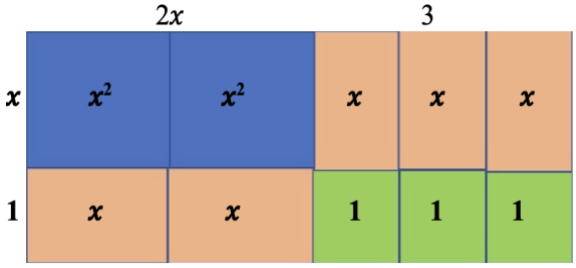
ALGEBRAREA JIGSAW PUZZLE GAME

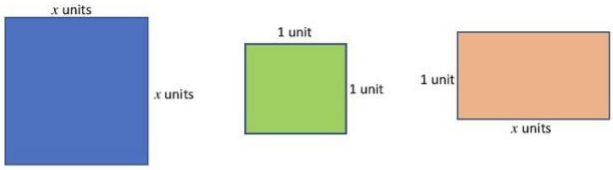
Product of two brackets and area

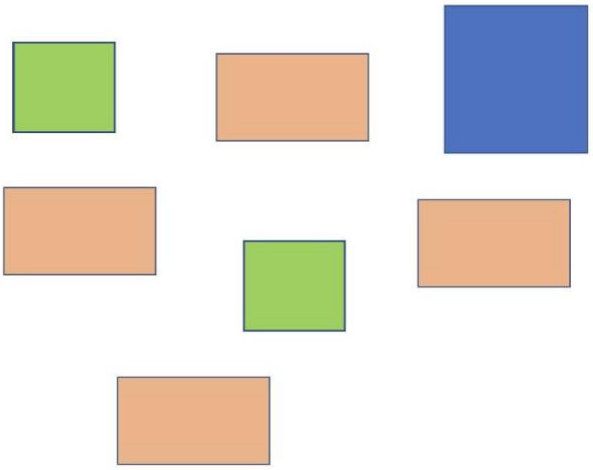
In this game players solve jigsaw puzzles and find the factors of quadratic expressions by fitting pieces together to make rectangles. To factorize the expression $ax^2 + bx + c$, where a, b and c are positive, players must arrange into a rectangle: ***a*** blue x^2 pieces, ***b*** brown x pieces and ***c*** green unit pieces.

This rectangle made from the pieces shown represents the algebraic expression $2x^2 + 5x + 3$ and its factors:

$$2x^2 + 5x + 3 = (2x + 3)(x + 1).$$







THE ALGEBRAREA PUZZLE GAME

Each pair or group of players needs an envelope with at least 20 blue x^2 pieces, 20 brown x pieces and 30 green unit pieces. The teacher needs to have a list of quadratic expressions that can be factorized and a few that cannot.

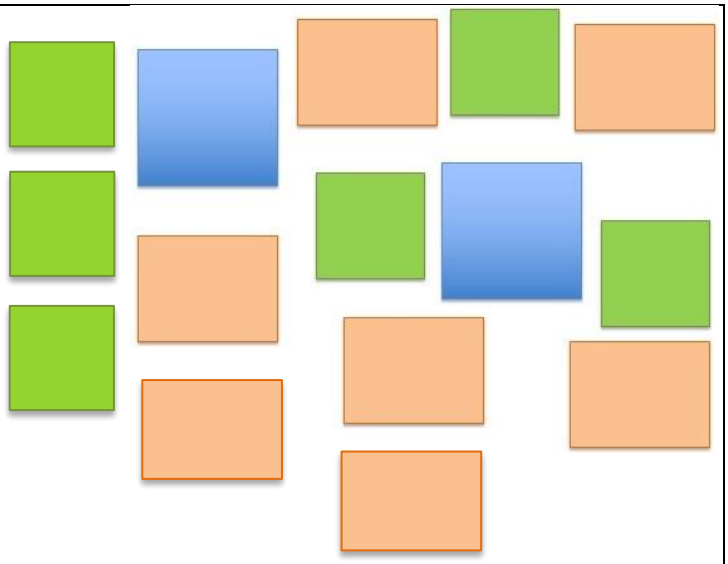
Rules of the Game:

1. The teacher writes a quadratic expression on the blackboard. The first pair or group to make a rectangle with the pieces and find the factors wins the round.
2. The winning pair should explain to the class how they solved the puzzle, and all the learners should copy the solution diagram into their notebooks.
3. This can be repeated for other quadratic expressions.

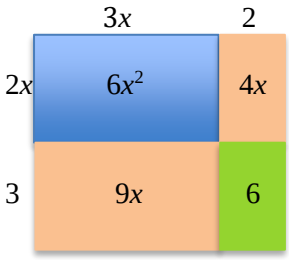
Extra challenge: The game can include a few quadratic expressions that cannot be factorized. In this case the winners are the players who can prove that the expression cannot be factored and explain why.

HELP

Visualise the finished rectangle as four smaller rectangles with the blue pieces in the top left, the green pieces in the bottom right, and the brown pieces at the top right and bottom left.
To put these pieces together to make a rectangle for $2x^2 + 7x + 6$ start with the blue x^2 pieces at the top left as for $2x^2 + 5x + 3$. Then arrange the green pieces into a rectangle, either 1 by 6 or 2 by 3, in the bottom right, with the brown pieces filling the rectangles at the top right and bottom left.



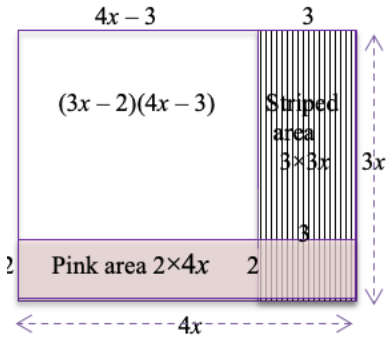
NEXT



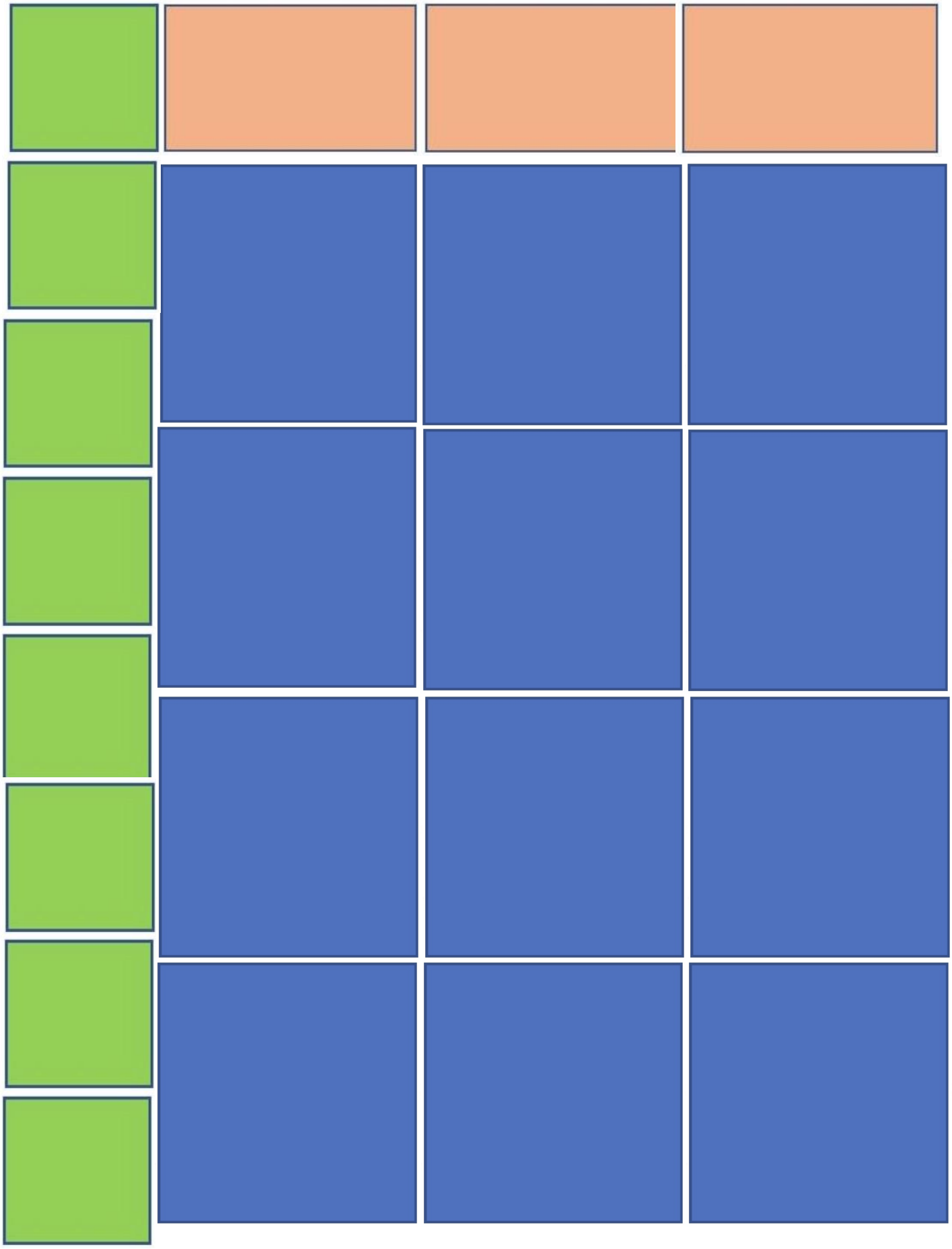
Make up some examples for yourself and work out more problems involving expansion of brackets where all the coefficients are positive, such as $(3x + 2)(2x + 3)$.

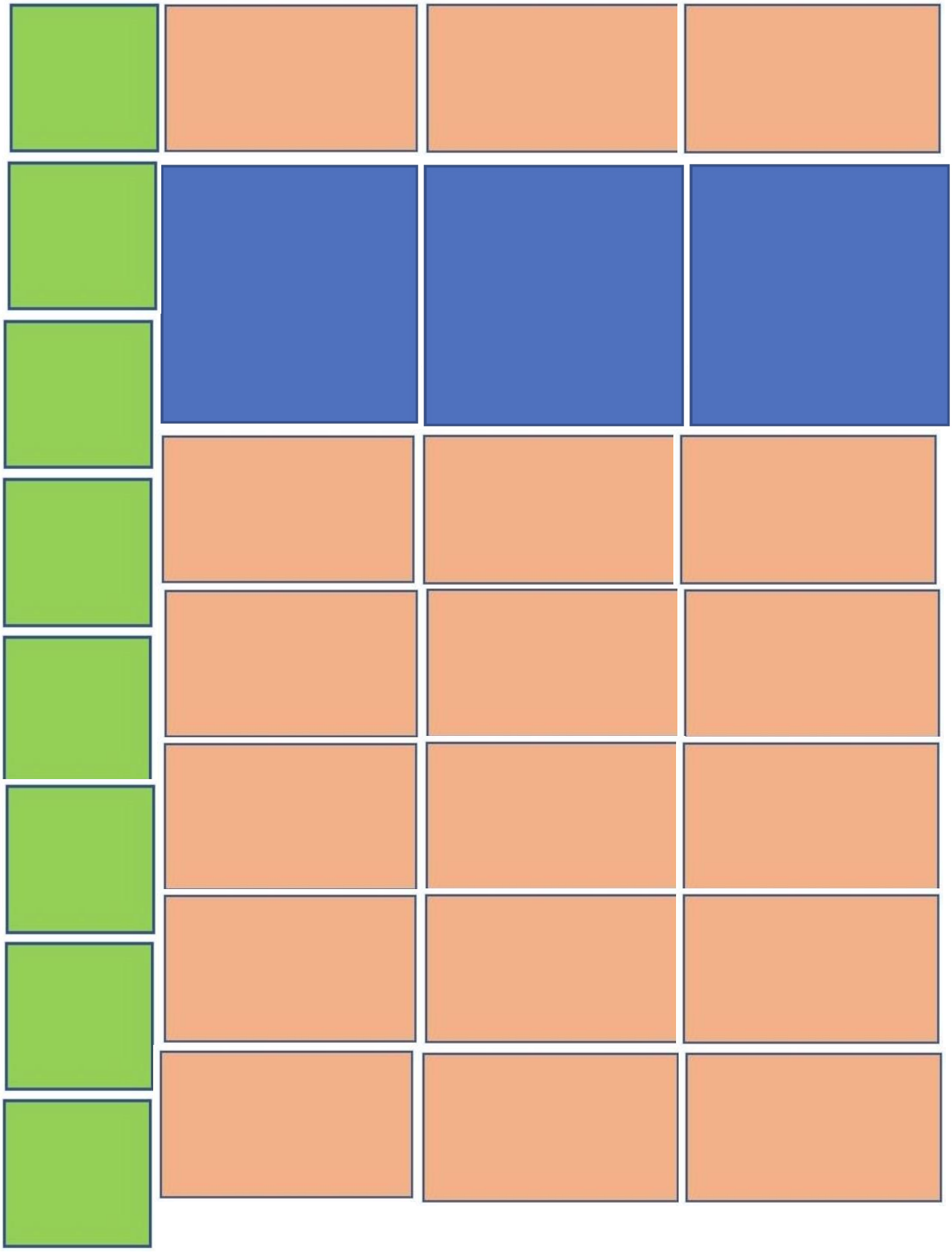
When you are confident about the method without needing to draw a diagram, then multiply binomials with negative terms such as $(2x - 3)(2x + 4)$ and $(3x - 2)(4x - 3)$ in a similar way. Remember the rules that multiplying two positive numbers or two negative numbers gives a positive number and multiplying a positive and a negative gives a negative number.

As the example below shows, it is possible to illustrate these examples, but it is not necessary nor particularly helpful to draw a diagram.



The unshaded area is $(3x - 2)(4x - 3)$,
given by the area of the large rectangle minus the pink area, minus the striped area plus the overlapping area that has been deducted twice.
So $(3x - 2)(4x - 3) = 3x \times 4x$
$-2 \times 4x$
$-3 \times 3x$
$+2 \times 3$





NOTES FOR TEACHERS

Why play this game?

This game links algebra with areas of rectangles and introduces the factorization of quadratic expressions in a practical way in order to develop conceptual understanding of the product of two binomials. Learners develop skills of manipulating a product of two binomials to obtain a quadratic expression; work which prepares them for solving quadratic equations.

To consolidate learner's knowledge and understanding of conservation of area, and of tessellation, in geometry, we focus on the connections between areas of simple geometrical shapes like squares and rectangles and the product of two binomials.

For those who will go into the construction industry or do their own home improvement, for example, the work will resonate with problems of tiling floors and laying patio tiles.

Learning objectives

In doing this activity students will have an opportunity to:

- construct quadratic expressions through areas of 2D shapes;
- calculate areas of squares and rectangles using dimensions given in algebraic form;
- establish a mathematical relationship between area (algebraic expression) and product of two binomials;
- find the product of two binomials by expanding the brackets;
- develop deep conceptual understanding of the construction of quadratic expressions by the product of two binomials.

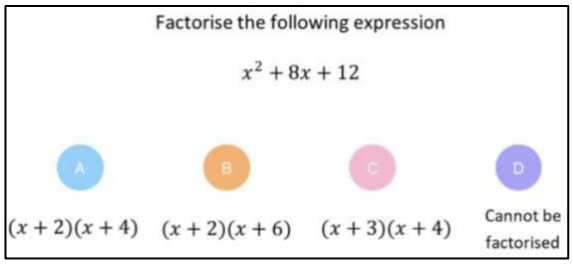
Generic competences

In doing this activity students will have an opportunity to:

- develop algebraic manipulative skills and to recognize the equivalence between different representations of the same relationship;
- co-operate, collaborate and work in a team;
- have empathy with others, listen to different points of view.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A,
2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
3. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
4. Ask the class to vote for the right answer again by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is **B.**

https://diagnosticquestions.com

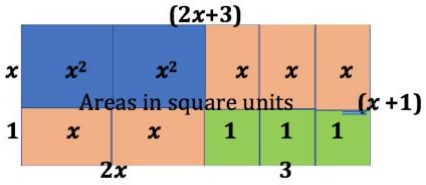
Suggestions for teaching

Resources needed: *An envelope with at least 20 blue x^2 pieces, 20 brown x pieces and 30 green unit pieces for each group of players. A list of quadratic expressions that can be factorized and a few that cannot.*

As preparation for this work you might like the class to do first the Partitioning learning activity https://aiminghigh.aimssec.ac.za/partitioning/ .

Turning this activity into a game makes the lesson practical and engaging. The whole class can play the game with groups of learners competing against other groups with two to four students per group.

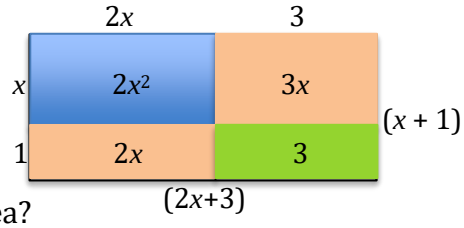
First explain, as on page 1, that they will play a game in which you will give them quadratic expressions, like for example $2x^2 + 5x + 3$, and the winners of that round will be the first group to make a rectangle with pieces that make up this area.



When trying to solve the problem, the learners select the pieces that give the required area and they only have to arrange the pieces to fit together to make a rectangular shape. Learners should easily notice that the rectangle can be used to calculate the area using the length and width of the shape which then give the factors of the quadratic expression.

Ask learners from the winning group to draw their solution on the board and to explain it to the class. Then the class should copy the solution into their workbooks.

It will be sufficient to make a simplified sketch of the solution as shown in this example.



Key questions

- Can you make a rectangle with pieces that have that total area?
- Can you subdivide the rectangle into 4 simpler subunits and find the 4 areas?
- Could you draw a rectangle to multiply 28 by 34 and multiply the tens separately from the units?
- Suppose you need to multiply $(p + q)$ by $(r + s)$, can you draw a rectangle with edges $(p + q)$ and $(r + s)$ and find 4 areas: and in this way find the product of the two expressions?
- Suppose you need to multiply $(3x + 2)$ by $(4x + 5)$, can you draw a rectangle with edges $(3x + 2)$ and $(4x + 5)$ and find 4 areas and in this way find the product of the two expressions?
- Can you apply the distributive law to multiply a binomial by another binomial?

Follow up

LOWER SECONDARY

MD https://aiminghigh.aimssec.ac.za/md/

Multiplication Squares
https://aiminghigh.aimssec.ac.za/multiplication-squares/

Two by Two Puzzle https://aiminghigh.aimssec.ac.za/two-by-two-puzzle/

Partitioning https://aiminghigh.aimssec.ac.za/partitioning/

Can you help these farmers?
https://aiminghigh.aimssec.ac.za/can-you-help-these-farmers/

Pair Products https://aiminghigh.aimssec.ac.za/pair-products/

Use Area to find x https://aiminghigh.aimssec.ac.za/use-area-to-find-x/

Muggles Magic https://aiminghigh.aimssec.ac.za/muggles-magic/

UPPER SECONDARY

Quadratic Matching 1 and Quadratic Matching 2
https://aiminghigh.aimssec.ac.za/quadratic-matching-1/
https://aiminghigh.aimssec.ac.za/quadratic-matching-2/

Quadratic Equations
https://aiminghigh.aimssec.ac.za/quadratic-equations/

Quadratic Functions
https://aiminghigh.aimssec.ac.za/quadratic-functions/

Graphing Quadratic Equations
https://aiminghigh.aimssec.ac.za/graphing-quadratic-equations/