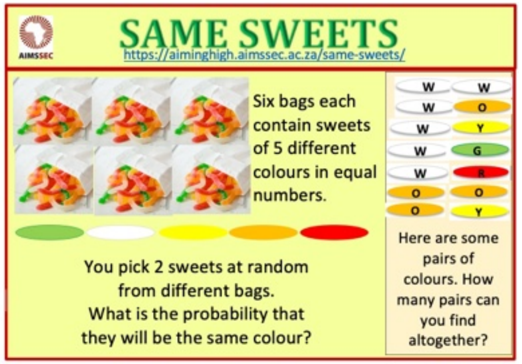
Is the probability the same for the other colours?

If you pick 6 sweets (one from each bag) what is the probability that you will get 2 of the same colour?

PIGEON-HOLE PRINCIPLE If there are more pigeons than boxes to go in then some boxes will have more than one pigeon.

14

If you pick 2 sweets from different bags without looking what is the probability that they will be the same colour?



WW	OW	YW	GW	RW
WO	OO	YO	GO	RO
Complete this 2-way table.				RY
				RG
				RR

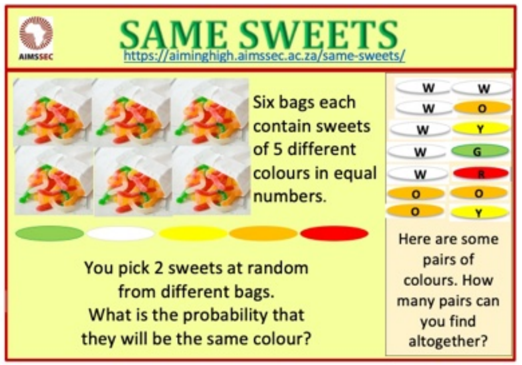
What is the probability that the 2nd sweet is a different colour from 1st?

What is the probability of 2 sweets of the same colour?

Then work out the probabilities for 3, 4, 5 and 6 sweets that 2 of the chosen sweets are the same colour.

15

If you pick 2 sweets from different bags without looking what is the probability that they will be the same colour?



WW	OW	YW	GW	RW
WO	OO	YO	GO	RO
WY	OY	YY	GY	RY
WG	OG	YG	GG	RG
WR	OR	YR	GR	RR

The probability that the second sweet is a different colour from the first is $\frac{20}{25} = \frac{4}{5}$.

The probability that the two sweets are of the same colour is $\frac{5}{25} = \frac{1}{5}$

16

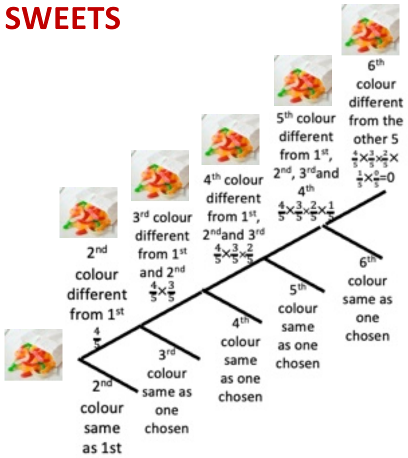
SAME SWEETS

Six bags of sweets each contain green, white, yellow, orange and red sweets with equal numbers of each colour (5 flavours).

You pick 2 sweets from different bags without looking (at random).

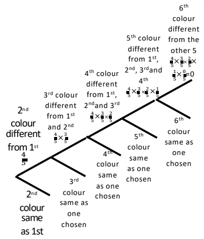
What is the probability that they will be the same colour?

What about picking 3, 4, 5 or 6 sweets from different bags? In these cases what is the probability that 2 sweets chosen will be the same colour?



17

If you pick sweets from different bags without looking what is the probability that two will be the same colour?

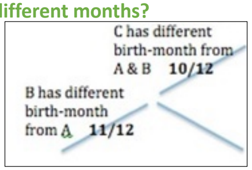


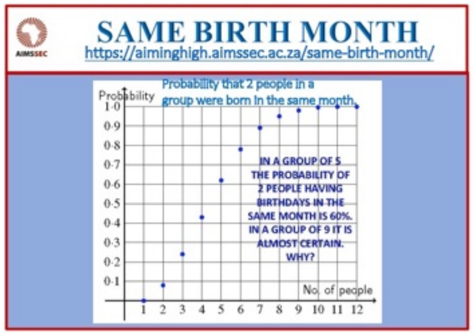
The probability that an event does not happen = 1 – probability it does happen. In the last two columns one probability is 1 minus the other.	Number of sweets picked	Probability that all sweets picked are different colours.	Probability that two sweets picked are the same colour
	2	$\frac{4}{5}=0.8$	0.2
	3	$\frac{4}{5}\times\frac{3}{5}=0.48$	0.52
	4	$\frac{4}{5}\times\frac{3}{5}\times\frac{2}{5}=0.192$	0.818
	5	$\frac{4}{5}\times\frac{3}{5}\times\frac{2}{5}\times\frac{1}{5}=0.0384$	0.9616
	6	0	1

18

In a group of 13 people it is certain that 2 people in the group were born in the same month. Why?

Starting with a simple case, in a group of 3 people what is the probability that they were all born in different months?





Pr(A, B and C are born in different months)

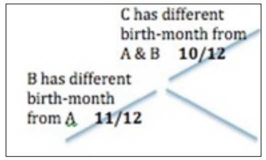
= Pr (A & B are born in different months) × Pr(C is born in a 3rd month different from both A & B) = ???

The probability that two of A, B and C were born in the same month is ???

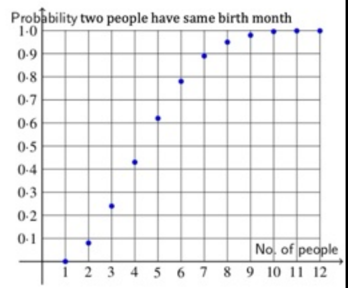
19

In a group of 13 people it is certain that 2 people in the group were born in the same month. Why?

In a group of 3 people what is the probability that they were all born in different months?



Number of people in group	Probability 2 people have same birth month
2	0.08
3	0.24
4	0.43
5	0.62
6	0.78
7	0.89
8	0.95
9	0.98
10	0.996
11	0.999
12	0.999
13	1



Pr(A, B and C are born in different months) =

Pr (A & B are born in different months)×Pr(C is born in a 3rd month different from both A & B)

$=\frac{11}{12}\times\frac{10}{12}=\frac{110}{144}=0.76$

The probability that two (or all 3) of A, B and C were born in the same month is 0.24

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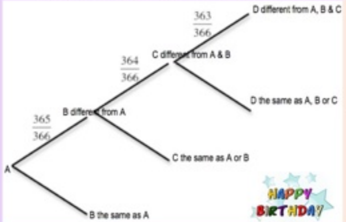
Answer in the chat:
In a group of 23 people, is the probability of 2 people having the same birthday more or less than 50%?

The probability that two people in a group have the same birthday
= 1 – probability that they all have different birthdays.

SAME BIRTHDAY

https://aiminghigh.aimssec.ac.za/same-birthday/

In a group of 23 people is the probability of 2 people having the same birthday more or less than 50%?



Try a simple case. For 4 people A, B, C, and D, what is the probability that two people have the same birthday?

For any number of people, a small part of the tree diagram is sufficient to show how to calculate the probability that 2 people in the group have the same birthday.

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SAME BIRTHDAY

For 4 people, A, B, C and D, the probability that A and B have different birthdays is 365/366.

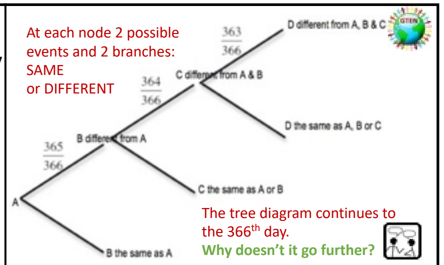
The probability that C has a different birthday from both A and B is $\frac{365}{366} \times \frac{364}{366}$.

The probability that A, B, C and D all have different birthdays
$= \frac{365}{366} \times \frac{364}{366} \times \frac{363}{366} = 0.98$.

The probability that 2 people have the same birthday = 1 - 0.98 = 0.02

Write down the calculation for 5 people.
Write down the calculation for 23 people.

At each node 2 possible events and 2 branches: SAME or DIFFERENT



The tree diagram continues to the 366th day.
Why doesn't it go further?

For 23 people, the probability that all have different birthdays
$= \frac{365 \times 364 \times 363 \times 362 \times \ldots \times 344}{366^{22}} = 0.49$
(calculated to 2 decimal places).
The probability that 2 have the same birthday = 1 - 0.49 = 0.51
More that 50%

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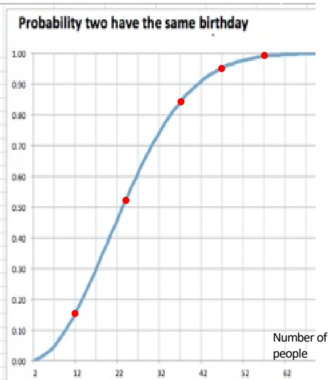
Probability all have different birthdays	Number of people in group	Probability 2 have the same birthday
1.00	2	0.00
0.99	3	0.01
0.98	4	0.02
0.97	5	0.03
0.96	6	0.04
0.94	7	0.06
0.93	8	0.07
0.91	9	0.09
0.88	10	0.12
0.86	11	0.14
0.83	12	0.17
0.81	13	0.19
0.78	14	0.22
0.75	15	0.25
0.72	16	0.28
0.69	17	0.31
0.65	18	0.35
0.62	19	0.38
0.59	20	0.41
0.56	21	0.44
0.53	22	0.47
0.49	23	0.51
0.46	24	0.54
0.43	25	0.57
0.40	26	0.60
0.37	27	0.63
0.35	28	0.65
0.32	29	0.68
0.29	30	0.71



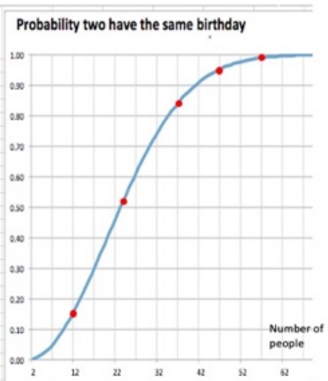
SAME BIRTHDAY

The calculations can be done, and the graph drawn, on a spreadsheet.

Write down the 5 readings from the graph at the marked points.

23

Probability all have different birthdays	Number of people in group	Probability 2 have the same birthday
1.00	2	0.00
0.99	3	0.01
0.98	4	0.02
0.97	5	0.03
0.96	6	0.04
0.94	7	0.06
0.93	8	0.07
0.91	9	0.09
0.88	10	0.12
0.86	11	0.14
0.83	12	0.17
0.81	13	0.19
0.78	14	0.22
0.75	15	0.25
0.72	16	0.28
0.69	17	0.31
0.65	18	0.35
0.62	19	0.38
0.59	20	0.41
0.56	21	0.44
0.53	22	0.47
0.49	23	0.51
0.46	24	0.54
0.43	25	0.57
0.40	26	0.60
0.37	27	0.63
0.35	28	0.65
0.32	29	0.68
0.29	30	0.71



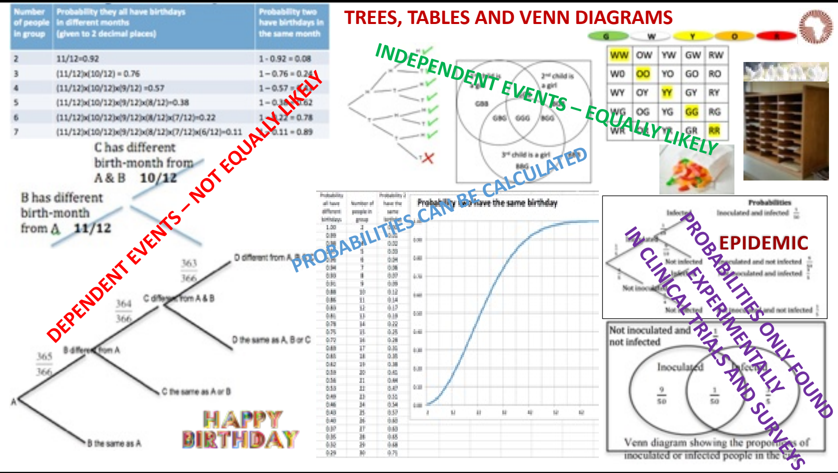
SAME BIRTHDAY

The calculations can be done, and the graph drawn, on a spreadsheet.

Write down the 5 readings from the graph at the marked points.

(number of people, probability)
(12, 0.17)
(23, 0.51)
(37, 0.83)
(47, 0.96)
(57, 0.99)

24



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AIMSSEC
AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE
6 MELROSE RD, MUIZENBERG 7975
CAPE TOWN, SOUTH AFRICA
T: +27 (0) 21 787 9326
GTEN

AN AIMING HIGH LEARNING PACK IS A WEBPAGE CONTAINING
A learning activity with links to:

a. PDF of the worksheet
b. Templates and instructions for making resources
c. Videos
d. Notes for Teachers with
- solutions
- curriculum links and learning objectives
- diagnostic quizzes
- suggestions for teaching
- key questions to guide learning
- follow up ideas and links

https://aiminghigh.aimssec.ac.za/

e. Inclusion Guides for School and Home Learning with
- a starter activity for a mixed–age group to do together
- a collection of learning activities to suit all ages from 4 to 18+
- Solutions with suggestions for teaching and assessment.

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE
GTEN

LINKS FOR LEARNING ACTIVITIES INVOLVING TREES AND CONTINGENCY TABLES

AT LEAST ONE https://aiminghigh.aimssec.ac.za/at-least-one/

SAME SWEETS https://aiminghigh.aimssec.ac.za/same-sweets/

SAME BIRTH MONTH https://aiminghigh.aimssec.ac.za/same-birth-month/

SAME BIRTHDAY https://aiminghigh.aimssec.ac.za/same-birthday/

EPIDEMIC https://aiminghigh.aimssec.ac.za/epidemic/

AIMSSEC YouTube Channel https://www.youtube.com/@MathsToys
GTEN website https://gtenmaths.org/

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LET'S PLAY MATHEMATICALLY AND LEARN

Order from AMAZON or TARQUIN https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically

Play Mathematically
- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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