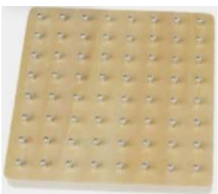
GEOBOARD NOTES

All the activities can be used in school and adapted for learners in different grades according to the learning objectives and time available.

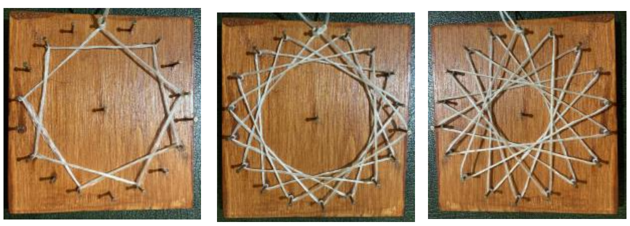
***Teaching ideas**. (1) With one geoboard for demonstration you will be able to use many of the ideas in these notes in your teaching. If learners do not have geoboards they can be asked to do the same tasks by joining points on dotty grids.
(2) Make a class set of geoboards so that learners can share one between two, work in pairs or small groups, then compare findings with other groups.
(3) Use and App https://www.mathlearningcenter.org/apps/geoboard

MAKING GEOBOARDS



8 by 8 commercially produced geoboard with nails and rubber bands

Make your own geoboard. Use a paper square with the grid marked so that you knock in your nails accurately at the points of the grid.
Maybe this could be a project for a Design Technology class.



Three circular 18 pin geoboards showing multiples of 4, 5 and 7.
See Paths to the Stars
https://aiminghigh.aimssec.ac.za/path-to-the-stars/

ACTIVITY 1 TRIANGLES ON A 25 PIN GEOBOARD

DO Make a **triangle** on your geoboard.

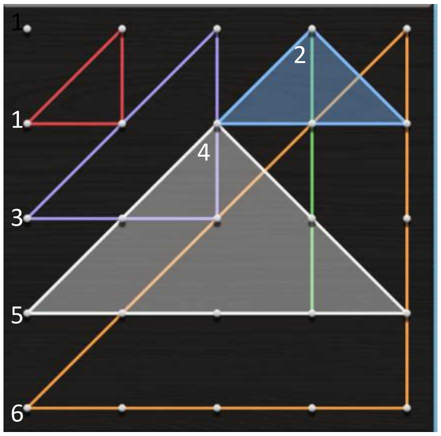
TALK Look at the geoboards of the people around you.
Are their triangles the same as yours?
Try to make a different triangle from theirs so that your group can see many different triangles.
What is the same? What is different?
Can you sort them into sets of similar triangles and sets of congruent triangles?

RECORD Draw some of the different triangles on square dotty paper.
To cut down this task just draw ONE triangle from each set of congruent triangles.
Note the properties of each triangle :–
Is it… Right-angled?
Isosceles (two sides equal)?
Scalene(unequal sides)?
If you make a triangle that looks almost equilateral measure the edges. Is it equilateral?
What is its area?
What are its symmetries?
All the right angled isosceles triangles are enlargements of the small red triangle (number 1). How much bigger are they, that is, what are the scale factors of the enlargements?

How can we extend this activity?

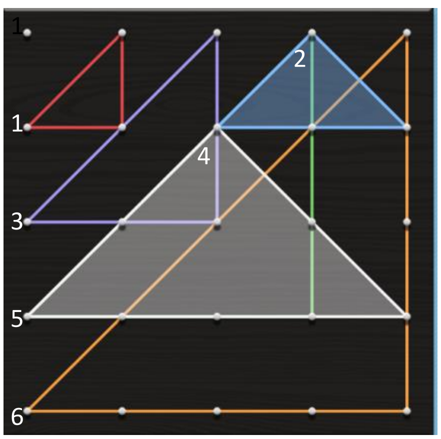
Example: Set of 6 right angled isosceles triangles

All similar to each other
All enlargements of the red triangle no. 1
All have line symmetry



Colour	Area	Lengths of Edges	Enlargements of red triangle 1	
			Linear scale factor	Area scale factor
1 red				
2 blue				
3 lilac				
4 white				
3 green				
6 orange				

All similar to each other
All enlargements of the red triangle
All have line symmetry



Colour	Area	Lengths of Edges	Enlargements of red triangle 1	
			Linear scale factor	Area scale factor
1 red	½	1, 1, √2	1	1
2 blue	1	√2, √2, 2	√2	2
3 lilac	2	2, 2, 2√2	2	4
4 white	4	2√2, 2√2, 4	2√2	8
3 green	4½	3, 3, 3√2	3	9
6 orange	8	4, 4, 4√2	4	16

*You could give learners the blank table above and ask them to fill in the information. Select some of the columns if you don't want the class to do them all.

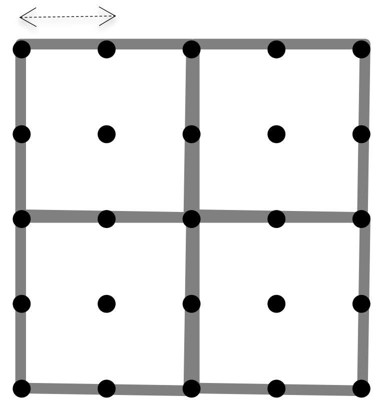
ACTIVITY 2 QUADRILATERALS ON A 25 PIN GEOBOARD

DO Make a **quadrilateral** on your geoboard.

TALK about and ***RECORD*** your quadrilaterals, as for the triangles in Activity 1.

You might like to make a 9-pin geoboard for these activities and photocopy 9-dot grids for the learners.

ACTIVITY 3 TRIANGLES ON A 9 PIN GEOBOARD



DO Use a 9 pin geoboard or split a 25 pin board into four equal squares
with elastic bands to make four 9 pin geoboards.
What is the area of each 9 pin geoboard?
Make a triangle with an area of **2** square units on one of the 9 pin geoboards.
Can you make a **different shaped** triangle with an area of **2** square units
on another of the 9-pin geoboards?
Can you make a different triangle with a **different area** on a 9 pin geoboard?

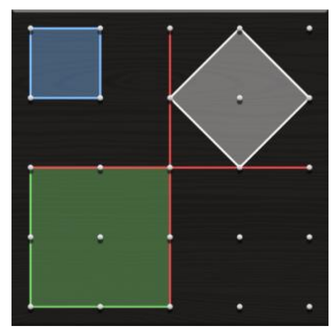
TALK Look at the geoboards of the people around you.
Are their triangles the same as yours?
How many **different shaped** triangles can you find on a 9 pin geoboard?

RECORD Draw all of the different triangles on a 9 pin geoboard on square dotty paper.
Note the properties of each triangle.
How can you be **sure** that you have found them **all**?

How can we extend this activity?

ACTIVITY 4 HOW MANY QUADRILATERALS USING 9 PIN GEOBOARD?

DO Use a 9 pin board or split your 25 pin geoboard into four 9 pin sections as in the diagram.
How many DIFFERENT quadrilaterals you find on a 9-pin geoboard?



TALK What makes 2 quadrilaterals the SAME in this activity?
You can change the activity by giving different answers! Your choice!

Either: a. Just include one square, one rectangle, one parallelogram, one trapezium and one scalene quadrilateral. Is a rhombus possible?

Or: b. include one copy from each set of congruent shapes.

Or: c. For each quadrilateral:

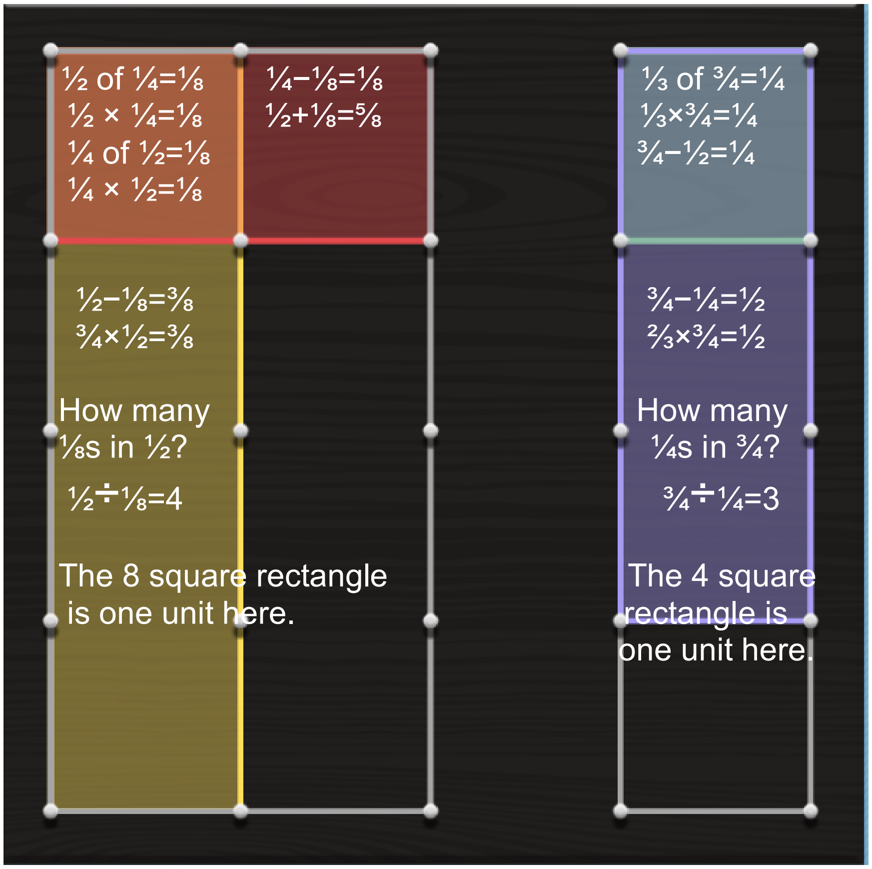
- How many congruent copies of that quadrilateral are there?
- How are they copied (translation, rotation, reflection)?
- How can you be sure that you have found ALL possible quadrilaterals?

*To find all the copies is a big challenge. You need to work systematically and think mathematically. This activity makes a good class project. Make a poster for your classroom wall. When one of your learners finds a new quadrilateral put it on the poster. This enables many learners to contribute.

For younger learners avoid the word **congruent** and talk about **shapes of the same size with the same angles.**

RECORD Draw each quadrilateral on square dotty paper.
Work out the area of each quadrilateral that you find.

ACTIVITY 5 FRACTIONS



DO Study the representations of fractions in this diagram.

Make some more of your own using different sized unit rectangles.

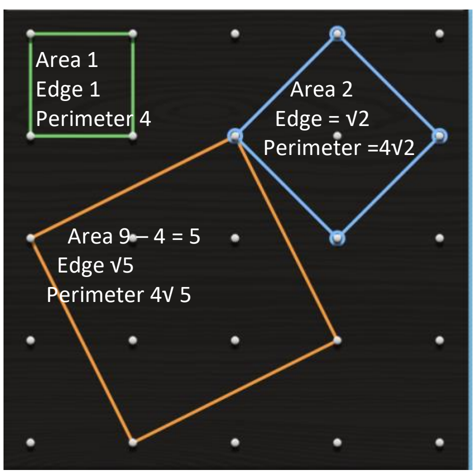
TALK about the different ways of explaining operations with fractions using a geoboard.

RECORD Your ideas.

* With learners it is best to concentrate on one operation at a time.

This diagram is a summary of some fraction ideas intended for discussion between teachers.

ACTIVITY 6 AREAS AND PERIMETERS OF SQUARES AND SQUARE ROOTS



DO Make some squares on the geoboard.
Work out their areas and perimeters.
You will need to use a subtraction method to find the areas of tilted squares and then find the edge length using the formula area = (edge)^2. For example: if the area is 2 square units the edge length is $\sqrt{2}$ units.
Share your findings with people around you.

TALK about the areas and the different ways of working out the areas, about perimeters and the different ways of working out perimeters
Can you make squares parallel to the edges of the geoboard with 4 different areas?
Can you make tilted squares with 4 different areas?

RECORD Your results noting the edge length, area and perimeter of each square.

*This is a good activity for reinforcing ideas of area and perimeter and at the same time reinforcing ideas about square roots. A **common misconception** is to count the pins rather than the distance between the pins. To avoid this misconception make it clear that a 25 pin board measures 4 units by 4 units (area 16).

ACTIVITY 7 INSIDE AND OUTSIDE

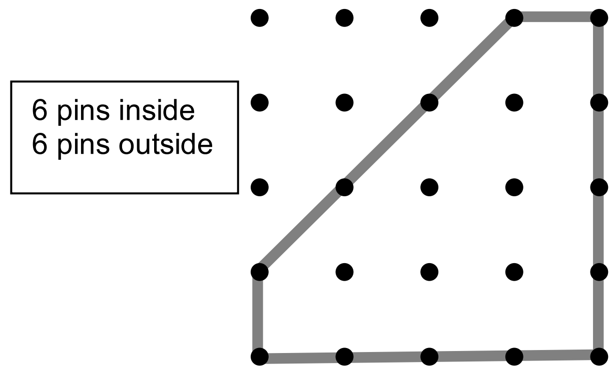
DO You are going to make some shapes on your geoboard which have the same number of pins inside and outside the boundary.

Here is one possibility on a 25-pin board.

Make it on your geoboard, and look at it carefully

This shape has **6** pins inside, and **6** pins outside.

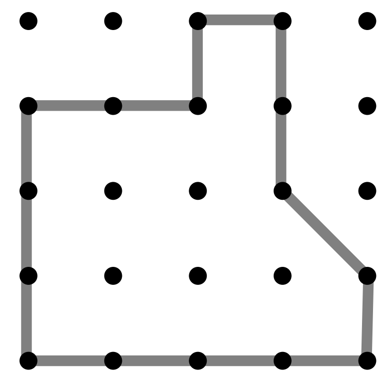
Can you see them?



Now make this shape on your geoboard, and look at it carefully.

How many pins has it got **inside**?

How many pins has it got **outside**?



Now can you find a shape with **2** pins inside and **2** pins outside?

Can you find some more shapes with the same number of pins inside and outside?

Find the **area** of each of your shapes. What do you notice?

Compare your results with your neighbours'.

TALK about your results.

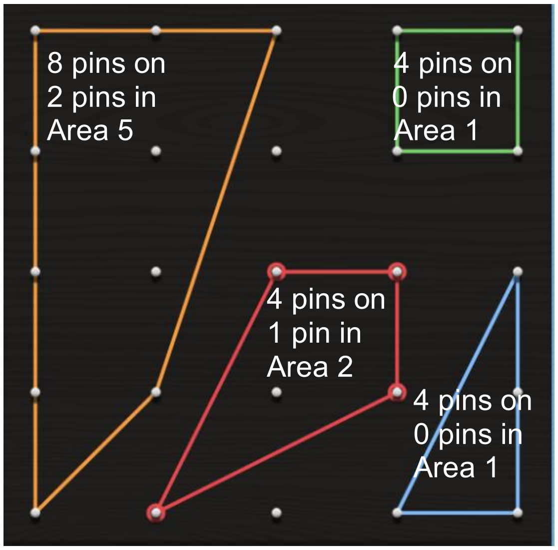
How can you ***RECORD*** your results?

You could use diagrams... a table of results... writing.

How can we extend this activity?

[Activity taken from ATM: ***A collection of ideas about Geoboards****.]*

ACTIVITY 8 ON AND IN



DO Make a polygon on your geoboard.
Count the number of pins **on the edge** (boundary)
Count the number of pins **inside.**
Work out the area of the polygon.
Make a note of these results.
Make a different polygon with the same number of pins on the edge and the same number of pins inside.
What do you notice?

TALK Compare your results with the other people around you
Check each other's results

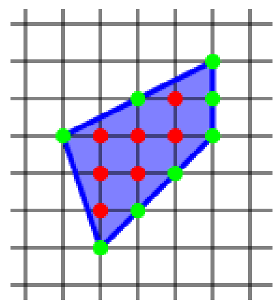
RECORD Record your results and the results of rest your group.
Be systematic – list together all the results with the same number of pins on the boundary and the same number of pins inside, like the square and the triangle in the diagram.

Pick's Theorem For a polygon constructed on a grid of points with integer coordinates such that the vertices of the polygon are grid points (as on a geoboard), **where i is the number of interior grid points and b is the number of grid points on the boundary**

the **Area A enclosed by the polygon** is given by the formula **A = i + ½b -1**

Example: In the diagram i = 7, b = 8, A = i + ½b – 1 = 10.

Show that the results you have found follow from Pick's Theorem.



*This activity gives learners practice in finding areas of different polygons. It enables learners who are not so confident to make simple shapes and those who are more able to challenge themselves by making more complex shapes. Some learners may discover that all the polygons with the same number of pins on the boundary and the same number of pins inside have the same area but if they don't make that discovery then they will still benefit from the exercise in counting, finding areas and recording results. The teacher can quickly check the areas using Pick's Theorem : The area A enclosed by any polygon with vertices at integer grid points, where i is the number of interior grid points and b is the number of grid points on the boundary, is given by the formula: A = i + ½b -1.

This is a simple exercise in algebra and it provides learners with an experience of making and proving a conjecture.

In mathematics the perimeter of a shape is THE LENGTH OF THE BOUNDARY. To avoid the misconception of confusion between the number of boundary points and the perimeter always use the words edge or boundary and do no not use the word perimeter in this activity.

ACTIVITY 9A   HOW MANY SQUARES?

In this activity squares on different pins are different even when they are congruent.

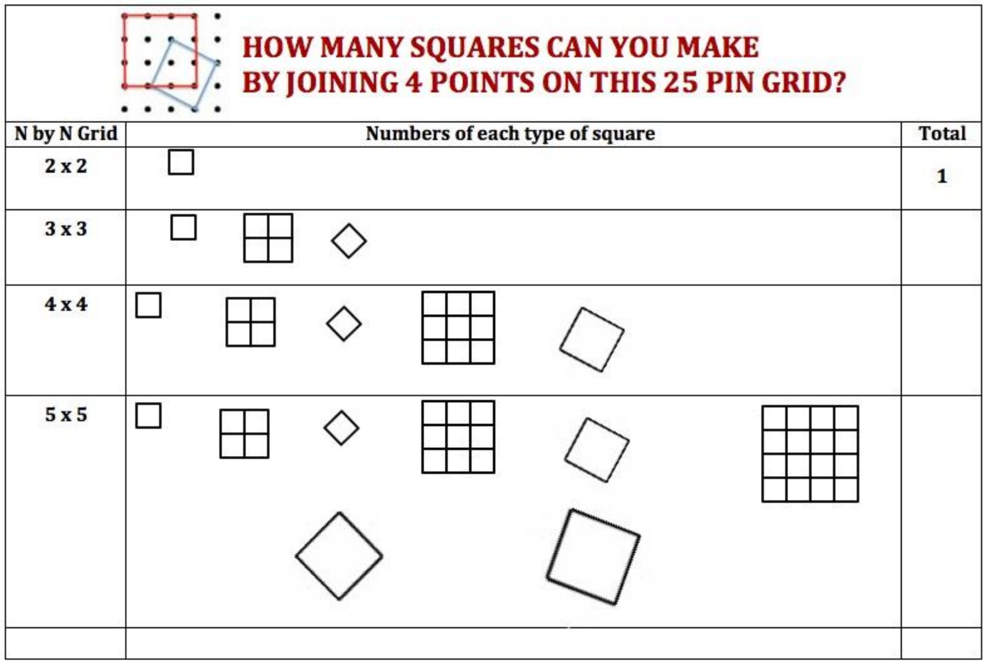
DO   Look at your geoboard**.**
How many pins are there on it?
The distance between the pins is ONE UNIT

How many different squares can you make?

TALK   How many of each different type of square can you make?

RECORD The numbers on this chart.

*In problem solving it is important to work systematically as we do here. It often helps to work on simple cases first so start with a 4-pin board, then a 9-pin, then a 16-pin and then a 25-pin.
What about bigger grids?



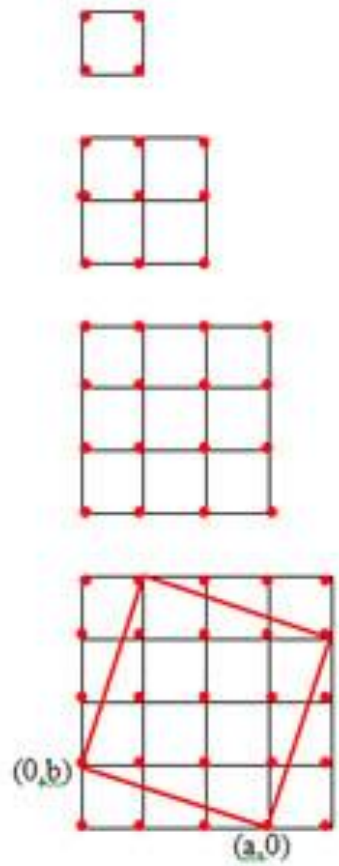
Learners will enjoy the game: SQUARE IT http://nrich.maths.org/2526 . This game gives practice in strategic thinking, and can lead on to work on gradients, on parallel and perpendicular lines, and on Pythagoras Theorem.

ACTIVITY 9B HOW MANY SQUARES AND WHAT ARE THEIR AREAS?

Another system for recording the squares is to classify them using the notation (x, y) for the square that has vertices at (x,0) and (0,y). For example: the square shown in the diagram below is a (3, 1) square and it has area 10 square units. The tables below can be used to record all the different squares according to their areas.

AIMSSEC HOW MANY SQUARES?

Find as many squares as you can with vertices on the intersection points. What are the areas of your squares? How many squares are there of each area? Are you sure you have found them all?



Area	
Number of squares	

TOTAL

Area								
Number of squares								

TOTAL

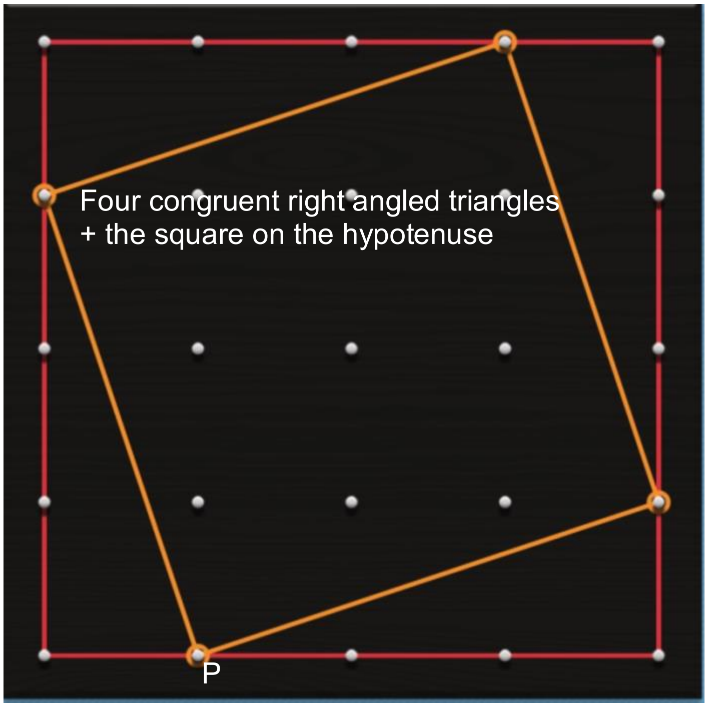
Area								
Number of squares								

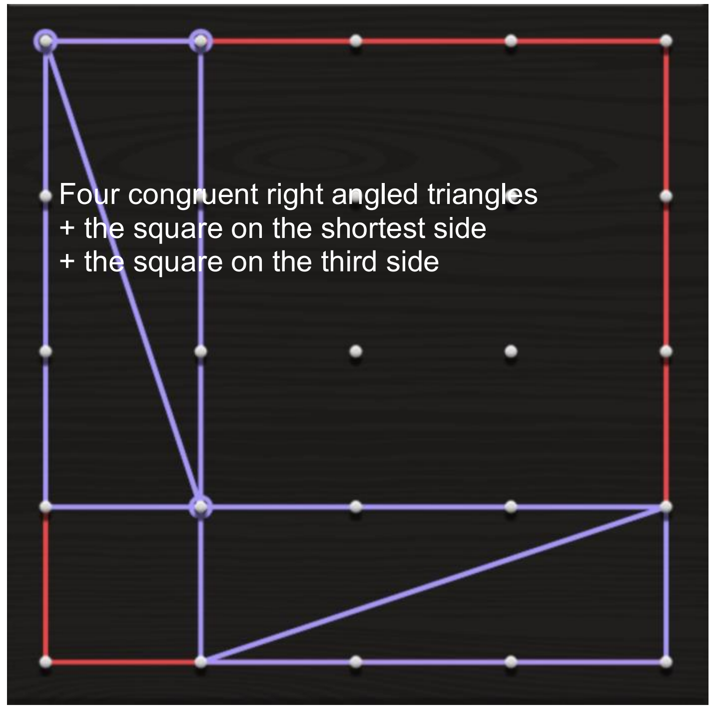
TOTAL

Area								
Number of squares								

TOTAL

ACTIVITY 10 PROOF OF PYTHAGORAS THEOREM





THE AREAS OF THE OUTER RED SQUARE ARE THE SAME WITH DIFFERENET ARRANGEMENTS OF THE PIECES INSIDE.   **TAKE AWAY THE AREAS OF THE FOUR CONGRUENT RIGHT ANGLED TRIANGLES AND EQUATE THE REMAINING AREAS.**

DO Make these diagrams with rubber bands on your geoboard

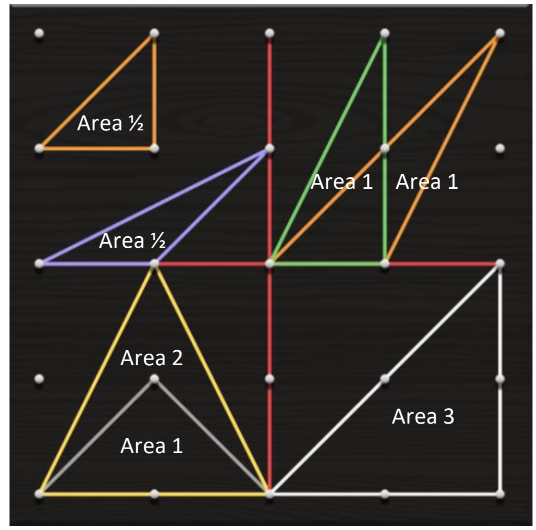
TALK about the areas of the shapes

and about how you can prove Pythagoras Theorem using these diagrams.

Does the same reasoning apply if you draw the diagrams with the point P anywhere on the edge of the outer square (not on one of the pins) so the sides are not integer lengths, or perhaps not a real lengths, for example lengths √2 and (4 - √2)?

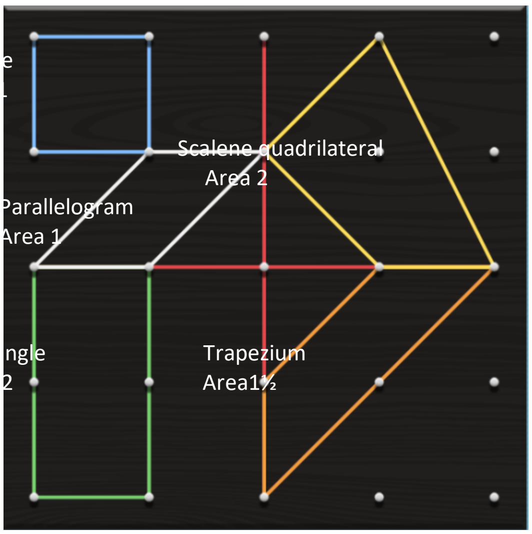
RECORD Write down your proof of Pythagoras Theorem based on these diagrams.

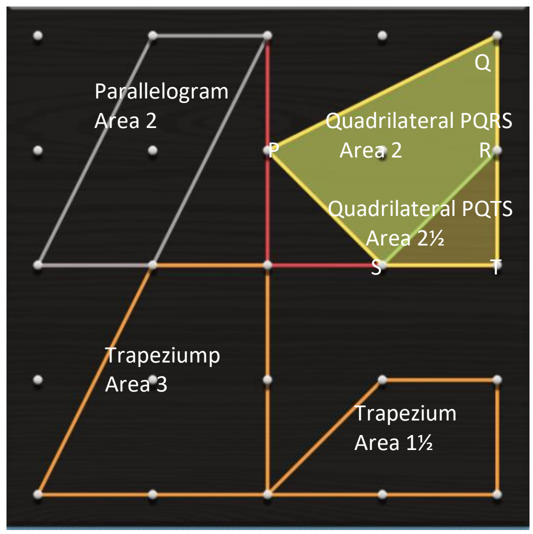
SOLUTION TO ACTIVITY 3 TRIANGLES ON 9 PIN BOARD



Have you found any other triangles?
Is your triangle an image of one of these triangles
by a translation
or a reflection
or a rotation
or enlargement?

To find all the triangles and all their images on a 25 pin board (Activity 1) you have to count these 7 triangles and all their images by translation, reflection, rotation and enlargement and by combinations of these transformations.





ACTIVITY 4 QUADRILATERALS ON A 9 PIN BOARD

NOTE ON ACTIVITY 7 INSIDE AND OUTSIDE

If the number of pins inside the polygon is equal to the number of pins outside the polygon then the area enclosed by the polygon is always 11½ square units. This result follows from

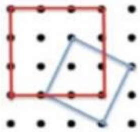
Pick's Theorem The area A enclosed by any polygon with vertices at integer grid points, where i is the number of interior grid points and b is the number of grid points on the boundary, is given by the formula:

$$A = i + \tfrac{1}{2}b - 1$$

As there are 25 grid points (pins) altogether, when the number of exterior and interior points are equal, the number of points on the boundary b = 25 – 2i

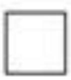
<u>So A = i + ½(25 – 2i) – 1 = 11½</u>

SOLUTION TO ACTIVITY 9 HOW MANY SQUARES?



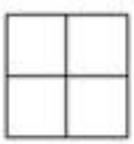
HOW MANY SQUARES CAN YOU MAKE BY JOINING 4 POINTS ON THIS 25 PIN GRID?

N by N Grid	Numbers of each type of square	Total
2 x 2	1	1
3 x 3	4, 1, 1	6
4 x 4	9, 4, 4, 1, 2	20
5 x 5	16, 9, 9, 4, 8, 1, 1, 2	50
	Can you find the areas of the squares? A connection to Pythagoras Theorem? What about bigger grids?	



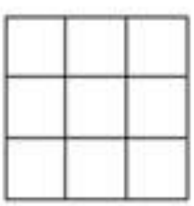
Area	1
Number of squares	1

TOTAL 1



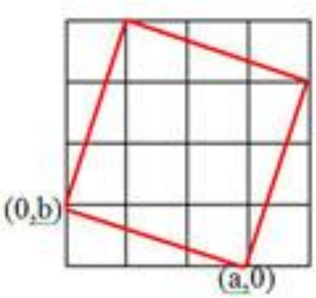
Area	1	2	4					
Number of squares	4	1	1					

TOTAL 6



Area	1	4	9	5	2			
Number of squares	9	4	1	2	4			

TOTAL 20



Area	1	4	9	16	10	5	2	8
a b	1 0	2 0	3 0	4 0	3 1 1 3	2 1 1 2	1 1	2 2
Number of squares	16	9	4	1	2	8	9	1

TOTAL 50

References

Fithian, E. (2005) Activity 3 Geoboards www.home.ptd.net/~efithian/Geometry/Activity-03.html

Furner M.J. & Marinas C.J.(2011) Geoboards to Geogebra: moving from the concrete to the abstract in geometry. Proceedings of the Twenty-third Annual International Conference on Technology in Collegiate Mathematics, ISBN 0-321-68984-4, Copyright (C) 2012 by Pearson Education, Inc. www.archives.math.utk.edu/ICTCM/VOL23/S088/paper.pdf

Scandrett, Hilary (2008) Using Geoboards in Primary Mathematics. *Australian Primary Mathematics Classroom*, v13 n2 p29-32 2008 www.files.eric.ed.gov/fulltext/EJ802704.pdf

Take a ..Geoboard http://nrich.maths.org/10674
Free Web Ap https://itunes.apple.com/gb/app/geoboard-by-math-learning/id519896952?mt=8
or http://www.mathlearningcenter.org/web-apps/geoboard/

Making a Geoboard

Video http://www.youtube.com/watch?v=TEw61i1veOg
http://www.notimeforflashcards.com/2010/08/diy-geoboard.html
http://www.education.com/activity/article/geoboard/
http://www.feelslikehomeblog.com/2010/02/how-to-make-a-geoboard/
http://engagingtoddleractivities.wordpress.com/2010/05/07/homemade-geoboard/

Free Printable Geoboard Masters

http://nrich.maths.org/public/viewer.php?obj_id=6676&part=index
(http://eclectichomeschool.org/pdf/10x10dot.pdf
http://etc.usf.edu/clipart/galleries/math/geoboards.php
http://www.teachervision.fen.com/tv/printables/scottforesman/Math_3_TTT_7.pdf

Lessons with Geoboards

Video http://www.youtube.com/watch?v=ikaSgNDnrv0&feature=related
Video http://www.youtube.com/watch?v=OFucbbgLGZQ&playnext=1&list=PL351EA69AAEE764C3&index=18
http://nrich.maths.org/2883
Tips for Manipulatives – Geoboards & list of sample activities http://oame.on.ca/lms**tips**/files/Manips/**Geoboards**.pdf
http://mathcentral.uregina.ca/RR/database/RR.09.98/loewen2.3.html (angles, perimeter and area)
http://mathforum.org/trscavo/geoboards/geobd6.html
http://www.cut-the-knot.org/Curriculum/Geometry/Geoboard.shtml
Exploring Symmetry
http://images.pcmac.org/Uploads/OnslowCounty/OnslowCounty/Departments/DocumentsCategories/Documents/Geoboard%20Challenge%20-%204th.pdf
Exploring Area and Perimeter
http://www.mathplayground.com/geoboard.html
Worksheet http://www.mathinscience.info/public/cover_up/geoboard_activities.pdf
National Library of Virtual Manipulative Activities
(grades prek-2) http://nlvm.usu.edu/en/nav/frames_asid_277_g_1_t_3.html?open=activities
(grades 3-5) http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html?open=activities
(grades 6-8) http://nlvm.usu.edu/en/nav/frames_asid_282_g_3_t_3.html?open=activities
(making 3-d shapes) http://nlvm.usu.edu/en/nav/frames_asid_129_g_1_t_3.html?open=activities
(circular geoboard grades 3-5) http://nlvm.usu.edu/en/nav/frames_asid_127_g_2_t_3.html?open=activities
(circular geoboard grades 6-8) http://nlvm.usu.edu/en/nav/frames_asid_284_g_3_t_3.html?open=activities
(coordinate grid activities 3-5) http://nlvm.usu.edu/en/nav/frames_asid_166_g_2_t_3.html?open=activities
(coordinate grid activities 6-8) http://nlvm.usu.edu/en/nav/frames_asid_303_g_3_t_3.html?open=activities