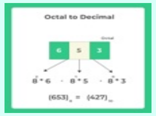
Global Teacher Empowerment Network GTEN

PART 2

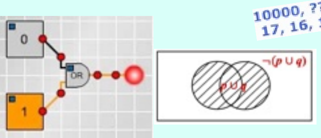
NUMBER BASES AND LOGIC

Base 8



Toni Beardon Caroline Ainslie Merilyn Buchanan

p	q	$p \cup q$
1	1	1
1	0	1
0	1	1
0	0	0



10000, ??, 100, 31, 24, 22, 20, 17, 16, 15, 14, 13, 12, 11, 10

Base 16

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THE POISON PILL

There are **8** pills that look the same – same size, same shape, same color.
One is **deadly** – it is a poison pill!
The poison pill is **heavier,** the others are the same weight.
You have a balance scale to weigh them

BUT you can only make two weighings…

How can you find and remove the poison pill?

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THE POISON PILL PROBLEM WITH 8 PILLS AND 2 WEIGHINGS

Weigh 3 pills against another 3 pills putting 2 aside temporarily

SAME

If the two sets of 3 pills weigh the same then the poison pill is one of the 2 put aside.
A second weighing will show which of the 2 pills is poisoned

DIFFERENT

If one of the two sets of 3 pills is heavier, a second weighing of one of those 3 pills (say pill A) against another of the 3 pills (say B) will determine which pill is poisoned.
If pill A is heavier it is the poison pill.
If pill B is heavier it is the poison pill.
If A and B weigh the same C is the poison pill

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OCTAL NUMBER SYSTEM

BUT WHAT IF…ON ANOTHER DISTANT PLANET, OCTAL, WHERE THE INHABITANTS HAVE 4 DIGITS ON EACH HAND, WHAT MIGHT THEIR COUNTING SYSTEM BE?

The octal or base 8 number system uses 8 symbols to represent 0, 1, 2, 3, 4, 5, 6 and 7. It is sometimes used by computers.
The decimal number 9 = 8+1 is 11 in the octal system.

DECIMAL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
OCTAL	1 8^0	2	3	4	5	6	7	10 8^1	11	12	13	14	15	16	17	20

DECIMAL	17	50	64 8^2	100	512 8^3	700	1000	4000	4096 8^4
OCTAL $8^4 = 4096$ $8^3 = 512$ $8^2 = 64$ $8^1 = 8$ $8^0 = 1$	21	62 =6x8 +2	100 =1x64	144 =1x64 +4x8 +4x1	1000 =1x512	1274 =1x512 +2x64 +7x8 +4	1750 =1x512 +7x64 +5x8	7640 =7x512 +6x64 +4x8	10000 =1x4096

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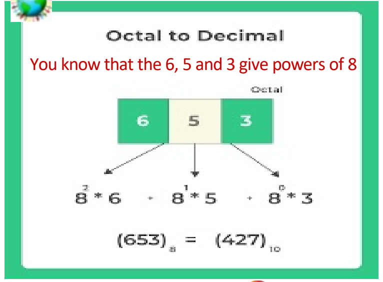
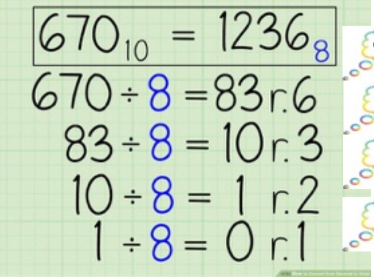


Write 670 as a sum of powers of 8



6 units

3 x 8

2 x 8^2
2 x 64

1 x 8^3
1 x 512

6x64 + 5x8 + 3 =
384 + 40 + 3 = 427

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THEN, WHAT IF…ON ANOTHER DISTANT PLANET INHABITANTS HAD 8 DIGITS ON EACH HAND? WHAT MIGHT THEIR COUNTING SYSTEM BE?

The table headings, or weights, are now in powers of 16,.				
16^3	16^2	16^1	1	**Powers of 16**
4096	256	16	1	**Decimal Value**
1000	**100**	**10**	**1**	**Recorded in Base 16**

Hands of an alien from a distant planet

THE HEXADECIMAL NUMBER SYSTEM is used by computers along with binary numbers.

Decimal	Hexadecimal	Decimal	Hexadecimal	Decimal	Hexadecimal
1	1	11	B	21	15
2	2	12	C	22	16
3	3	13	D	23	17
4	4	14	E	24	18
5	5	15	F	25	19
6	6	16	10	26	1A
7	7	17	11	27	1B
8	8	18	12	28	1C
9	9	19	13	29	1D
10	A	20	14	30	1E

The **hex**adecimal numeral system, often called "**hex**", is an **alphanumeric** system made up of 16 symbols (**base 16**). Hex uses the 10 decimal digits : 0,1,2,3,4,5,6,7,8,9, plus six extra symbols: A, B, C, D, E, and F.

To Convert Decimal to Hexadecimal

Divide the decimal number by 16. The **remainders** are the hexadecimal digits starting with the most significant digit first. If the remainder is 10 or greater it must be converted to its hexadecimal letter symbol (A, B, C, D, E, F).

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SUMMARY

Number Systems		
System	**Base**	**Digits**
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

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BOOLEAN ALGEBRA, BINARY NUMBERS, CIRCUITS AND TRUTH TABLES

George Boole (1815-1864), the son of a cobbler, was a school-teacher who enjoyed exploring mathematical ideas.
In 1844 Boole was awarded the Royal Society medal for his paper *On a general method of analysis* and he was appointed professor at Cork University.
Boole combined logical deduction with algebra and extended the logic of the ancient Greek philosopher Aristotle. Boole developed a new branch of algebra and its associated arithmetical rules, now called Boolean Algebra.
Individual statements, or propositions are either true or false, and are combined according to the rules of logic,
e.g.: Statement p: 'The sun is the centre of the solar system' (True) and
Statement q: 'The Earth is flat' (False).
The statement p OR q *is:*
(The sun is the centre of the solar system)'OR '(The Earth is flat).

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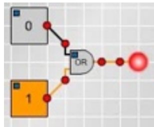
BOOLEAN ALGEBRA, BINARY NUMBERS, CIRCUITS AND TRUTH TABLES
OR

Boole gave true statements a truth value 1 and false statements a truth value 0.

The statement p OR q is also written $p \cup q$.

For p OR q to be true only one of p or q needs to have a truth value of 1.



This corresponds to the circuit and truth table shown.

If current flows from either switch the light on the right lights up.

We can write it as a kind of addition:

$p + q$ stands for p OR q , and if p is true and q is false, we see that $p + q$ has the truth value 0+1 = 1.

When is p OR q **TRUE and when is it FALSE?**

Replace the question marks

p	q	$p \cup q$
1	1	?
1	0	?
0	1	?
0	0	?

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BOOLEAN ALGEBRA, CIRCUITS AND TRUTH TABLES
NOT

Statement p: 'The sun is the centre of the solar system' is true

Statement NOTp 'The sun is not the centre of the solar system' is false.

Statement q: 'The Earth is flat' is false.

Statement NOTq: 'The Earth is not flat' is true.

In general, if statement p is true then statement NOTp is false

NOTp

p	$\neg p$
1	0
0	1

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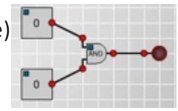
BOOLEAN ALGEBRA, CIRCUITS AND TRUTH TABLES
AND



Statement p 'The sun is the centre of the solar system' (True)

Statement q: 'The Earth is flat' (False)

Statement p AND q:

(The Earth is flat) AND (The sun is the centre of the solar system) (False)

For p AND q to be true, both p and q must have the truth value 1.

If current flows from both of the two switches then the light on the right lights up.

But if either switch is off (no current flows) then the light is off.

We can write it as a kind of multiplication:

$p \times q$ stands for p AND q, also written $p \cap q$

$0 \times 0 = 1 \times 0 = 0 \times 1 = 0$ and $1 \times 1 = 1$.

This corresponds to the circuit and truth table shown.

Replace the question marks

p	q	$p \cap q$
1	1	?
1	0	?
0	1	?
0	0	?

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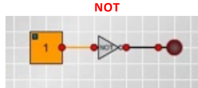
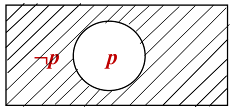
BOOLEAN ALGEBRA, CIRCUITS, TRUTH TABLES & VENN DIAGRAMS

NOTp

p	$\neg p$
1	0
0	1

NOT

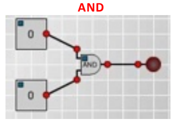


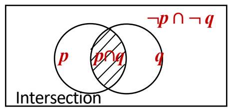


Statement and Negation
NOT

p	q	$p \cap q$
1	1	1
1	0	0
0	1	0
0	0	0

AND

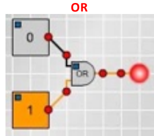


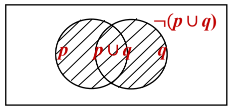


Intersection
AND

p	q	$p \cup q$
1	1	1
1	0	1
0	1	1
0	0	0

OR





Union
OR

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01010111 01101001 01101011
01101001 01110000 01100101
01100100 01101001 01100001

The word 'Wikipedia' in ASCII binary code, made up of 9 bytes (72 bits).

The lower-case *a*, is represented by the bit string 01100001 in the standard ASCII code.
It can also be represented as the decimal number "97".

A **binary code** represents text, computer processor instructions, or any other data using a two-symbol system. The two-symbol system used is often "0" and "1" from the binary number system.
The binary code assigns a pattern of binary digits, also known as bits, to each character, instruction, etc.
For example, a binary string of eight bits (which is also called a byte) can represent any of 256 possible values and can, therefore, represent a wide variety of different items such as letters of the alphabet.
https://en.wikipedia.org/wiki/Binary_code

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FIND THE POISON PILL IN 3 WEIGHINGS

There are **16** pills that look the same – same size, same shape, same color.
One is **deadly** – it is a poison pill!
It is **heavier** than the other 15.

You have a balance scale to weigh them
BUT you can only make three weighings…

How can you find and remove the poison pill?

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THE POISON PILL PROBLEM WITH 16 PILLS AND 3 WEIGHINGS

Weigh 7 pills against another 7 pills putting 2 aside temporarily

SAME

If the two sets of 7 pills weigh the same then the poison pill is one of the 2 put aside.
A second weighing will show which of the 2 pills is poisoned.

DIFFERENT

If one of sets of 7 pills is heavier weigh 3 against 3 and put one aside.
If the two sets of 3 are the same, then the pill put aside is poisoned.
If you can identify a set of three and know that one is poisoned, then a third weighing will find the poisoned pill by weighing one of those 3 pills (say pill A) against another of the 3 pills (say B) as with the 8 pills problem.

For 32 (2^5) pills the problem can be solved with 4 weighings.
For 64 (2^6) pills you need 5 weighings.
In general, for 2^n pills where n>3, you need 2^{n-1} weighings.

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Let's return to those alien worlds and think of other number bases…

What is the missing number in this set?

10000, ???, 100,
31, 24, 22, 20,
17, 16, 15, 14, 13, 12, 11, 10

This set of numbers have one property in common.

What do you notice?

Replace the ??? using digits chosen from 0 to 9

What might the elements of the set have in common?

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SOLUTION

These are all the number 16 in different bases

10000_2 ??? 100_4

31_5 24_6 22_7 20_8

17_9 16_{10} 15_{11} 14_{12} 13_{13} 12_{14} 11_{15} 10_{16}

Base	2	3	4
Number	10000_2	121_3	100_4
	16	9+6+1	16

Base	5	6	7	8
Number	31_5	24_6	22_7	20_8
	15+1	12+4	14+2	16

Base	9	10	11	12	13	14	15	16
Number	17_9	16_{10}	15_{11}	14_{12}	13_{13}	12_{14}	11_{15}	10_{16}
	9+7	16	15+1	14+2	13+3	12+4	15+1	16

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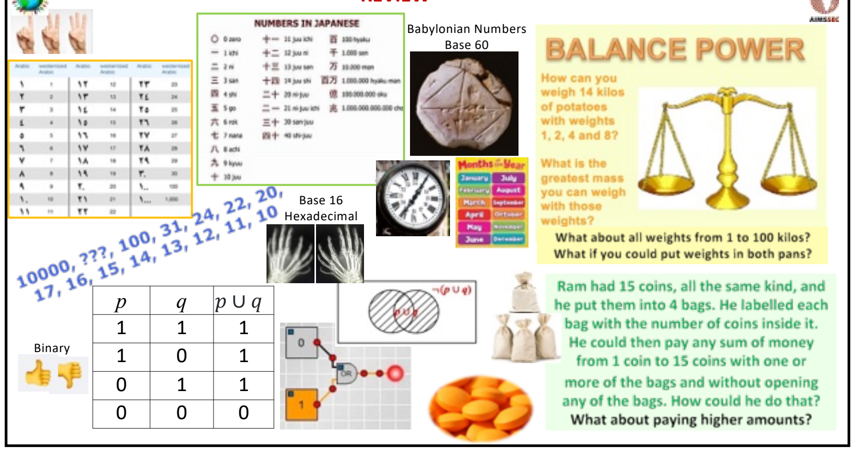
REVIEW



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AIMINGHIGH TEACHER NETWORK

AIMSSEC

WHICH SCRIPTS? https://aiminghigh.aimssec.ac.za/which-scripts/

BALANCE POWER https://aiminghigh.aimssec.ac.za/balance-power/

MONEY BAGES https://aiminghigh.aimssec.ac.za/money-bags/

BEAUTIFUL NUMBERS https://aiminghigh.aimssec.ac.za/beautiful-numbers

Wiki article on binary codes https://en.wikipedia.org/wiki/Binary_code

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LET'S PLAY MATHEMATICALLY AND LEARN

Order from AMAZON or TARQUIN https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically

Play Mathematically
- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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GTEN

Thanks for coming to this workshop.

Use the AIMSSEC ideas
on AIMING HIGH and add comments.

Share what you have learned
with other teachers.

Try to help all your learners to have a
'YES I CAN'
attitude to mathematics.

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