

Global Teacher Empowerment Network GTEN
 Saturday 29 July 2023 16.00 – 18.00 London Time

NUMBER BASES AND LOGIC

Base 8

Octal to Decimal

Months - Year

January	July	MONDAY
February	August	TUESDAY
March	September	WEDNESDAY
April	October	THURSDAY
May	November	FRIDAY
June	December	SATURDAY
		SUNDAY

p	q	$p \cup q$
1	1	1
1	0	1
0	1	1
0	0	0

10000, ??, 100, 31, 24, 22, 20, 17, 16, 15, 14, 13, 12, 11, 10

Base 16

Toni Beardon Caroline Ainslie Marilyn Buchanan

THE POISON PILL

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THE POISON PILL

There are **8** pills that look the same – same size, same shape, same color. One is **deadly** – it is a poison pill! The poison pill is **heavier**, the others are the same weight. You have a balance scale to weigh them

BUT you can only make two weighings...

How can you find and remove the poison pill?

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THE POISON PILL PROBLEM WITH 8 PILLS AND 2 WEIGHINGS

Weigh 3 pills against another 3 pills putting 2 aside temporarily

SAME DIFFERENT

If the two sets of 3 pills weigh the same then the poison pill is one of the 2 put aside. A **second weighing** will show which of the 2 pills is poisoned

If one of the two sets of 3 pills is heavier, a **second weighing** of one of those 3 pills (say pill A) against another of the 3 pills (say B) will determine which pill is poisoned. If pill A is heavier it is the poison pill. If pill B is heavier it is the poison pill. If A and B weigh the same C is the poison pill

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OCTAL NUMBER SYSTEM

BUT WHAT IF...ON ANOTHER DISTANT PLANET, OCTAL, WHERE THE INHABITANTS HAVE 4 DIGITS ON EACH HAND, WHAT MIGHT THEIR COUNTING SYSTEM BE?

The octal or base 8 number system uses 8 symbols to represent 0, 1, 2, 3, 4, 5, 6 and 7. It is sometimes used by computers. The decimal number $9 = 8+1$ is 11 in the octal system.

DECIMAL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
OCTAL	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17	20
	8^0							8^1								

DECIMAL	17	50	64	100	512	700	1000	4000	4096
OCTAL	21	62	100	144	1000	1274	1750	7640	10000
	$8^4 = 4096$				$8^3 = 512$				$8^4 = 4096$
	$8^2 = 64$								
	$8^1 = 8$								
	$8^0 = 1$								
		$=6 \times 8$	$=1 \times 64$	$=1 \times 64$	$=1 \times 512$	$=1 \times 512$	$=1 \times 512$	$=7 \times 512$	$=1 \times 4096$
		$+2$		$+4 \times 8$		$+7 \times 8$	$+5 \times 8$	$+6 \times 64$	
				$+4 \times 1$		$+4$		$+4 \times 8$	

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Octal to Decimal
You know that the 6, 5 and 3 give powers of 8

Octal

6	5	3
---	---	---

$8^2 \times 6 + 8^1 \times 5 + 8^0 \times 3$
 $(653)_8 = (427)_{10}$

Write 670 as a sum of powers of 8

$670_{10} = 1236_8$

$670 \div 8 = 83 \text{ r. } 6$
 $83 \div 8 = 10 \text{ r. } 3$
 $10 \div 8 = 1 \text{ r. } 2$
 $1 \div 8 = 0 \text{ r. } 1$

$6 \times 64 + 5 \times 8 + 3 = 384 + 40 + 3 = 427$

Hands of an alien from a distant planet

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THEN, WHAT IF...ON ANOTHER DISTANT PLANET INHABITANTS HAD 8 DIGITS ON EACH HAND? WHAT MIGHT THEIR COUNTING SYSTEM BE?

The table headings, or weights, are now in powers of 16.

16^3	16^2	16^1	1	Powers of 16
4096	256	16	1	Decimal Value
1000	100	10	1	Recorded in Base 16

Hands of an alien from a distant planet

THE HEXADECIMAL NUMBER SYSTEM is used by computers along with binary numbers.

The hexadecimal numeral system, often called "hex", is an alphanumeric system made up of 16 symbols (base 16). Hex uses the 10 decimal digits : 0,1,2,3,4,5,6,7,8,9, plus six extra symbols: A, B, C, D, E, and F.

To Convert Decimal to Hexadecimal
Divide the decimal number by 16. The remainders are the hexadecimal digits starting with the most significant digit first. If the remainder is 10 or greater it must be converted to its hexadecimal letter symbol (A, B, C, D, E, F).

Decimal	Hexadecimal	Decimal	Hexadecimal	Decimal	Hexadecimal
1	1	11	B	21	15
2	2	12	C	22	16
3	3	13	D	23	17
4	4	14	E	24	18
5	5	15	F	25	19
6	6	16	10	26	1A
7	7	17	11	27	1B
8	8	18	12	28	1C
9	9	19	13	29	1D
10	A	20	14	30	1E

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SUMMARY

Number Systems		
System	Base	Digits
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

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BOOLEAN ALGEBRA, BINARY NUMBERS, CIRCUITS AND TRUTH TABLES

George Boole (1815-1864), the son of a cobbler, was a school-teacher who enjoyed exploring mathematical ideas. In 1844 Boole was awarded the Royal Society medal for his paper *On a general method of analysis* and he was appointed professor at Cork University.

Boole combined logical deduction with algebra and extended the logic of the ancient Greek philosopher Aristotle. Boole developed a new branch of algebra and its associated arithmetical rules, now called Boolean Algebra. Individual statements, or propositions are either true or false, and are combined according to the rules of logic,

e.g.: Statement *p*: 'The sun is the centre of the solar system' (True) and Statement *q*: 'The Earth is flat' (False).

The statement *p* OR *q* is:
(The sun is the centre of the solar system)'OR '(The Earth is flat).

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BOOLEAN ALGEBRA, BINARY NUMBERS, CIRCUITS AND TRUTH TABLES

OR

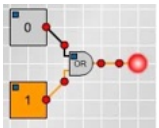

Boole gave true statements a truth value 1 and false statements a truth value 0.

The statement p OR q is also written $p \cup q$.

For p OR q to be true only one of p or q needs to have a truth value of 1.

This corresponds to the circuit and truth table shown.

If current flows from either switch the light on the right lights up.

p	q	$p \cup q$
1	1	?
1	0	?
0	1	?
0	0	?

We can write it as a kind of addition:
 $p + q$ stands for p OR q , and if p is true and q is false, we see that $p + q$ has the truth value $0+1 = 1$.

When is p OR q TRUE and when is it FALSE?
 Replace the question marks

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BOOLEAN ALGEBRA, CIRCUITS AND TRUTH TABLES

NOT

Statement p : 'The sun is the centre of the solar system' is true

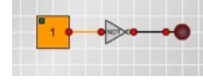
Statement NOT p : 'The sun is not the centre of the solar system' is false.

Statement q : 'The Earth is flat' is false.

Statement NOT q : 'The Earth is not flat' is true.

In general, if statement p is true then statement NOT p is false

p	NOT p
1	0
0	1



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BOOLEAN ALGEBRA, CIRCUITS AND TRUTH TABLES

AND

Statement p : 'The sun is the centre of the solar system' (True)

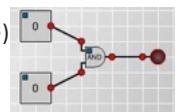
Statement q : 'The Earth is flat' (False)

Statement p AND q :
 (The Earth is flat) AND (The sun is the centre of the solar system) (False)

For p AND q to be true, both p and q must have the truth value 1.

If current flows from both of the two switches then the light on the right lights up.

But if either switch is off (no current flows) then the light is off.



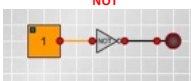
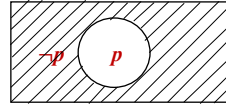
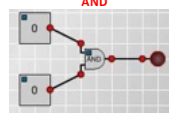
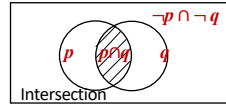

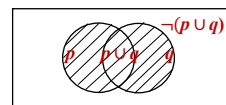
p	q	$p \cap q$
1	1	?
1	0	?
0	1	?
0	0	?

We can write it as a kind of multiplication:
 $p \times q$ stands for p AND q , also written $p \cap q$
 $0 \times 0 = 1 \times 0 = 0 \times 1 = 0$ and $1 \times 1 = 1$.


This corresponds to the circuit and truth table shown.
 Replace the question marks

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BOOLEAN ALGEBRA, CIRCUITS, TRUTH TABLES & VENN DIAGRAMS

<table border="1"> <thead> <tr> <th>p</th> <th>q</th> <th>NOT p</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table>	p	q	NOT p	1	1	0	1	0	1	0	1	0	0	0	1		 <p>Statement and Negation NOT</p>
p	q	NOT p															
1	1	0															
1	0	1															
0	1	0															
0	0	1															
<table border="1"> <thead> <tr> <th>p</th> <th>q</th> <th>$p \cap q$</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table>	p	q	$p \cap q$	1	1	1	1	0	0	0	1	0	0	0	0		 <p>Intersection AND</p>
p	q	$p \cap q$															
1	1	1															
1	0	0															
0	1	0															
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<table border="1"> <thead> <tr> <th>p</th> <th>q</th> <th>$p \cup q$</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table>	p	q	$p \cup q$	1	1	1	1	0	1	0	1	1	0	0	0		 <p>Union OR</p>
p	q	$p \cup q$															
1	1	1															
1	0	1															
0	1	1															
0	0	0															

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The word 'Wikipedia' in [ASCII](#) binary code, made up of 9 bytes (72 bits).

01010111 01101001 01101011
 01101001 01110000 01100101
 01100100 01101001 01100001


The lower-case *a*, is represented by the bit string 01100001 in the standard ASCII code. It can also be represented as the decimal number "97".

A **binary code** represents text, computer processor instructions, or any other data using a two-symbol system. The two-symbol system used is often "0" and "1" from the binary number system.


The binary code assigns a pattern of binary digits, also known as bits, to each character, instruction, etc.

For example, a binary string of eight bits (which is also called a byte) can represent any of 256 possible values and can, therefore, represent a wide variety of different items such as letters of the alphabet.


https://en.wikipedia.org/wiki/Binary_code



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FIND THE POISON PILL IN 3 WEIGHINGS




There are **16** pills that look the same – same size, same shape, same color. One is **deadly** – it is a poison pill! It is **heavier** than the other 15.


You have a balance scale to weigh them
BUT you can only make three weighings...




How can you find and remove the poison pill?



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THE POISON PILL PROBLEM WITH 16 PILLS AND 3 WEIGHINGS




Weigh 7 pills against another 7 pills putting 2 aside temporarily

SAME	DIFFERENT
------	-----------


If the two sets of 7 pills weigh the same then the poison pill is one of the 2 put aside.
A second weighing will show which of the 2 pills is poisoned.

If one of sets of 7 pills is heavier weigh 3 against 3 and put one aside.
 If the two sets of 3 are the same, then the pill put aside is poisoned.
 If you can identify a set of three and know that one is poisoned, then a **third weighing** will find the poisoned pill by weighing one of those 3 pills (say pill A) against another of the 3 pills (say B) as with the 8 pills problem.

For 32 (2^5) pills the problem can be solved with 4 weighings.
 For 64 (2^6) pills you need 5 weighings.
 In general, for 2^n pills where $n > 3$, you need 2^{n-1} weighings.



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Let's return to those alien worlds and think of other number bases...

What is the missing number in this set?

10000, ???, 100,
 31, 24, 22, 20,
 17, 16, 15, 14, 13, 12, 11, 10

This set of numbers have one property in common.

What do you notice?
 Replace the ??? using digits chosen from 0 to 9
 What might the elements of the set have in common?

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SOLUTION

These are all the number 16 in different bases

10000_2 ??? 100_4

31_5 24_6 22_7 20_8

17_9 16_{10} 15_{11} 14_{12} 13_{13} 12_{14} 11_{15} 10_{16}

Base	2	3	4
Number	10000_2	121_3	100_4
	16	9+6+1	16

Base	5	6	7	8
Number	31_5	24_6	22_7	20_8
	15+1	12+4	14+2	16

Base	9	10	11	12	13	14	15	16
Number	17_9	16_{10}	15_{11}	14_{12}	13_{13}	12_{14}	11_{15}	10_{16}
	9+7	16	15+1	14+2	13+3	12+4	15+1	16

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REVIEW

NUMBERS IN JAPANESE

0	0	十	10	百	100
一	1	十	10	千	1,000
二	2	十	10	万	10,000
三	3	十	10	百万	1,000,000
四	4	十	10	千万	10,000,000
五	5	十	10	亿	100,000,000
六	6	十	10	兆	1,000,000,000,000
七	7	十	10	京	10,000,000,000,000,000
八	8	十	10	垓	100,000,000,000,000,000,000
九	9	十	10	穰	1,000,000,000,000,000,000,000,000
十	10	十	10	十	10,000,000,000,000,000,000,000,000,000

BABYLONIAN NUMBERS

Base 60

BALANCE POWER

How can you weigh 14 kilos of potatoes with weights 1, 2, 4 and 8?

What is the greatest mass you can weigh with those weights?

What about all weights from 1 to 100 kilos? What if you could put weights in both pans?

Ram had 15 coins, all the same kind, and he put them into 4 bags. He labelled each bag with the number of coins inside it. He could then pay any sum of money from 1 coin to 15 coins with one or more of the bags and without opening any of the bags. How could he do that? What about paying higher amounts?

Base 16 Hexadecimal

10000_2 ???, 100_4 , 31_5 , 24_6 , 22_7 , 20_8 , 17_9 , 16_{10} , 15_{11} , 14_{12} , 13_{13} , 12_{14} , 11_{15} , 10_{16}

	p	q	$p \cup q$
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	0

Binary

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AIMINGHIGH TEACHER NETWORK

AIMSSEC African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

WHICH SCRIPTS? <https://aiminghigh.aimssec.ac.za/which-scripts/>

BALANCE POWER <https://aiminghigh.aimssec.ac.za/balance-power/>

MONEY BAGS <https://aiminghigh.aimssec.ac.za/money-bags/>

BEAUTIFUL NUMBERS <https://aiminghigh.aimssec.ac.za/beautiful-numbers>

Wiki article on binary codes https://en.wikipedia.org/wiki/Binary_code

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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE

On the AIMING HIGH website there are freely downloadable worksheets, Inclusion and Home Learning Guides with activities for learners of all ages and attainment levels, Notes for Teachers with solutions, Key Questions and Diagnostic Quizzes for formative assessment.

AIMSSEC GTEN YouTube Channel



<https://www.youtube.com/c/MathsToys/videos>

AIMSSEC FACEBOOK <https://www.facebook.com/aimssecsa/>

Collaborative Professional Development

<https://aiminghigh.aimssec.ac.za/category/cpd>

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SCHOOLS ENRICHMENT CENTRE 

Thanks for coming to this workshop.

Use the AIMSSEC ideas
on AIMING HIGH and add comments.

Share what you have learned
with other teachers.

Try to help all your learners to have a
'YES I CAN'
attitude to mathematics.

