

Global Teacher Empowerment Network GTEN
 Saturday 20 March 2021 16.00 – 18.00 London Time

ALGEBRA AREA

Same area

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Make a rectangle

	400	60	2
300	120 000	18 000	600
10	4 000	600	20
2	800	120	4

Not to scale

Multiplication
Area ↔ Algebra

$2x + 3$
 $x + 1$
 Area $3x$ square units Area $2x$ square units
 Area $3x$ square units Area $2x^2$ square units
 3 $2x$

1

MACY'S GREAT AMERICAN MARCHING BAND (2010)
 HOW MANY IN THE BAND?

10 majorettes in red and 2 leaders at the front.
 Others in rows of 8 one behind the other forming 8 columns in a rectangular array.
 If we were in the street we could count the number of rows from the front of the marching band to the back.

**MULTIPLY
 THE NUMBER OF ROWS
 BY THE NUMBER OF COLUMNS**

Imagine each person occupying 1 square unit of area. Then the number of people in the rectangle is the same as the area of the rectangle in square units.

2

YOUR LEARNERS CAN ARRANGE THEMSELVES IN RECTANGULAR ARRAYS.

For different rectangles count the numbers of learners.
 If the total number of learners is a prime number, then ask some of the learners to count the others so they will not be in the rectangle.
 Ask what's different about the rectangles they make when the total number of learners is even and when it is odd.

3

Each small square is one unit of area.

What is the area of the big square?

How is it split into rectangles of different colours?

4

$(4 + 1) \times (2 + 3) = 5 \times 5 = 25$
 $(4 + 1) \times (3 + 2) = 12 + 8 + 3 + 2 = 25$

ALGEBRAREA

$41 \times 32 = (4 \text{ tens} + 1 \text{ unit})(3 \text{ tens} + 2 \text{ units})$

5

$41 \times 32 = (4 \text{ tens} + 1 \text{ unit})(3 \text{ tens} + 2 \text{ units})$
 $= (40 + 1) \times (30 + 2)$
 $= 1200 + 80 + 30 + 2$
 $= 1312$

ALGEBRAREA

NOT TO SCALE

6

HOW TO INVOLVE EVERYONE IN YOUR LESSONS

Students giving feedback

SHOWBOARDS

laminated A4 sheets of paper
 blank one side
 square grid other side

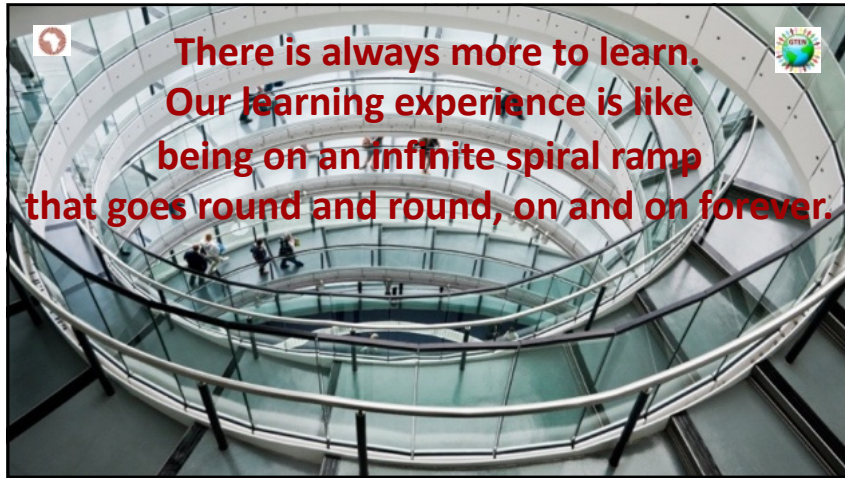
<http://bit.ly/AIMSSEPrintableResources>

7



gc girl cymbal players	bc boy cymbal players	c cymbal players
gd girl drummers	bd boy drummers	d drummers

$(g + b)(c + d)$
 $= gc + gd + bc + bd$


8



9

The spiral continues



ALGEBRAREA

Year 11-12

Simplifying algebraic expressions, multiplying algebraic expressions, solving equations and applying the algebra to solving problems...

Y9-10

Application of multiplication to find area, connection to algebra and applications of algebra

Y7-8

Understanding the nature of multiplication as repeated addition and the connection to multiplication and area
Application to 'long multiplication' using place value



Y4-6

Count the number of elements in a rectangular array by multiplication
Connection between counting squares and multiplication





Y1-3

Informal understanding of the basics of rectangles and area Pre-school

10

EARLY YEARS









What is the same and what is different about the patterns?



Look out for tiling patterns, for example in arts and crafts, also on floors and walls in shopping malls.

Describe the patterns you see.


Arrange the red and blue tiles into patterns.

11

EARLY YEARS

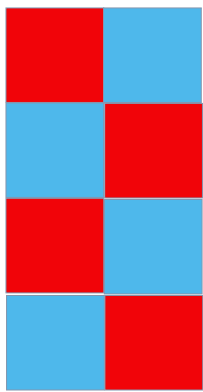


3 tiles.
1 row of 3.
2 red tiles, 1 blue tile

Make different rectangles with red and blue tiles.

Count the squares in the rectangles.

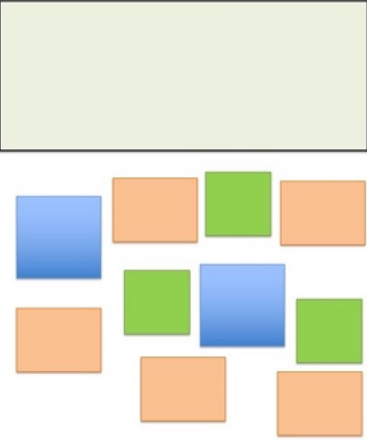
How many rectangles can you make?



8 tiles.
4 rows of 2.
4 red tiles, 4 blue tiles

12

Years 1 and 2



FREE PLAY What can you do with these pieces?

TALK about the shapes and colours
ASK How many pieces have you got?

FORMAL TASK Can you fit all 10 small pieces into the frame?

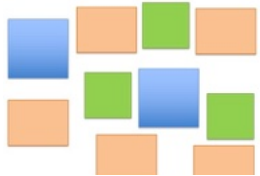
TALK What do you notice?

Edges must be made to match. This is not allowed.

When you cut out the small pieces make sure that the edges of the rectangle exactly match the edges of the squares.

13

Years 3 and 4



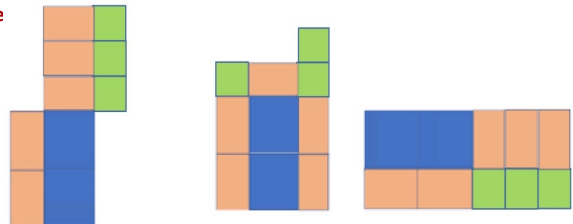
Describe the shapes.

How many other shapes can you make with these 10 pieces matching them edge to edge?

Is it true that whatever way you arrange the 10 shapes they will together make the same total area? If so, how can you be sure?

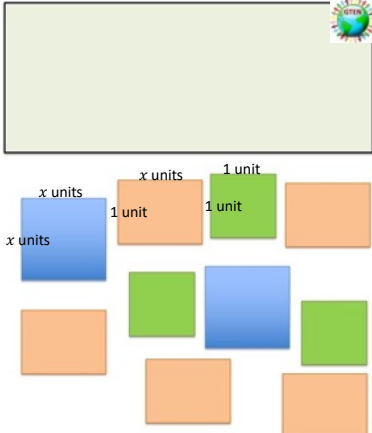
Draw the shapes you make

Here are some examples.



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Years 5 and 6



Draw and cut out 10 shapes with squares of different sizes and rectangles to match. Label your smaller squares 1 by 1 making them 1 square unit of area (whatever size they are) and label your bigger squares as x by x .

We say x is a variable because it can take different values.

Ask learners to solve the puzzle by arranging their pieces to make a rectangle.

Talk about the different shapes.

How many *other* shapes can learners make with these 10 pieces matching pieces edge to edge?

Draw the different shapes and write down their areas in square units in terms of x .

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Break between Primary and Secondary

SERIOUS MATHEMATICAL HEALTH WARNING

AREA IS NOT LENGTH TIMES BREADTH

What is the area of a triangle?
 What is the area of a circle?

16

Multiply 462 by 312 using this rectangle

	400	60	2	
300	120 000	18 000	600	462×312 $= 400 \times 300 + 400 \times 10 + 400 \times 2$ $+ 60 \times 300 + 60 \times 10 + 60 \times 2$ $+ 2 \times 300 + 2 \times 10 + 2 \times 2$ $= 120\,000 + 4\,000 + 800$ $+ 18\,000 + 600 + 120$ $+ 600 + 20 + 4$ $= 144\,144$
10	4 000	600	20	
2	800	120	4	

Not to scale

17

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Imagine each person occupying 1 square unit of area. Then the number of people in the rectangle is the same as the area of the rectangle in square units.



18

The spiral continues



ALGEBRAREA

Year 11-12
Simplifying algebraic expressions, multiplying algebraic expressions, solving equations and applying the algebra to solving problems...

Y9-10
Application of multiplication to find area, connection to algebra and applications of algebra

Y7-8
Understanding the nature of multiplication as repeated addition and the connection to multiplication and area
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Y4-6
Count the number of elements in a rectangular array by multiplication
Connection between counting squares and multiplication

Y1-3
Informal understanding of the basics of rectangles and area

Pre-school

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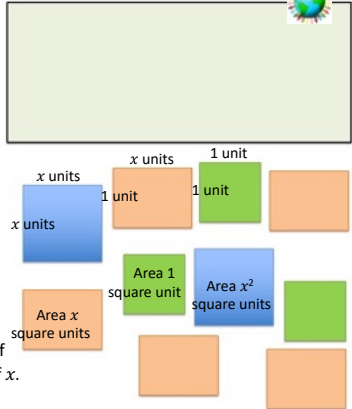
Year 7 – Solve the puzzle

Draw your own shapes as in the picture choosing your own length for 1 unit and your own length x and cut them out.

Solve the puzzle by making a rectangle with the 10 pieces.

Talk about the different shapes and their areas.

How many *other* shapes can you make with the 10 pieces matching the pieces edge to edge.



Multiplication

Area ↔ **Algebra**

Draw the shapes that you make and write down the areas of the parts and of the whole shape in square units in terms of x .

20

Four versions of the same solution

Reflection in horizontal mirror

Reflection in vertical mirror

Rotation by half turn

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MULTIPLICATION

$$(2x + 3)(x + 1) = 2x^2 + 5x + 3$$

Reflection in horizontal mirror

Reflection in vertical mirror

Arrange the 10 pieces into a rectangle:
Four versions of the same solution to the puzzle.

ALGEBRA

TRANSFORMATIONS

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Years 7, 8 and 9

Expression	Value
$(7 + 2)(5 + 3)$	20
$(7 + 2)5 + 3$	23
$7 + 2 \times 5 + 3$	72
$7 + 2 \times (5 + 3)$	48

Match the expressions with their values.

B rackets
I ndex
D ivision
M ultiplication
A ddition
S ubtraction

23

Years 8 and 9


CONSERVATION OF AREA

$$\begin{aligned} \text{Area} &= (2x + 3)(x + 1) \\ &= 2x^2 + 2x + 3x + 3 \\ &= 2x^2 + 5x + 3 \end{aligned}$$


Blue Peach Green

All areas are in square units

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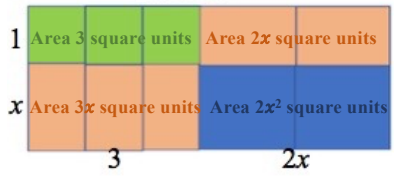


ALGEBRAREA



$(2x + 3)(x + 1) = 2x^2 + 5x + 3$

$2x + 3$




$462 \times 312 = 144\ 144$


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Not to scale

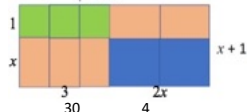
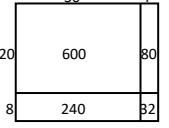
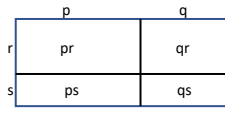
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Key questions




1. Show me how to make a rectangle with the 10 pieces?
2. Show me how to subdivide the rectangle into 4 parts and find the 4 areas?
3. You have made some different shapes using the 10 pieces. What do you notice about the areas?
4. Show me how to draw a rectangle to multiply 28 by 34 and multiply the tens separately from the units? $28 \times 34 = 600 + 80 + 240 + 32 = 952$
5. Suppose you need to multiply $(p + q)$ by $(r + s)$, show me how to draw a rectangle with edges $(p + q)$ and $(r + s)$ and find 4 areas: and in this way find the product of the two expressions?

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ASSESSMENT




Suppose you need to multiply $(3x + 2)$ by $(4x + 5)$.


Show me what you would draw to find 4 areas that give you the product of the two expressions?



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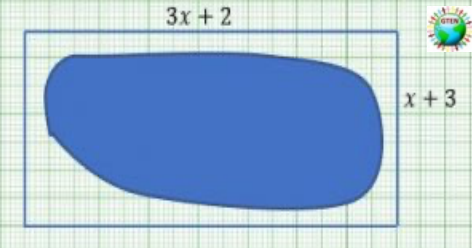


Upper Secondary





The area of the rectangle is 66 square metres. Find the length x .

$(3x + 2)(x + 3) = 3x^2 + 11x + 6$
 So $3x^2 + 11x + 6 = 66$
 $3x^2 + 11x - 60 = 0$
 $3x^2 - 9x + 20x - 60 = 0$
 $3x(x - 3) + 20(x - 3) = 0$
 $(3x + 20)(x - 3) = 0$
 So $x = 3$ metres (it cannot be negative)




The dimensions are 11 metres by 6 metres. The PERIMETER is 34 metres. To fence the area requires 34 metres of fencing. Suppose the problem was changed to ask what length of fencing would be needed to make a square enclosure of the same area?

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 **LET'S PLAY MATHEMATICALLY AND LEARN** 

Order from AMAZON or TARQUIN <https://www.tarquingroup.com/products/aiming-high-lets-play-mathematically>



Play Mathematically

- to develop a love for mathematics
- to unlock knowledge and understanding
- to improve numeracy and visualisation skills
- to practise mathematical procedures
- to motivate concentration and critical thinking
- to boost confidence in mathematical ability.

This **first book** in this AIMING HIGH series provides 36 games that are easy to learn and enjoyable to play for any age. Each comes with reflective questions and materials designed to bring out mathematical thinking and provide a deeper understanding of the topic that underlies the game. Even for the youngest players, this can be transformational.

The **second book** offers suggestions for teachers for using games and puzzles in lessons to teach the regular curriculum with different ideas for different age groups.. It is due to be published in mid 2026.

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 **AIMS** African Institute for Mathematical Sciences
SCHOOLS ENRICHMENT CENTRE 



Thanks for coming to this workshop.
Use the AIMSSEC ideas on AIMING HIGH and add comments.
Share what you have learned with other teachers.
Try to help all your learners to have a 'Yes I Can' attitude to mathematics.



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