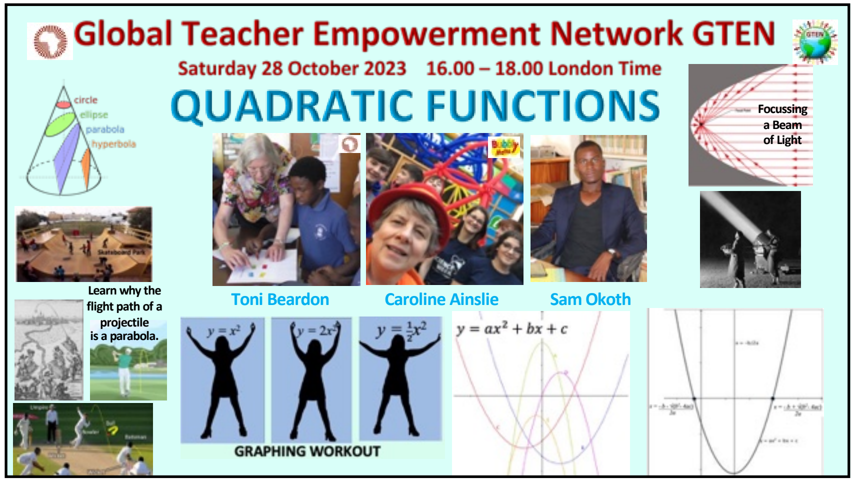
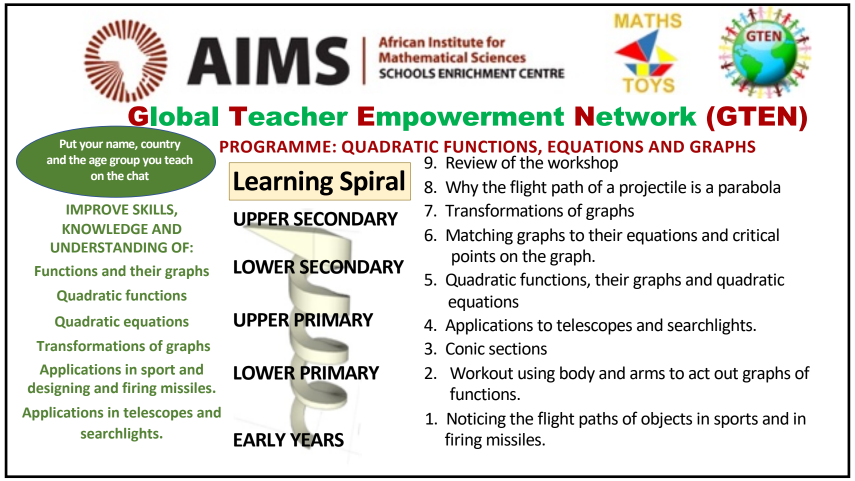
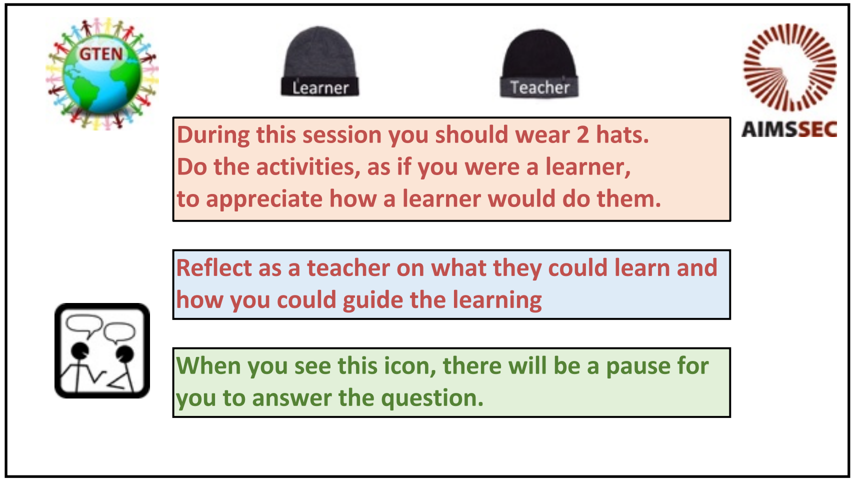

Global Teacher Empowerment Network GTEN
Saturday 28 October 2023 16.00 – 18.00 London Time
QUADRATIC FUNCTIONS
circle
ellipse
parabola
hyperbola
Focussing a Beam of Light
Learn why the flight path of a projectile is a parabola.
Toni Beardon
Caroline Ainslie
Sam Okoth
$y = x^2$
$y = 2x^2$
$y = \frac{1}{2}x^2$
$y = ax^2 + bx + c$
GRAPHING WORKOUT

1


AIMS
African Institute for Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE
MATHS TOYS
GTEN
Global Teacher Empowerment Network (GTEN)
Put your name, country and the age group you teach on the chat
PROGRAMME: QUADRATIC FUNCTIONS, EQUATIONS AND GRAPHS
Learning Spiral
UPPER SECONDARY
LOWER SECONDARY
UPPER PRIMARY
LOWER PRIMARY
EARLY YEARS
IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:
Functions and their graphs
Quadratic functions
Quadratic equations
Transformations of graphs
Applications in sport and designing and firing missiles.
Applications in telescopes and searchlights.
9. Review of the workshop
8. Why the flight path of a projectile is a parabola
7. Transformations of graphs
6. Matching graphs to their equations and critical points on the graph.
5. Quadratic functions, their graphs and quadratic equations
4. Applications to telescopes and searchlights.
3. Conic sections
2. Workout using body and arms to act out graphs of functions.
1. Noticing the flight paths of objects in sports and in firing missiles.

2

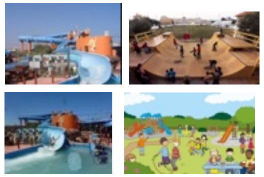

GTEN
Learner
Teacher
AIMSSEC
During this session you should wear 2 hats.
Do the activities, as if you were a learner,
to appreciate how a learner would do them.
Reflect as a teacher on what they could learn and how you could guide the learning
When you see this icon, there will be a pause for you to answer the question.

3


UPS AND DOWNS * NOTICE THE FLIGHT PATHS
Umpire
Ball
Bowler
Wicket
Wicket
Fielders

4

UPS AND DOWNS – CURVES AND SLOPES



Talk about the UPS and DOWNS of rides on water slides, roller coasters, playground slides and skateboard ramps. The children should look at the pictures and describe the rides.

What are the similarities in the shapes of water slides and roller coasters?

What are the differences? Does a ride usually start at the top or at the bottom, and is this a difference between water slide and roller coaster rides? Why is that?

Is it exciting when a ride climbs steeply or drops steeply (suddenly)?

The water slides are actually in Muizenberg where the AIMSSEC office is located.

Walking up or down hills is part of everyday experience so, although they might not have experienced rides like this, the discussion introduces ideas of curves and steepness (gradient) which are important concepts in mathematics.

This discussion will help children to develop a vocabulary about curves and movement on curved tracks, that will be used later in describing graphs.

5

UPS AND DOWNS – CURVES AND SLOPES

With your children use scrap cardboard to make a marble run or a ramp for toy cars. The one illustrated on the left is made from paper plates and the cylinder from inside a roll of kitchen paper. You need scissors, sticky tape and imagination and you can improvise with other scrap materials.

To help your young learners with their mathematics in school when they are older, talk about the spiral shapes, curves and slopes and how steep the slopes are.

Can you even make a run that has some ups as well as downs.

You could do experiments. Time how long it takes for a marble or car to travel down the ramp from different heights, and how far it goes on different surfaces once it reaches the bottom.

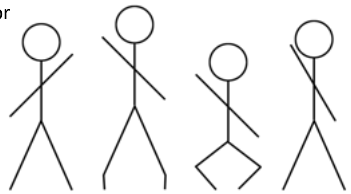
6

GRAPHS WORKOUT - LINES

Talk about which graphs the stick men demonstrate, how and why the arm positions show different lines, and what the equations could be.

All face the front and follow the leader who has his or her back to the group. The leader defines the x- and y- axes, then calls out equations in turn. Everyone uses their arms to make the graph for that equation.

What lines are these stick people demonstrating?



Start with easy equations, For example:

$y = x$, $y = -x$, $y = 2x$, $y = -2x$,

$y = -x + 3$ (stand on tiptoe!),

$y = -x - 2$ (crouch down),

$y = -2x - 2$ (still crouching down)....

Talk about the graphs and how you have acted them out.

For a challenge try: $y = 2$ (easy), $x=2$, $x+y=3$...

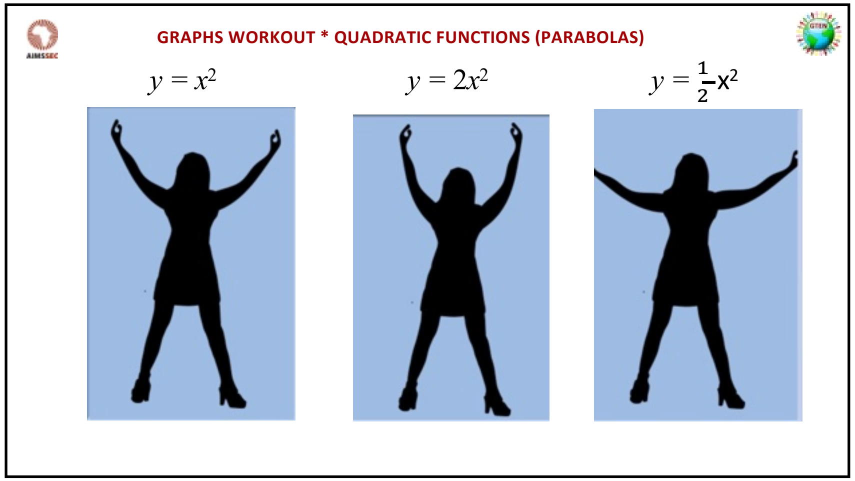
Let other members of the group act as the leader.

7

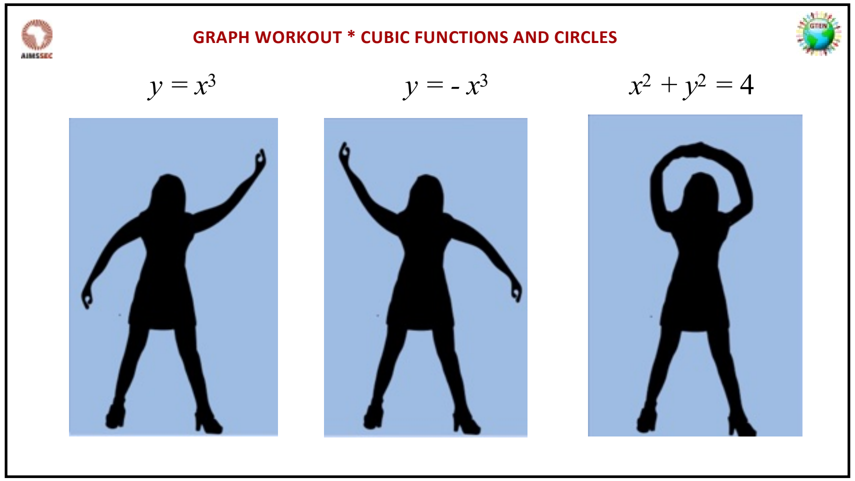
GRAPHS WORKOUT * THE MODULUS FUNCTION

Mod x: $y = |x|$ $y = -|x|$ $y = |-x|$

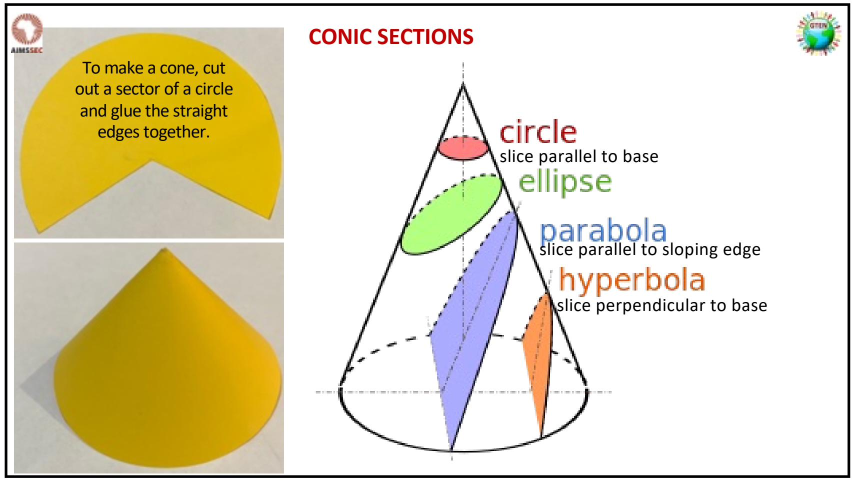
8


GRAPHS WORKOUT * QUADRATIC FUNCTIONS (PARABOLAS)
$y = x^2$
$y = 2x^2$
$y = \frac{1}{2}x^2$

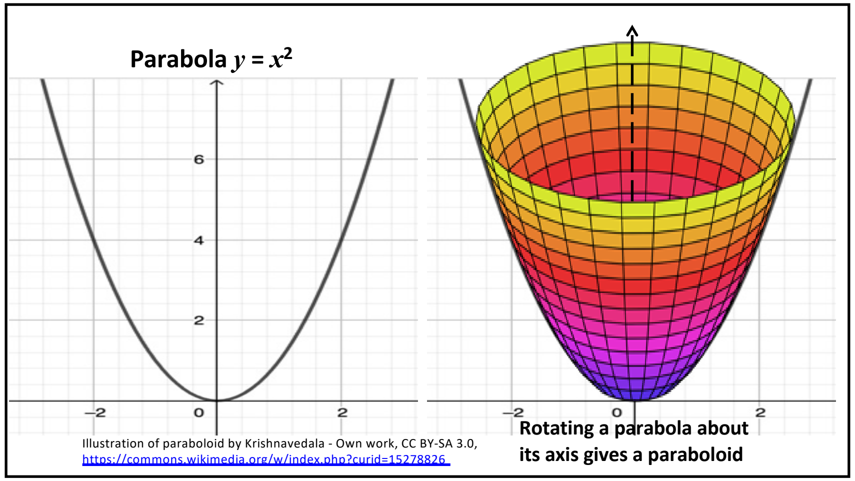
9


GRAPH WORKOUT * CUBIC FUNCTIONS AND CIRCLES
$y = x^3$
$y = -x^3$
$x^2 + y^2 = 4$

10


CONIC SECTIONS
To make a cone, cut out a sector of a circle and glue the straight edges together.
circle
slice parallel to base
ellipse
parabola
slice parallel to sloping edge
hyperbola
slice perpendicular to base

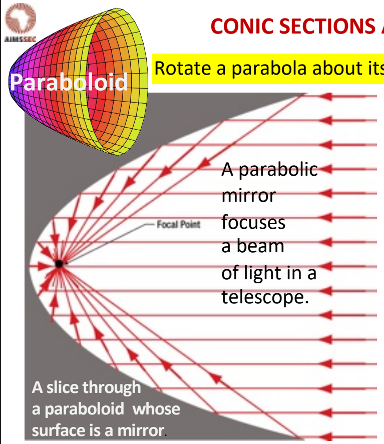
11


Parabola $y = x^2$
Rotating a parabola about its axis gives a paraboloid
Illustration of paraboloid by Krishnavedala - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=15278826

12

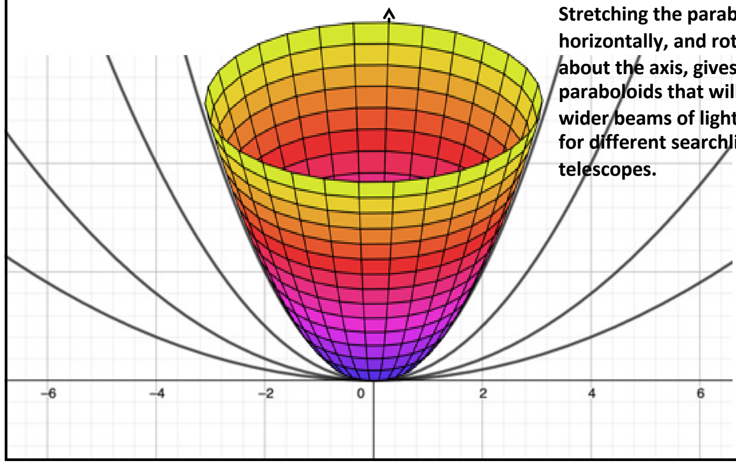
CONIC SECTIONS AND APPLICATIONS

Rotate a parabola about its axis and you get a paraboloid



Put a light source at the focal point of a paraboloid mirror so that the direction of the light is reversed (reverse the arrows) and you get a searchlight.

13



Stretching the parabola horizontally, and rotating it about the axis, gives 'wider' paraboloids that will reflect wider beams of light suitable for different searchlights and telescopes.

14

LET'S PLAY MATHEMATICALLY AND LEARN

https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically

This first book in the AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

Play Mathematically

- to develop a love for mathematics
- to unlock knowledge
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

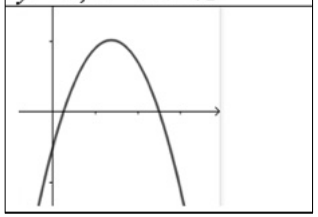
Play the games with the book and equipment you have to hand - or buy with the AIMING HIGH Family Games Kit which has everything you might need pre-prepared.

15

QUADRATIC MATCHING

https://aiminghigh.aimssec.ac.za/quadratic-matching-1/

EXAMPLE
$y = 11x - 5 - 2x^2$
$y = (2x - 1)(5 - x)$
$x = 0, y = -5$
$y = 0, x = 5$ or ½



The solutions of a quadratic equation
$ax^2 + bx + c = 0$
correspond to the x-coordinates of the points where the corresponding graph
$y = ax^2 + bx + c$ crosses the x-axis.

For example, the quadratic equation
$-2x^2 + 11x - 5 = 0$
or $11x - 5 - 2x^2 = 0$ (rearranging the equation)
can be factorized as $(2x - 1)(5 - x) = 0$.
The equation has solutions $x = 5$ or $\frac{1}{2}$

The graph of
y= $-2x^2 + 11x - 5$
cuts the x-axis at $y = 0$, $x = 5$ or $\frac{1}{2}$

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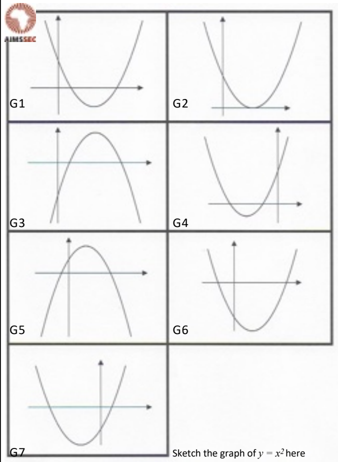
QUADRATIC MATCHING - INTERCEPTS WITH AXES



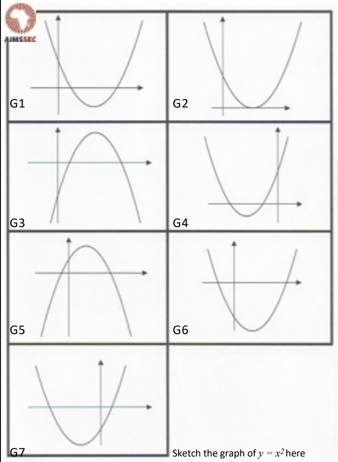
Match each of the graphs to the points where it cuts the x-axis, that is the line $y = 0$.
For example, G1 matches with X5 and Y2.

Y1 $x = 0, y = -16$	Y2 $x = 0, y = 16$
Y3 $x = 0, y = 16$	Y4 $x = 0, y = -8$
Y5 $x = 0, y = 8$	Y6 $x = 0, y = 8$
Y7 $x = 0, y = -16$	X1 $y = 0, x = 8$ or -2
X2 $y = 0, x = -4$ or -2	X3 $y = 0, x = -2$ or 4
X4 $y = 0, x = 4$ or 2	X5 $y = 0, x = 8$ or 2
X6 $y = 0, x = 4$	X7 $y = 0, x = -8$ or 2

G1	X5	Y2
G2		Y2
G3		Y3
G4		Y4
G5		Y4
G6		Y1
G7		Y1

17

QUADRATIC MATCHING



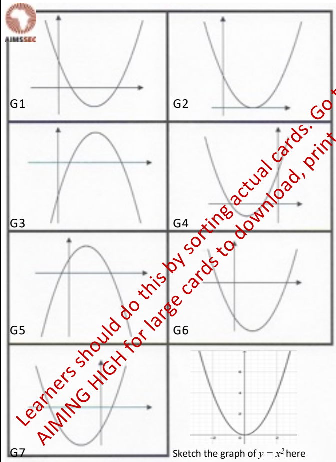
Match each of the graphs to its equation both in its quadratic polynomial form (A cards) and in its factorized form (B cards). For example, the graph G1 cuts the x-axis at two positive values of x, two positive roots or solutions. G1 matches with A5 and B5.

A1 $y = x^2 + 6x - 16$	A2 $y = x^2 - 8x + 16$
A3 $y = 8 - x^2 + 2x$	A4 $y = 6x - x^2 - 8$
A5 $y = x^2 - 10x + 16$	A6 $y = x^2 + 6x + 8$
A7 $y = x^2 - 6x - 16$	B1 $y = (x - 8)(x + 2)$
B2 $y = (x + 4)(x + 2)$	B3 $y = (x + 2)(4 - x)$
B4 $y = (x - 4)(2 - x)$	B5 $y = (x - 8)(x - 2)$
B6 $y = (x - 4)(x - 4)$	B7 $y = (x + 8)(x - 2)$

G1	A5	B5
G2		
G3		
G4		
G5		
G6		
G7		

18

QUADRATIC MATCHING - INTERCEPTS WITH AXES



Learners should do this by sorting actual cards. Go to AIMING HIGH for large cards to download, print and cut out

Match each of the graphs to the points where it cuts the axes. For example G1 matches with X4 and Y4.

Y1 $x = 0, y = -16$	Y2 $x = 0, y = 16$
Y2 $x = 0, y = 16$	Y3 $x = 0, y = -8$
Y4 $x = 0, y = 8$	Y4 $x = 0, y = 8$
Y1 $x = 0, y = -16$	X1 $y = 0, x = 8$ or -2
X2 $y = 0, x = -4$ or -2	X3 $y = 0, x = -2$ or 4
X4 $y = 0, x = 4$ or 2	X5 $y = 0, x = 8$ or 2
X6 $y = 0, x = 4$	X7 $y = 0, x = -8$ or 2

G1	X5	Y2
G2	X6	Y2
G3	X4	Y3
G4	X2	Y4
G5	X3	Y4
G6	X1	Y1
G7	X7	Y1

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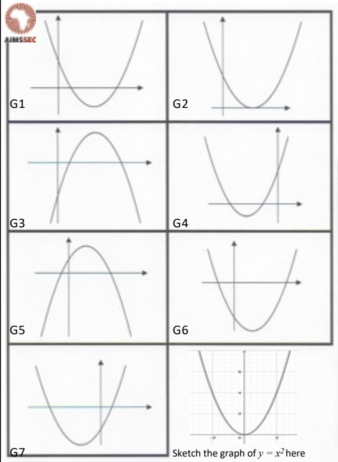
QUADRATIC MATCHING



Match each of the graphs to its equation both in its factorized form and its quadratic polynomial form.
For example, the graph G1 cuts the x-axis at two positive values of x, two positive roots or solutions.
G1 matches with A5 and B5.

A1 $y = x^2 + 6x - 16$	A2 $y = x^2 - 8x + 16$
A3 $y = 8 - x^2 + 2x$	A4 $y = 6x - x^2 - 8$
A5 $y = x^2 - 10x + 16$	A6 $y = x^2 + 6x + 8$
A7 $y = x^2 - 6x - 16$	B1 $y = (x - 8)(x + 2)$
B2 $y = (x + 4)(x + 2)$	B3 $y = (x + 2)(4 - x)$
B4 $y = (x - 4)(2 - x)$	B5 $y = (x - 8)(x - 2)$
B6 $y = (x - 4)(x - 4)$	B7 $y = (x + 8)(x - 2)$

G1	A5	B5
G2	A2	B6
G3	A4	B4
G4	A6	B2
G5	A3	B3
G6	A7	B1
G7	A1	B7

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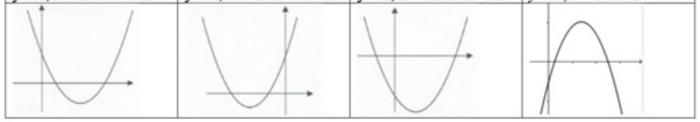
QUADRATIC MATCHING SUMMARY

$y = x^2 + 6x - 16$ G7 A1	$y = x^2 - 8x + 16$ G2 A2	$y = 8 - x^2 + 2x$ G5 A3	$y = 6x - x^2 - 8$ G3 A4
$y = (x + 8)(x - 2)$ B7	$y = (x - 4)(x - 4)$ B6	$y = (x + 2)(4 - x)$ B3	$y = (x - 2)(4 - x)$ B4
$x = 0, y = -16$ Y1	$x = 0, y = 16$ Y2	$x = 0, y = 8$ Y4	$x = 0, y = -8$ Y3
$y = 0, x = -8$ or 2 X7	$y = 0, x = 4$ X6	$y = 0, x = -2$ or 4 X3	$y = 0, x = 2$ or 4 X4

			EXAMPLE
$y = x^2 - 10x + 16$ G1 A5	$y = x^2 + 6x + 8$ G4 A6	$y = x^2 - 6x - 16$ G6 A7	$y = 11x - 5 - 2x^2$
$y = (x - 2)(x - 8)$ B5	$y = (x + 2)(x + 4)$ B2	$y = (x + 2)(x - 8)$ B1	$y = (2x - 1)(5 - x)$
$x = 0, y = 16$ Y2	$x = 0, y = 8$ Y4	$x = 0, y = -16$ Y1	$x = 0, y = -5$
$y = 0, x = 2$ or 8 X5	$y = 0, x = -2$ or -4 X2	$y = 0, x = -2$ or 8 X1	$y = 0, x = 5$ or ½



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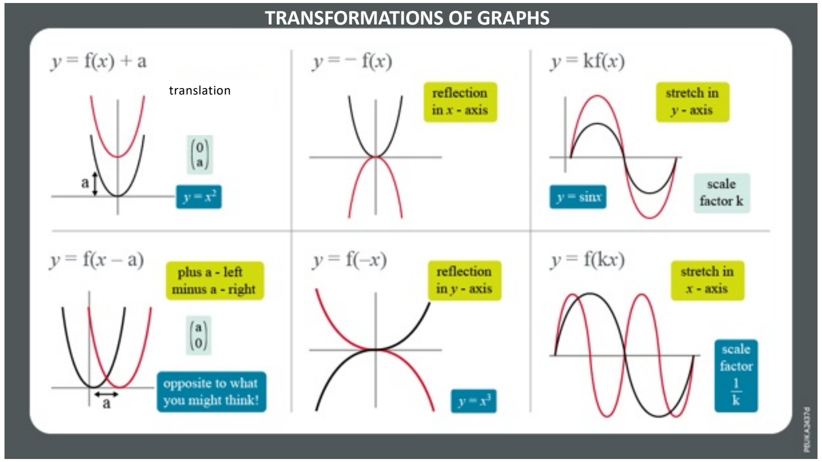
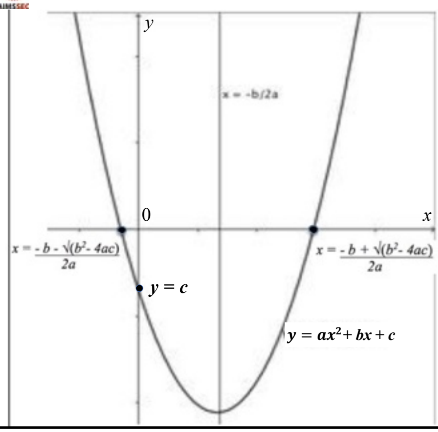
TRANSFORMATIONS OF GRAPHS



22

TRANSFORMATIONS OF GRAPH OF $y = x^2$ AND SOLUTIONS OF QUADRATIC EQUATIONS



The family of graphs of quadratic functions (parabolas) are all transformations of the graph of $y = x^2$ (the parent graph).

To find out how the parent graph is transformed to give the graph of the quadratic function $y = ax^2 + bx + c$ the equation can be written as

$y = a(x + p)^2 + q$ where

$p = \frac{b}{2a}$ and $q = c - \frac{b^2}{4a}$.

This diagram shows how the graph of $y = x^2$ is **stretched** in the y direction by a factor a, and **translated** p units in the positive x direction and q units in the y direction.

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UPS AND DOWNS – Equations of Motion using calculus

The horizontal and vertical distances travelled are *x* and *y*, and g the constant of acceleration vertically downwards due to gravity. The initial horizontal velocity is *u* metres per second and the initial vertical velocity is *v* metres per second upwards.

The *x*-coordinate and *y*-coordinate of the object at time *t* are:

$x = ut$ (1) (distance = velocity × time)

$y = vt - \frac{1}{2}gt^2$ (2)

Differentiate equations (1) and (2) to find the velocity

$\frac{dx}{dt} = u$ (Horizontal velocity - rate of change of position)

$\frac{dy}{dt} = v - gt$ (Vertical velocity - rate of change of position)

Differentiate again to find the acceleration

$\frac{d^2x}{dt^2} = 0$ (Horizontal acceleration - rate of change of velocity)

$\frac{d^2y}{dt^2} = -g$ (Vertical acceleration - rate of change of velocity)

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UPS AND DOWNS – Why the flight path (trajectory) of a projectile is a parabola

We take x and y to be the horizontal and vertical distances travelled and g the constant of acceleration vertically downwards due to gravity.

If the initial horizontal velocity is u metres per second then the x coordinate of the object at time t is $x = ut$ (1).

If the initial vertical velocity is v metres per second upwards, then the y coordinate of the object at time t is $y = vt - \frac{1}{2}gt^2$ (2).

These equations are given by Newton's Laws of Motion. Even if you don't understand the physics and calculus involved, it's easy to see why these equations give a parabolic flight path.

From equation (1): $t = ?$

To eliminate t and get the equation of the flight path, substitute this value of t in equation (2): $y = ?$

25

UPS AND DOWNS – Why the flight path of a projectile is a parabola

Take x and y to be the horizontal and vertical distances travelled and g the constant of acceleration due to gravity vertically downwards. If the initial horizontal velocity is u metres per second, then the x coordinate of the object at time t is $x = ut$ (1).

If the initial vertical velocity is v metres per second upwards, then the y coordinate of the object at time t is $y = vt - \frac{1}{2}gt^2$ (2).

Eliminate t to get the equation of the flight path. From (1): $t = \frac{x}{u}$

By substituting this value of t in equation (2): $y = \frac{v}{u}x - \frac{g}{2u^2}x^2$

This is a quadratic equation. It is the equation of a parabola $y = ax^2 + bx + c$

with the constants $a = -\frac{g}{2u^2}$, $b = \frac{v}{u}$ and $c = 0$.

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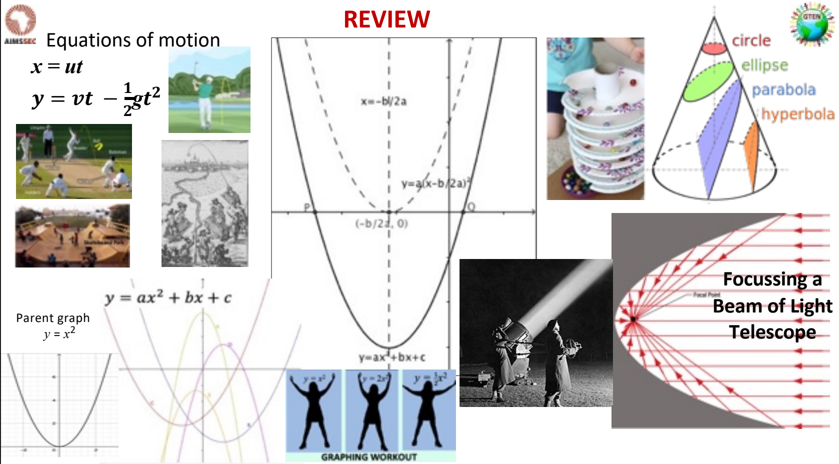
REVIEW

Equations of motion

$x = ut$

$y = vt - \frac{1}{2}gt^2$



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AIMINGHIGH TEACHER NETWORK

AIMSSEC

See the Learning Packs on AIMING HIGH

QUADRATIC FUNCTIONS https://aiminghigh.aimssec.ac.za/quadratic-functions/

QUADRATIC MATCHING 1 AND 2 https://aiminghigh.aimssec.ac.za/quadratic-matching-1/

https://aiminghigh.aimssec.ac.za/quadratic-matching-2/

GRAPHING QUADRATIC EQUATIONS

https://aiminghigh.aimssec.ac.za/graphing-quadratic-equations/

QUADRATIC EQUATIONS https://aiminghigh.aimssec.ac.za/quadratic-equations/

ANIMATION OF PROJECTILE MOTION https://phet.colorado.edu/en/simulations/projectile-motion

Most of the 'Let's Play Mathematically' games are on AIMING HIGH so play the games with your learners, and join the discussion to win https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically

Interactive demonstration of projectile motion https://phet.colorado.edu/en/simulations/projectile-motion

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