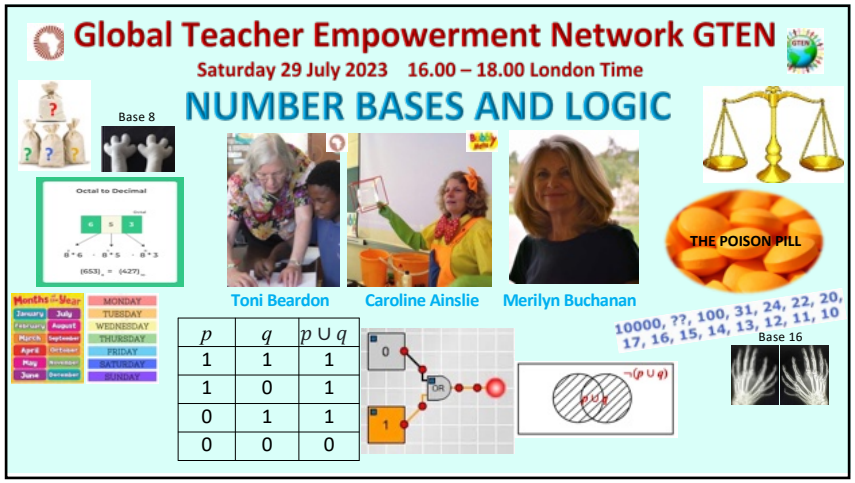
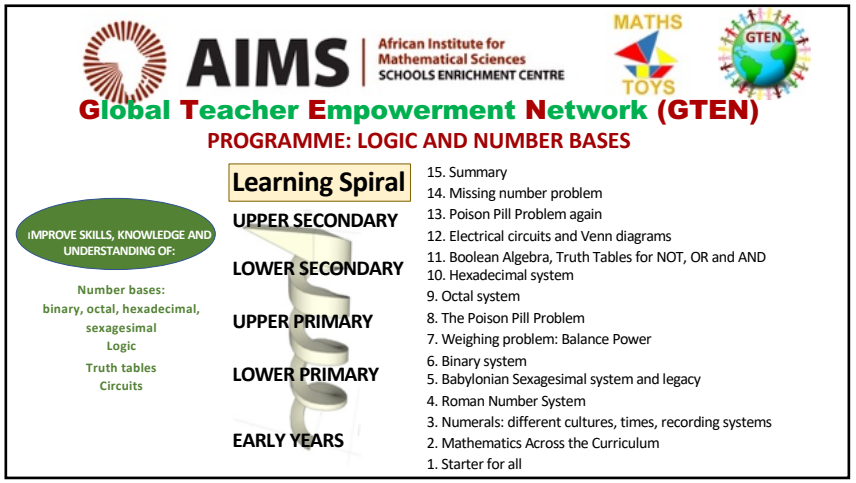
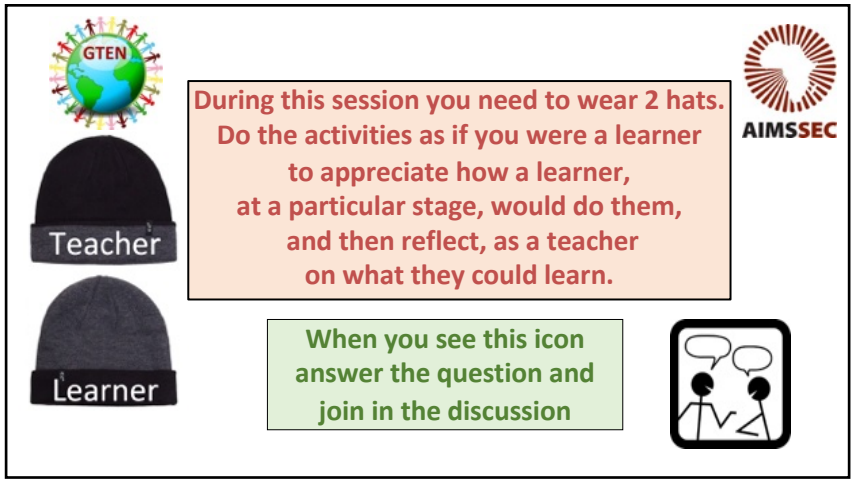

Global Teacher Empowerment Network GTEN
Saturday 29 July 2023 16.00 – 18.00 London Time
NUMBER BASES AND LOGIC
Base 8
Toni Beardon
Caroline Ainslie
Merilyn Buchanan
THE POISON PILL
Base 16
10000, ??, 100, 31, 24, 22, 20, 17, 16, 15, 14, 13, 12, 11, 10
p q p ∪ q
1 1 1
1 0 1
0 1 1
0 0 0

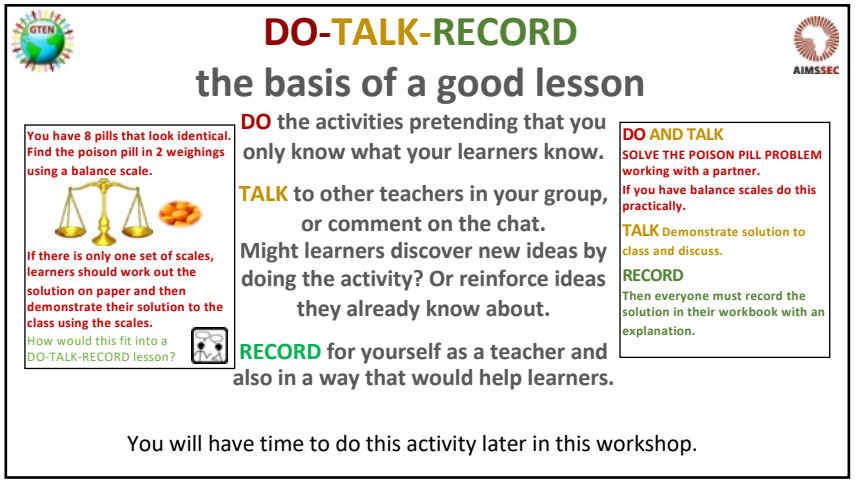
1


AIMS
African Institute for Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE
MATHS TOYS
Global Teacher Empowerment Network (GTEN)
PROGRAMME: LOGIC AND NUMBER BASES
IMPROVE SKILLS, KNOWLEDGE AND UNDERSTANDING OF:
Number bases: binary, octal, hexadecimal, sexagesimal
Logic
Truth tables
Circuits
Learning Spiral
UPPER SECONDARY
LOWER SECONDARY
UPPER PRIMARY
LOWER PRIMARY
EARLY YEARS
15. Summary
14. Missing number problem
13. Poison Pill Problem again
12. Electrical circuits and Venn diagrams
11. Boolean Algebra, Truth Tables for NOT, OR and AND
10. Hexadecimal system
9. Octal system
8. The Poison Pill Problem
7. Weighing problem: Balance Power
6. Binary system
5. Babylonian Sexagesimal system and legacy
4. Roman Number System
3. Numerals: different cultures, times, recording systems
2. Mathematics Across the Curriculum
1. Starter for all

2


GTEN
Teacher
Learner
AIMSSEC
During this session you need to wear 2 hats.
Do the activities as if you were a learner
to appreciate how a learner,
at a particular stage, would do them,
and then reflect, as a teacher
on what they could learn.
When you see this icon
answer the question and
join in the discussion

3


DO-TALK-RECORD
the basis of a good lesson
AIMSSEC
You have 8 pills that look identical. Find the poison pill in 2 weighings using a balance scale.
If there is only one set of scales, learners should work out the solution on paper and then demonstrate their solution to the class using the scales.
How would this fit into a DO-TALK-RECORD lesson?
DO the activities pretending that you only know what your learners know.
TALK to other teachers in your group, or comment on the chat.
Might learners discover new ideas by doing the activity? Or reinforce ideas they already know about.
RECORD for yourself as a teacher and also in a way that would help learners.
DO AND TALK
SOLVE THE POISON PILL PROBLEM working with a partner.
If you have balance scales do this practically.
TALK Demonstrate solution to class and discuss.
RECORD
Then everyone must record the solution in their workbook with an explanation.
You will have time to do this activity later in this workshop.

4

MATHEMATICS ACROSS THE CURRICULUM

If you teach young learners talk about this picture and ask them to count people and objects in it.

People have recorded numbers for more than 40000 years.

Numbering systems have progressed from the use of fingers and tally marks, to the use of sets of glyphs to represent numbers and allow efficient methods of calculation.

Number systems have evolved to serve different functions, to keep count, for trading and for time and astronomical calculations.

The earliest known notations for numbers emerged in Mesopotamia about 6000 years ago.

Roman numerals are written with combinations of letters from the Latin alphabet, each with a fixed integer value. They originated in ancient Rome and were the most familiar numeration system in Europe until the 14th century, although more efficient number systems were in use in Asia.

5

6

WHICH SCRIPTS?

Here are six numbers written in five different scripts.

Can you sort out which is which?

Write 51 in each script.

੧੦੦	13	৫৮	۱۳	二十五
੮੩	۲	੨੫	83	১০০
二	五十八	2	১৩	一百
۲۵	۸۳	੧੩	২	58
25	৮৩	੨	八十三	۱۰۰
۵۸	十三	100	੫੮	২৫

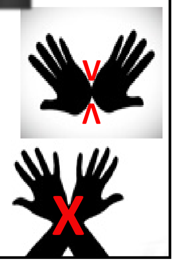
2	25	58	83	13	100	Hindu Arabic
২	২৫	৫৮	৮৩	১৩	১০০	Bengali Assamese
੨	੨੫	੫੮	੮੩	੧੩	੧੦੦	Gurmukhi (Sikh)
二	二十五	五十八	八十三	十三	一百	Chinese
۲	۲۵	۵۸	۸۳	۱۳	۱۰۰	Urdu (similar to Arabic)

https://aiminghigh.aimssec.ac.za/which-scripts/

7

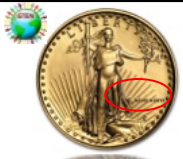
As easy as one, two, three

Make that ten

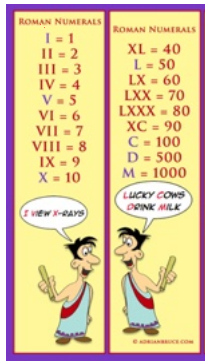


Give me 5

8



Where might you still see Roman Numerals used today?



- How would you write 55 in Roman numerals?
- What do you think is the value of XCVII?
- How would you write 274 in Roman numerals?

Look at the chart.

- How does the Roman system differ from the Arabic?
- What do you notice about Roman numerals?
- Can you see from the examples how the Roman numbering system works?

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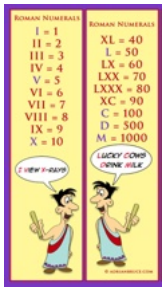
- How would you write 55 in Roman numerals? **LV**
- What do you think is the value of XCVII? **97**
- How would you write 274 in Roman numerals? **CCLXXIV**



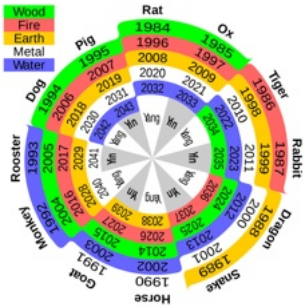
- ✓ **No zero.**
- ✓ **Not based on place value.**
- ✓ Roman Numbers written with combinations of letters from the Latin alphabet, each with a fixed integer value.
- ✓ Only seven characters are used: I, V, X, L, C, D, M
- ✓ Add numerals of the same character that come together. *For example, III = 3, CCC = 300.*
- ✓ Can't use the same symbol more than three times in a row.
- ✓ **A numeral of lesser value before a numeral of greater value decreases the second numeral by the amount of the first. For example, IV equals 4 because V (5) is decreased by I.**
- ✓ **A numeral of lesser value after a numeral of greater value to increase the first numeral by the amount of the second.**
  For example, VI equals 11 because V (10) is increased by I.

10

CALENDARS AND THE BASE 60 OR SEXAGESIMAL NUMBER SYSTEM

The Babylonian calendar had 12 months each of 30 days.

The Hindu or Vedic calendar follows the lunar months, but inserts an extra full month, once every 32–33 months, to ensure that the festivals and crop-related rituals fall in the appropriate season, unlike our Gregorian calendar which adds additional days to the month to adjust for the mismatch between twelve lunar cycles (354 lunar days) and 365 solar days.



Chinese 60-year cycles

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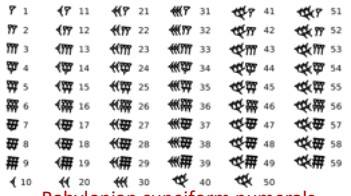
BASE 60 OR SEXAGESIMAL NUMBER SYSTEM

- The sexagesimal number system originated with the ancient Sumerians in the 3rd millennium BCE.
- It was used by the ancient Babylonians for calculations in astronomy, time and trade.
- It is still used today, in a modified form.

Can you think how it is still used today?



Babylonian cuneiform numerals

TIME, ANGLES AND GEOGRAPHIC COORDINATES OF LATITUDE AND LONGITUDE.

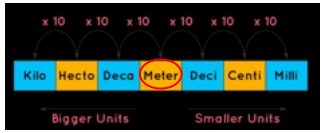
60 seconds in a minute
60 minutes in an hour

12

Mathematics Across the Curriculum

It wasn't until the Late Middle Ages, that Roman numerals began to be replaced in Europe by **Arabic numerals**.

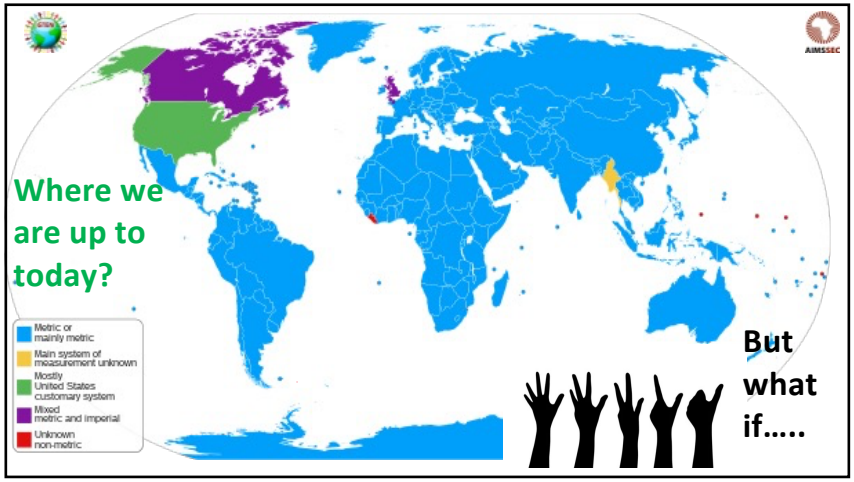
Leonard of Pisa, now known as **Fibonacci,** promoted the use of Arabic numerals for trading. He introduced the Hindu-Arabic place-value decimal system into Europe in 1202 in his book Liber Abaci (which means 'book of calculations').



At the time of the French Revolution, 1799, the existing systems of measures, with their multiple bases, were regarded as impractical for trade and were replaced by the decimal system.

Can you name some of those 'old' measurement bases?

13



14

Inhabitants of Planet Uno do 'one finger counting' Thumbs up 👍 or thumbs down 👎

1. How would Unitarians communicate numbers?
2. How would this differ to ten fingered Earthlings?
3. When they record numbers how many symbols would they use?
4. How could they use thumbs up thumbs down for the number 3?

Let's name their system

When it's just one thing or the other (2 choices) in this case 1 or 0 we call it a BINARY SYSTEM.

Can you figure out from this table how inhabitants of Planet Uno record numbers?

1010	?	?	?	?	101	100	11	10	1

UNO MAS

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BINARY CODE

DECIMAL	1	2 $=2^1$	3	4 $=2^2$	5	6	7	8 $=2^3$	9	10
BINARY	1	10	11	100	101	110	111	1000	1001	1010

If 10 represents the number 2 in binary code (and not the number ten)

and 100 represents the number 4 (and not the number one hundred)

and 101 represents the number 5 (and not the number one hundred and one).

What does 111 represent?

A. 6 B. 3 C. 7 D. 8

What does the number 10001 represent?

A. 14 B. 15 C. 16 D. 17

16

MONEY BAGS

Ram had 15 coins, all the same kind, and he put them into 4 bags.

He labelled each bag with the number of coins inside it.

He could then pay any sum of money from 1 coin to 15 coins using one or more of the bags without opening any of the bags.

How many coins did he put in each bag?

What could this have to do with the way computers work?

The bags have 1, 2, 4 and 8 coins in them.
You can check that all amounts can be paid with these numbers.

https://aiminghigh.aimssec.ac.za/money-bags/

17

BALANCE POWER

How would you weigh all integer masses from 1 to 60 with 6 weights putting the weights in one pan and the object in the other pan? Which weights are used?

This list shows how to weigh masses 1, 2, 3, 4, 5, 6, and 7 units.

1
2
3=1 + 2
4
5=1 + 4
6=2 + 4
7=1 + 2 + 4

Let's answer and extend the questions above in stages:

1. What additional weight is needed to weigh 1, 2, 3,....15 units?
2. What pattern do you notice in the weights used? Can you explain it?
3. When you include the weight 8 units, what objects can now be weighed?
4. What weights are needed to weigh objects of 1, 2, 3 ... 100 units?
5. What weights are needed to weigh 1 to 1000 units or 1 to n units?

https://aiminghigh.aimssec.ac.za/balance-power/

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What if we had a different Base? For a bigger challenge can you find how many weights are needed if you can put weights in both pans?

Try it.

Weights	Object	Left Pan	Right Pan
1	1	1	Object
1, 3	2	3	1, Object
1, 3	3	3	Object
1, 3	4	1, 3	Object
1, 3, 9	5	9	1, 3, Object
1, 3, 9	6	9	3, Object
1, 3, 9	7	1, 9	3, Object
1, 3, 9	8	9	1, Object
1, 3, 9	9	9	Object
1, 3, 9	10	1, 9	Object
1, 3, 9	11	3, 9	1, Object
1, 3, 9	12	3, 9	Object
1, 3, 9	13	1, 3, 9	Object
1, 3, 9, 27	14	27	1, 3, 9, Object

1. What weights are needed to weigh 1, 2, 3,....13 units? What pattern do you notice in the weights used? Can you explain it?
2. How many objects can be weighed when you include another weight in the set used?
3. What is that 4th weight and what is the heaviest object that can be weighed with the 4 weights.
4. How many weights are needed to weigh 1 to 1000 units or 1 to n units?

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