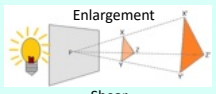
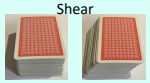
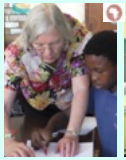
Global Teacher Empowerment Network GTEN

Saturday 18 May 2024 16.00 – 18.00 London Time

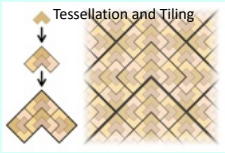
SHADOWS SIMILARITY AND ENLARGEMENT

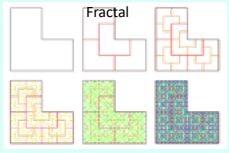


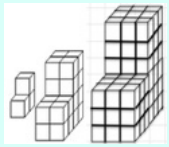


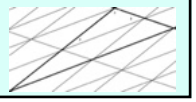


Toni Beardon Caroline Ainslie Sam Okoth





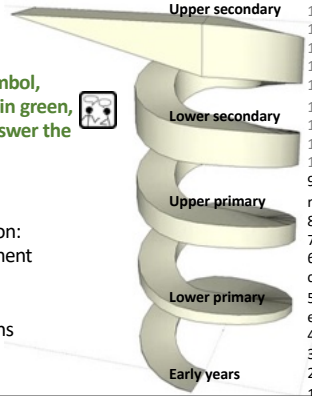




1

Programme for the GTEN workshop

SHADOWS SIMILARITY AND ENLARGEMENT



When you see this symbol, or you see a question in green, do the activity and answer the question.

Today we are working on:
Similarity and Enlargement
Scale factors
Tessellation and Tiling
Trigonometry definitions

18. Cubic centimetres and cubic metres.
17. Proof of Ratio Theorem
16. Introduction to trigonometry
15. Shears
14. Reptiles forming Fractals
13. The Sphynx. Enlargement
12. Tessellation and Reptiles
11. Tessellation of triangles
10. **Talking Points. Lesson Starters.**
9. Standard paper. Enlargement scale factor root2
8. Scale factors and paper airplanes. Video
7. Ratios
6. Puzzles involving reptiles. How many cubes.
5.Projection of images on a screen. Centre of enlargement.
4.Sundials and Roman Numerals
3. The 5 Regular solids
2. Identifying objects from their shadows
1. Making shadows

2

SHADOWS SIMILARITY & SHEARS

Is your shadow always the same length?

Why is your shadow sometimes longer and sometimes shorter?

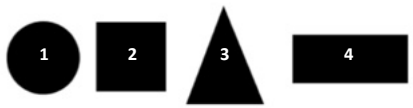
https://aiminghigh.aimssec.ac.za/shadows-activity /

3

SHADOWS ACTIVITY

https://aiminghigh.aimssec.ac.za/shadows-

Here are some shadows cast by solid (3 dimensional) objects and flat (2 dimensional) shapes.
Describe the 3D objects that made the shadows.



There is more than one answer for each shadow.
Can you draw different shadows that could be made by the same objects?

MAKE SHADOW PICTURES

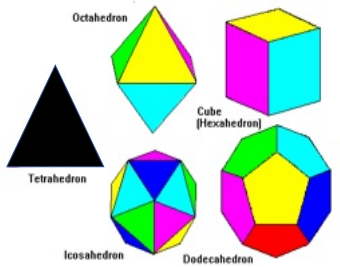
Create some images and share them in the comments

What objects that might have made these shadows.

Make some shadows using the torch on your phone. Share your photos with #aiminghigh

4

SHADOWS



What shadows can the 5 regular polyhedra make?

The shadow of a regular tetrahedron can be an equilateral triangle but, as the light source moves, the shadow changes to other triangles and to quadrilaterals.

Experiment for yourself with these polyhedra and other solid shapes you have available.

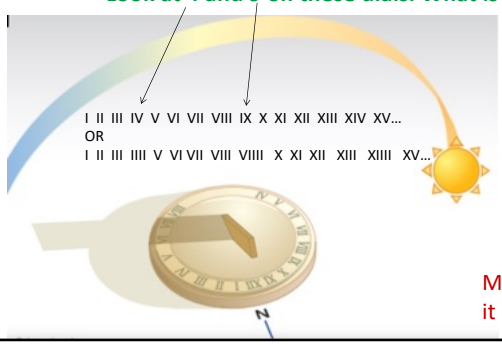
5

SHADOWS APPLICATION *THE 24-HOUR SUNDIAL CLOCK

https://www.britannica.com/technology/sundial

Roman Numerals: I = 1, V=5, X = 10, C = 100

Look at 4 and 9 on these dials. What is the difference?



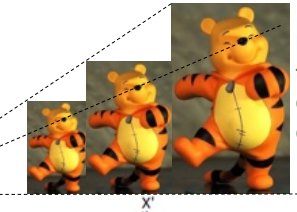
Make your own sundial and set it up at noon on a sunny day.

6

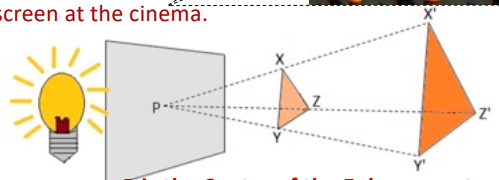
SIMILARITY

The Pooh Bear images are SIMILAR to each other.

This is how films are projected onto a screen at the cinema.



How do you find the centre of enlargement?

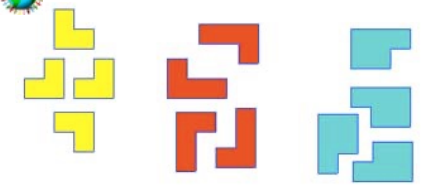


Notice that corresponding angles are equal and there is no distortion of the image.

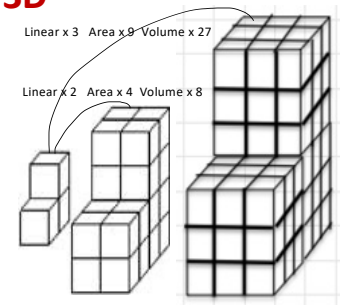
P is the Centre of the Enlargement

7

Similarity, Enlargement and Scale Factors 2D & 3D



Make enlargements of the coloured shapes by fitting together four identical smaller shapes.
Either use squared paper or cut out the pieces to make the 3 puzzles.



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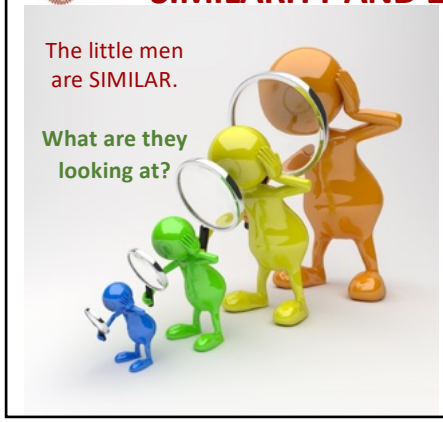
SIMILARITY AND ENLARGEMENT



The little men are equidistant from each other in line. The orange man is 3 times the height of the blue man.

Linear scale factors:

Blue : Green : Yellow : Orange

1 : 1.44 : 1.44^2=2.08 : 1.44^3=3

Surface area scale factors:

1 : 1.44^2=2.08 : 2.08^2=4.28 : 3^2=9

Volume scale factors:

1 : 1.44^3=3 : 2.08^3=9 : 3^3=27

(numbers given correct to 3 significant figures)

9

Scale factors, A4 paper and paper airplanes

https://youtu.be/nIp1Kv138yM

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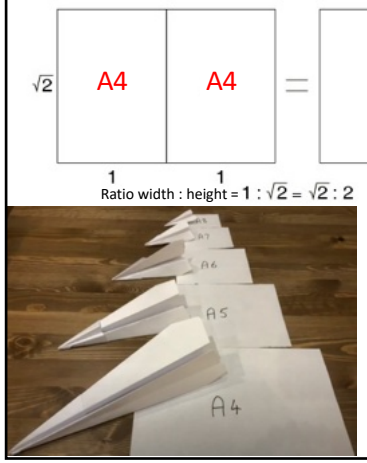

Linear Scale Factor =

Area Scale Factor =

Measurements in centimetres

4A0	1682 × 2378
2A0	1189 × 1682
A0	841 × 1189
A1	594 × 841
A2	420 × 594
A3	297 × 420
A4	**210 × 297**
A5	148 × 210
A6	105 × 148
A7	74 × 105
A8	52 × 74
A9	37 × 52
A10	26 × 37

A0 paper has an area of 1 square metre.

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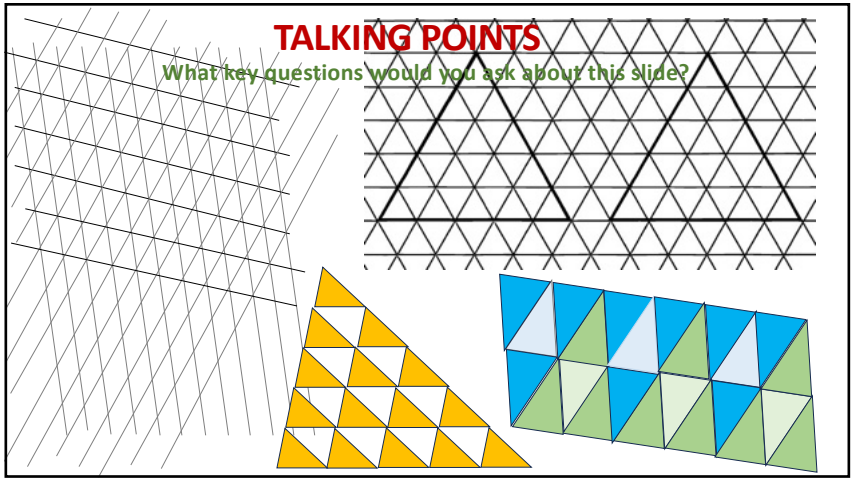
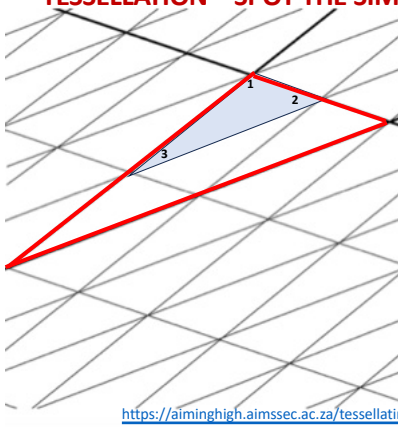
TALKING POINTS

What key questions would you ask about this slide?



12

TESSELLATION * SPOT THE SIMILAR TRIANGLES



What do you see in this diagram?
Look at the triangle with angles marked 1, 2 and 3. Call this T.
Label angles equal to these angles.
Why are they equal?
Look at the larger heavily outlined triangle.
How many copies of triangle T can you see in this triangle?

https://aiminghigh.aimssec.ac.za/tessellating-triangles/

See the worksheet

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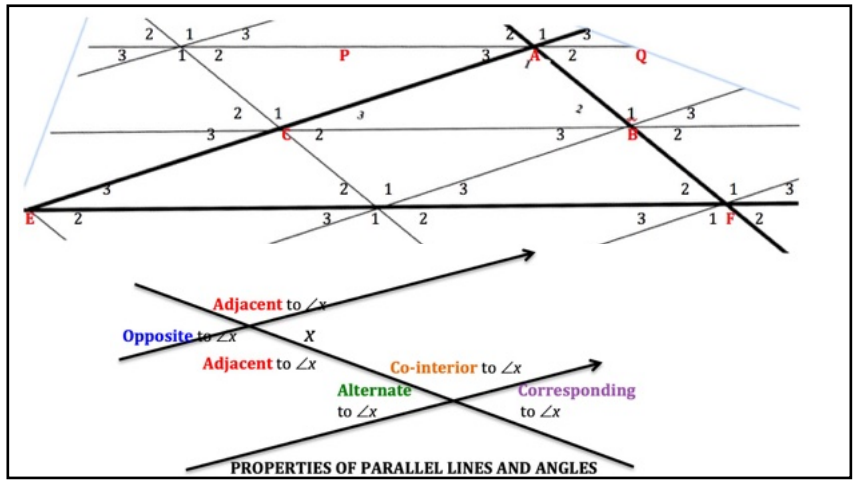
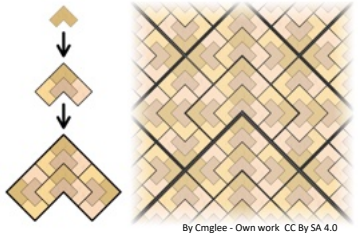


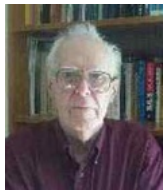
14

Tessellations and Tilings

In the geometry of tessellations, a reptile, or rep-tile, is a shape that can be dissected into smaller copies of the same shape.



By Cmglee - Own work CC By SA 4.0



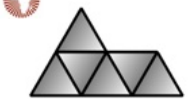
Martin Garner 1914 – 2010

Solomon Golomb 1932 - 2016

The term was coined as a pun on animal reptiles by the mathematician Solomon W. Golomb and popularized by Martin Gardner in his Mathematical Games column in the Scientific American.

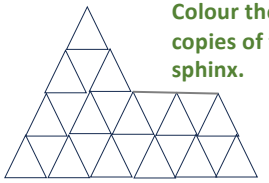
15

Fit together 4 copies of this shape to make an enlargement



This reptile is called The Sphinx.

Colour the outline below to show how 4 copies of the sphinx fit inside the larger sphinx.



What are the scale factors of this enlargement?

Repeat the process to make an even larger sphinx with 16 copies of the sphinx above.

What are the scale factors?
See the worksheet.

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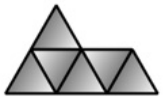
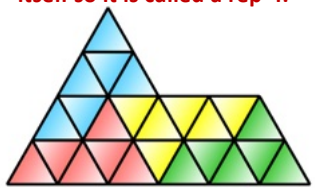
THE SPHYNX



The 2D Sphynx can be made up of 4 smaller copies of itself so it is called a rep-4.

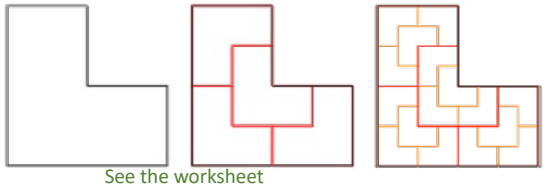


A reptile is a shape that can be dissected into smaller identical copies of the same shape.

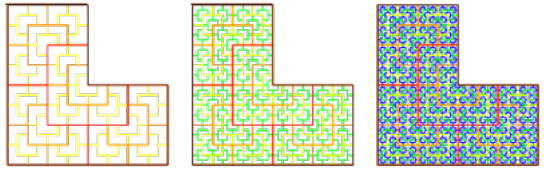
Identical copies of a reptile can be put together to make an enlargement of the shape.

17

Reptiles and Fractals * Triomino or Trisquare



See the worksheet

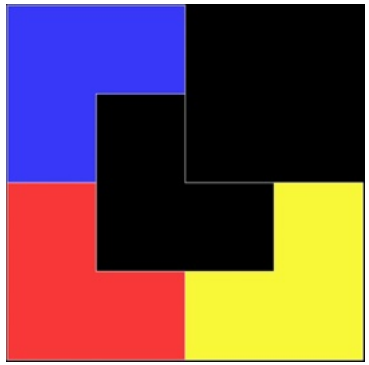


Making smaller copies of the shape inside every copy of the shape, and repeating this process infinitely often, produces a fractal.

The diagrams show stages 0 and 1 of the fractal process based on the shape made up of 3 squares. Draw the next one on the worksheet.

18

Reptile Fractal * Triomino or Trisquare



Why does this fractal iteration produce a white pattern?

Primary colour mix rainbow wheel. Johannes Itten 1961

The white pattern in the fractal occurs through mixing primary colours.

By MagistraMundi - Using an animated-gif editor, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=29791584

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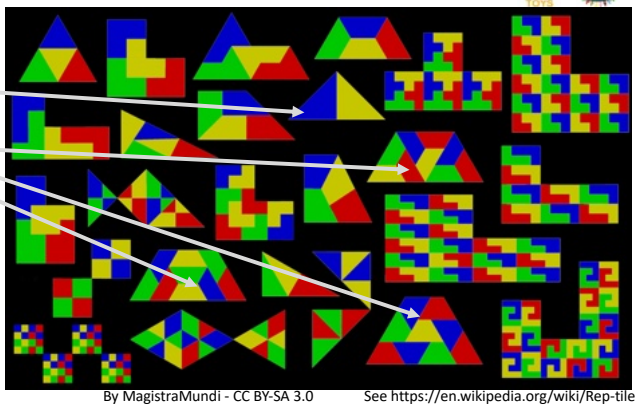
Selection of Reptiles

Can you spot a rep-2?

And some rep-9s?

How would you classify the other reptiles?



By MagistraMundi - CC BY-SA 3.0 See https://en.wikipedia.org/wiki/Rep-tile

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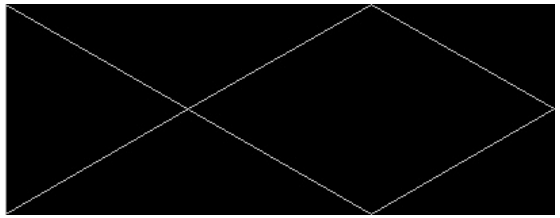
Self Intersecting Reptiles

Fish reptile made up of 3 equilateral triangles



Each fish can be replaced by 9 smaller fish inside itself over and over again infinitely often to produce a fractal.

It's easy to make these patterns yourself, especially if you use isometric paper.

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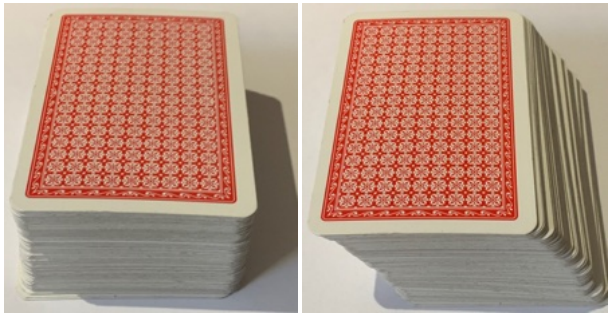
SHEARS



What do these two pictures show?

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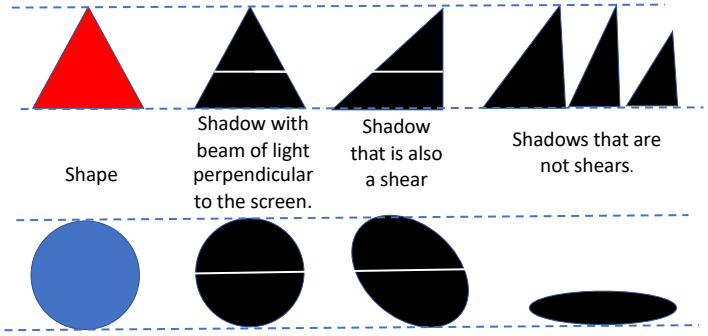
SHADOWS AND SHEARS

A shear is a change in shape with no change in area (2D shapes) and no change in volume (3D shapes).



Shape	Shadow with beam of light perpendicular to the screen.	Shadow that is also a shear	Shadows that are not shears.

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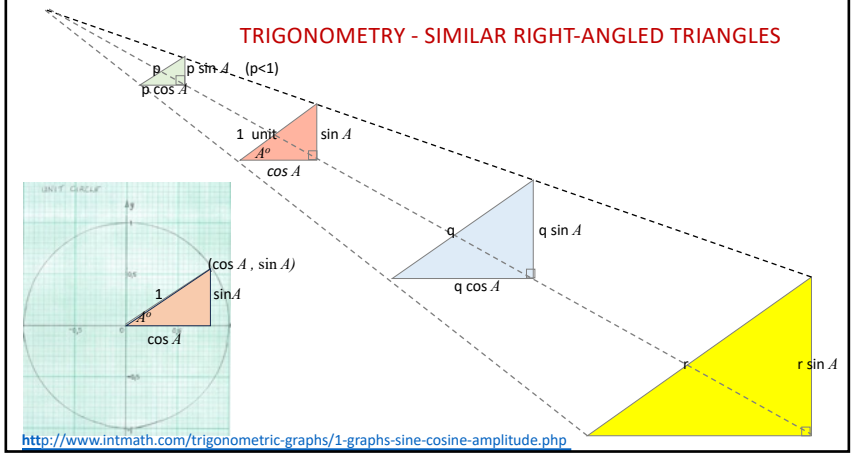
TRIGONOMETRY - SIMILAR RIGHT-ANGLED TRIANGLES



http://www.intmath.com/trigonometric-graphs/1-graphs-sine-cosine-amplitude.php

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TRI-FOLD – TO PROVE THE RATIO THEOREM

1. Draw ΔABC.
2. Fold the baseline BC back onto itself so that the fold line goes through point A. Mark the point R at the base of the fold. AR is called an altitude of the triangle.
3. Make a fold by placing point A anywhere on AR. Mark the fold-line PQ with P and Q on the edges of Δ*ABC*.
4. What do you notice about the lines AR and PQ?

(*Answer: By construction, AR and PQ are perpendicular.*)

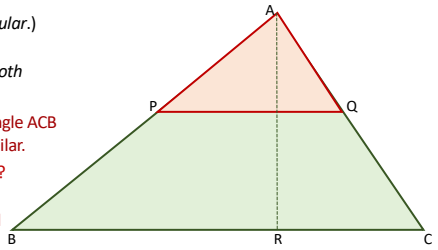
5. What do you notice about the lines PQ and BC?

(*Answer: By construction, PQ and BC are parallel, both perpendicular to AR.*)

6. Notice angle APQ = angle ABC and angle AQP=angle ACB (corresponding angles) so Δ*ABC* and Δ*APQ* are similar.
7. What else do you notice about ΔAPQ and ΔABC?

(*Answer: ΔABC is an enlargement of ΔAPQ).*

8. **We have shown that *PQ* and *BC* are parallel and Δ*APQ* and Δ*ABC* are similar, so it follows that the three ratios *AP:AB, AQ:AC* and *PQ:BC* are equal (each ratio is the SCALE FACTOR of the enlargement).**



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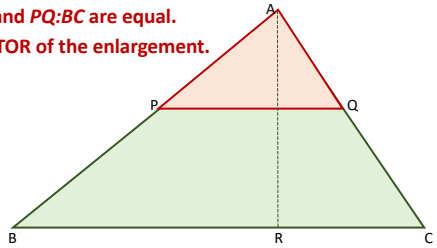
TRI-FOLD – PROOF OF THE RATIO THEOREM SUMMARY

By construction, AR and PQ are perpendicular.
By construction, PQ and BC are parallel, both perpendicular to AR.
ΔABC is an enlargement of ΔAPQ .
We have shown that *PQ* and *BC* are parallel and Δ*APQ* and Δ*ABC* are similar, so it follows that the three ratios *AP:AB, AQ:AC* and *PQ:BC* are equal.
Each ratio is the SCALE FACTOR of the enlargement.



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Scale and the cubic metre

https://youtu.be/ISnGpHQTE-o

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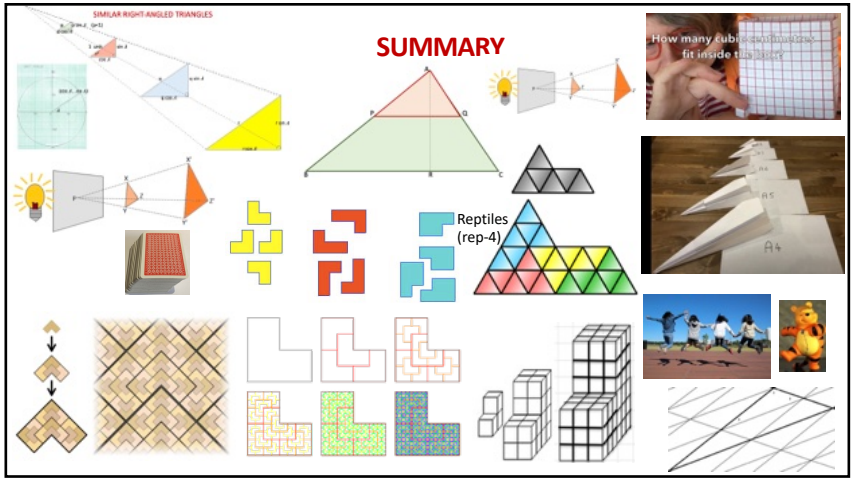
SUMMARY



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AIMS African Institute for Mathematical Sciences SCHOOLS ENRICHMENT CENTRE
AIMSSEC
6 MELROSE RD, MUIZENBERG 7975 CAPE TOWN, SOUTH AFRICA T: +27 (0) 21 787 9326
GTEN

AN AIMING HIGH LEARNING PACK IS A WEBPAGE CONTAINING

A learning activity with links to:

a. PDF of the worksheet
b. Templates and instructions for making resources
c. Videos
d. Notes for Teachers with
- solutions
- curriculum links and learning objectives
- diagnostic quizzes
- suggestions for teaching
- key questions to guide learning
- follow up ideas and links

https://aiminghigh.aimssec.ac.za/

e. Inclusion Guides for School and Home Learning with
- a starter activity for a mixed–age group to do together
- a collection of learning activities to suit all ages from 4 to 18+
- Solutions with suggestions for teaching and assessment.

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LINKS FOR LEARNING ACTIVITIES INVOLVING SIMILARITY & ENLARGEMENT

SHADOWS https://aiminghigh.aimssec.ac.za/shadows-activity/
ENLARGEMENT https://aiminghigh.aimssec.ac.za/enlargement/
TRISQUARES https://aiminghigh.aimssec.ac.za/trisquares/
SIMSETS https://aiminghigh.aimssec.ac.za/simsets/
TESSELLATING TRIANGLES https://aiminghigh.aimssec.ac.za/tessellating-triangles//
VIDEO: SCALE FACTORS, A4 AND PAPER AIRPLANES https://youtu.be/nIp1Kv138yM
VIDEO: SCALE AND CUBIC METRES https://youtu.be/ISnGpHQTE-o
ANIMATIONS http://nrich.maths.org/5671
http://www.intmath.com/trigonometric-graphs/1-graphs-sine-cosine-amplitude.php

AIMSSEC YouTube Channel https://www.youtube.com/@MathsToys

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LET'S PLAY MATHEMATICALLY AND LEARN

Order from AMAZON or TARQUIN https://www.tarquingroup.com/products/aiming-

Play Mathematically
- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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Thanks for coming to this workshop.
Use the AIMSSEC ideas on AIMING HIGH and add comments.
Share what you have learned with other teachers.
Try to help all your learners to have a 'Yes I Can' attitude to mathematics.

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