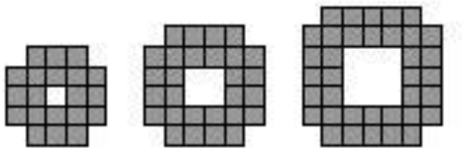
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

EXPANDING PATTERN



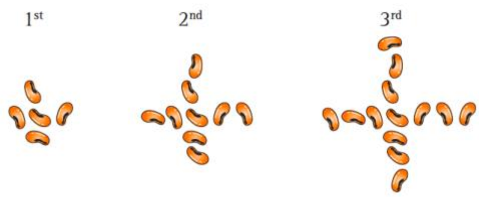
The diagram shows the first three patterns in a sequence in which each pattern has a hole in the middle.
Draw the 4th and 5th pattern in the sequence.

How many small squares are needed to build the tenth pattern in the sequence?

What about the 100th pattern?

HELP

If you find the Expanding Pattern activity tricky then answer the same questions for this pattern made from dried beans. It is easier.



NEXT

Create your own sequences of patterns with dried beans and on squared paper, and work out the rule for the number of beans in each pattern or the number of squares. Then exchange your patterns and solve each other's problems.

NOTES FOR TEACHERS

Why do this activity?

This activity challenges learners to draw a sequence of patterns and to find the rule for making the pattern. For some learners drawing the 4^{th} and 5^{th} patterns accurately will be all they can manage but they should be asked to describe in their own words how they did this.

Other learners will be able to find the formula for the nth pattern in the form 8n + 12.

Intended Learning Objectives (Years 7 to 9)

- To investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns represented in physical or diagram form, and not limited to sequences involving a constant difference or ratio.
- To describe and justify the general rules for observed relationships between numbers in own words or in algebraic language

Possible approach

Give the learners a copy of the question as on page 1 or draw the diagrams on the board. Give the learners squared paper so that they can copy the diagrams and count the small squares and then draw the 4^{th} and 5^{th} patterns. Let the learners work in pairs to find the rule for building the patterns.

Some learners will choose one of the methods given in the solution and some will choose the other. They may even find another method. It is important to let them choose their own method. If they find a formula they should check that it works for the first five patterns.

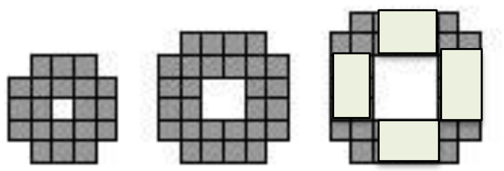
You might get pairs to compare their results. Make it clear that different methods can be used but they should all give the same number of squares. If the learners find a formula then they should be able to simplify it to give 8n + 12 small squares for the nth pattern.

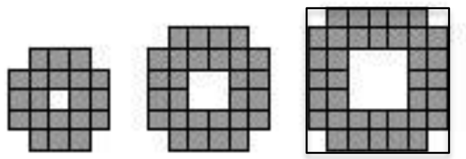
When most of the learners have found some answers have a class discussion in which learners explain their methods and their answers.

Key questions

What is the same about the patterns?
What is different about the patterns?
What is the size of the hole in the middle of the 1^{st} pattern? The 2^{nd} pattern? The 3^{rd} pattern? …
Can you find a rule for working out how many small squares there will be in each pattern?

SOLUTION



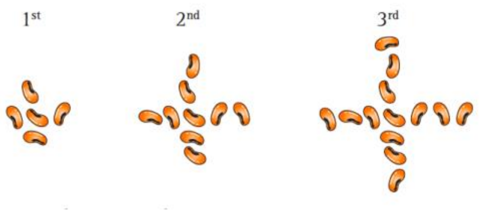


METHOD 1
One way to build the pattern is to surround the central hole with four 2 by n rectangles (as shown above) and then to add four L shapes at the corners each having 3 squares. This gives
$(4\times2\times n) + (4\times3) = 8n + 12$ small squares.

The 1st three patterns have
$8 + 12 = 20$, $16 + 12 = 28$ and $24+12 = 36$ squares respectively.
The 10th pattern is made up of 92 squares.
The 100th pattern has 812 small squares.

METHOD 2
Another way to build the pattern is to see it as a (n+4) by (n+4) square with 4 small squares cut from the corners and an n by n hole in the middle.

The number of small squares is
$(n + 4)^2 - 4 - n^2 = 8n + 12$



The rule for this sequence is n -> 4n +1