AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

The Answer Is 2025, but WHAT IS THE QUESTION?

This **Inclusion and Home Learning Guide** is part of a Learning Pack
downloadable from the AIMING HIGH website https://aiminghigh.aimssec.ac.za
It provides related activities for all ages and learning stages from pre-school to school-leaving,
together with guidance for inclusion in school lessons and for home-learning,
all on the Common Theme 'Working Backwards from 2025'
Choose what seems suitable for the age or attainment level of your learners.

THE ANSWER IS 2025, BUT WHAT IS THE QUESTION?

You can be a creative mathematician; you can be someone who has your own mathematical ideas. Find your own interesting facts about 2025 and calculations that have the answer 2025.

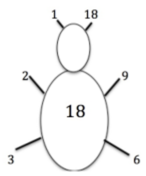
What different questions can you find with 2025 as the answer? Perhaps you can make up an easy question, a harder one and one that is very hard. Compare your questions with other people's. For example you might ask: 'Do you find anything interesting if you reverse the digits of 2025 and add the two numbers together?'

What interesting facts can you find about the year that you were born?

Is it correct to say "twenty twenty-five" or should we say "two thousand and twenty-five" or are both correct? Why?

People say "twenty twenty five" so do the 2 twenties mean – 20 thousands, 20 hundreds, 20 tens or 20 units?

Find the prime factors of 2025 and write 2025 as a product of its prime factors. Draw the factor bug for 2025 in which each pair of legs gives 2 factors whose product is 2025. How many legs does it have?



Here is the factor bug for 18. The antennae show 1 × 18 = 18.

The pairs of legs show and 2 × 9 = 18 and 3 × 6 = 18.

Factor bugs for other numbers can have more legs.

HELP

How old are you? If you are 9 years old then write down some interesting calculations that have the answer 9 (or whatever your age is, do the same for your age).

For example, all these have the answer 9:

3 × 3 ; half of 18 ; 10 – 1 ; 20 – 11; 16 -7 ; 3^2 ; square root of 81 etc.

See the problem 'I'm Eight'

https://aiminghigh.aimssec.ac.za/years-3-10-i-am-eight/

NEXT

How many ways can you arrange the digits 2, 2, 0 and 5 to get different numbers? What is the sum of those numbers?

Explore Wild and Wonderful Number Patterns, see http://nrich.maths.org/33
Make up some of your own number patterns.
You've probably come across number patterns before, ones like :-
2 4 6 8 10 12 . . .
512 256 128 64 32 . . .
220 210 200 190 180 170 . . .
11 14 17 20 23 26 . . .
Work out the rules that produced each of the patterns.
What is the reason for the series of dots appearing after each one?
Now make up some of your own number patterns.

INCLUSION AND HOME LEARNING GUIDE
THEME: WORKING BACKWARDS from 2025

Early Years and Lower Primary

Understanding time and related vocabulary.

Use a calendar for 2025. If your calendar has pictures, look at them and ask the children to tell you what they see in the pictures. Do they like the pictures? Why?
Ask about their birthdays. Find their birthdays on the calendar.
Talk about the weeks, count the 7 the days of the week and talk about their names.
On what days of the week will their birthdays occur in 2025?
Talk about the months, count the 12 months of the year and talk about the names of the months.
In what months will their birthdays occur in 2025?

January					
Mon		6	13	20	27
Tue		7	14	21	28
Wed	1	8	15	22	29
Thu	2	9	16	23	30
Fri	3	10	17	24	31
Sat	4	11	18	25	
Sun	5	12	19	26	
Week No.	1	2	3	4	5

February					
Mon		3	10	17	24
Tue		4	11	18	25
Wed		5	12	19	26
Thu		6	13	20	27
Fri		7	14	21	28
Sat	1	8	15	22	
Sun	2	9	16	23	
Week No.	5	6	7	8	9

March						
Mon		3	10	17	24	31
Tue		4	11	18	25	
Wed		5	12	19	26	
Thu		6	13	20	27	
Fri		7	14	21	28	
Sat	1	8	15	22	29	
Sun	2	9	16	23	30	
Week No.	9	10	11	12	13	14

April					
Mon		7	14	**21**	28
Tue	1	8	15	22	29
Wed	2	9	16	23	30
Thu	3	10	17	24	
Fri	4	11	**18**	25	
Sat	5	12	19	26	
Sun	6	13	20	27	
Week No.	14	15	16	17	18

May					
Mon		5	12	19	**26**
Tue		6	13	20	27
Wed		7	14	21	28
Thu	1	8	15	22	29
Fri	2	9	16	23	30
Sat	3	10	17	24	31
Sun	4	11	18	25	
Week No.	18	19	20	21	22

June						
Mon		2	9	16	23	30
Tue		3	10	17	24	
Wed		4	11	18	25	
Thu		5	12	19	26	
Fri		6	13	20	27	
Sat		7	14	21	28	
Sun	1	8	15	22	29	
Week No.	22	23	24	25	26	27

July					
Mon		7	14	21	28
Tue	1	8	15	22	29
Wed	2	9	16	23	30
Thu	3	10	17	24	31
Fri	4	11	18	25	
Sat	5	12	19	26	
Sun	6	13	20	27	
Week No.	27	28	29	30	31

August					
Mon		4	11	18	**25**
Tue		5	12	19	26
Wed		6	13	20	27
Thu		7	14	21	28
Fri	1	8	15	22	29
Sat	2	9	16	23	30
Sun	3	10	17	24	31
Week No.	31	32	33	34	35

September					
Mon	1	8	15	22	29
Tue	2	9	16	23	30
Wed	3	10	17	24	
Thu	4	11	18	25	
Fri	5	12	19	26	
Sat	6	13	20	27	
Sun	7	14	21	28	
Week No.	36	37	38	39	40

October					
Mon		6	13	20	27
Tue		7	14	21	28
Wed	1	8	15	22	29
Thu	2	9	16	23	30
Fri	3	10	17	24	31
Sat	4	11	18	25	
Sun	5	12	19	26	
Week No.	40	41	42	43	44

November					
Mon		3	10	17	24
Tue		4	11	18	25
Wed		5	12	19	26
Thu		6	13	20	27
Fri		7	14	21	28
Sat	1	8	15	22	29
Sun	2	9	16	23	30
Week No.	44	45	46	47	48

December					
Mon	1	8	15	22	29
Tue	2	9	16	23	30
Wed	3	10	17	24	31
Thu	4	11	18	**25**	
Fri	5	12	19	**26**	
Sat	6	13	20	27	
Sun	7	14	21	28	
Week No.	49	50	51	52	1

Look for patterns in the numbers on the calendar. The numbers in the rows increase by 7 and in the columns by 1.

Ask learners how old they will be on their birthday in 2025?

In which year were they born? Count from the year of their birth up to 2025: for example 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025. So someone born in 2016 will be 9 years old in 2025.

Upper Primary and Lower Secondary Years 4 – 10

Developing independent learning and understanding of factors.

2025

Copy this question on the board:

THE ANSWER IS 2025, BUT WHAT IS THE QUESTION?

It is worthwhile to have a preliminary discussion about the number 2025, what words we use to talk about it and place value.

There is no need for assessment of prior knowledge for this activity, but teachers can use the activity to assess what understanding of number operations and what number sense and creativity the individual learners have. It is a good activity for the start of the school year. It can be adapted year by year to give 'The answer is 2025…' or 'The answer is 2026…' etc.

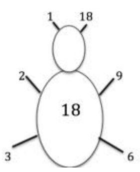
Explain to learners that they can and should be able to think for themselves like mathematicians and not just follow instructions and copy other people. Explain that computers now do all the routine jobs and, in this century, higher skills are needed. Tell them that sometimes you are going to give them problems without telling them what to do to solve them but that you will teach them a lot of new mathematics so that they will get better and better at problem solving.

You might like to use the 'One-Two-Four-More' strategy getting the learners to work individually until each learner has a calculation that gives the answer 2025. Then tell the learners to work in pairs to check that their partner's calculation does have the answer 2025. Then ask the learners to work in fours. Perhaps each group could make a poster showing some calculations that have the answer 2025. You could have a class discussion as to what would make one of these 'made-up questions' interesting.

Ask each group to contribute one of their 'calculations' and the class could vote on which is the 'most interesting'.

Another challenge would be to investigate polite numbers. A number is said to be **polite** if it can be expressed as the sum of at least two consecutive natural numbers. It is easy to write 2025 as the sum of 2 consecutive natural numbers.

If it can be written as the sum of 3 consecutive natural numbers then they will be less than, and also greater than, **2025 ÷ 3 = 675.** So the sum is **674 + 675 + 676 = 2025**. Using a similar method you can show that 2025 is not the sum of 4 consecutive natural numbers but it is the sum of 5 consecutive natural numbers: **2025 ÷ 5 = 405** and **403 + 404 + 405 + 406 + 407 = 2025**. Can the learners share the work between them of finding all 14 sums of consecutive numbers that add up to 2025?



Here is the factor bug for 18. The antennae show 1 × 18 = 18.
The pairs of legs show and 2 × 9 = 18 and 3 × 6 = 18.
Factor bugs for other numbers can have more legs.
Draw the factor bug for 2025. How many legs does it have?

Key questions

- Can you explain your method to me and how it works?
- What answer does your calculation give when you work it out?
- What could you add to your answer to get 2025?
- Can you use other operations (such as subtraction, multiplication, division, squaring etc)?

Secondary Years 9 to 12

1. You might ask: 'How many 25s are there in 2025?' or 'What are the prime factors of 2025?' or 'How many divisors of 2025 are there?'
2. Can you write 2025 as the product of 3 squares? Can you write it as the product of 2 squares?
3. If you investigate the prime factors of 2025 and write 2025 as a product of its prime factors, you will find that 2025 is a square number. When was the last year that was a square number? When will the next year be that is a square number?
4. The sum of the first n odd numbers is $S_n = n^2$ so 2025 is the sum of the first 45 odd numbers. Check it out! **$1 + 3 + 5 + 7 + 9 + \ldots + 89 = 2025$.**
5. 2025 is the sum of two squares **$27^2 + 36^2$**. Check it out.
6. 2025 is the sum of 3 squares. One is 40^2. Can you find the other two?
7. The sum of the natural numbers from 1 to 9 (one of the triangle numbers) is $(9 \times 10)/2 = 45$ so **$(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)^2 = 2025$.**
8. You might like to investigate the sum of cubes of natural numbers given by the formula $S = [n^2 (n + 1)^2]/4$, where S is the sum and n is the number of natural numbers taken. You will find that **$1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3 = (9^2 \times 10^2)/4 = 2025$**

Upper Secondary Years 12 and 13 – addressed directly to students

1. Create some calculations using powers of whole numbers that give the answer 2025.
2. The number 2025 can be written as the sum of 2 squares in one way $27^2 + 36^2$ and it can be written as the sum of 3 squares in 11 different ways. The French mathematician Lagrange proved, in 1770, that all natural numbers can be written as the sum of 4 squares. Can you show that 2025 can be written as the sum of four squares in 80 different ways?

If you know how to do simple coding then adapt the pseudo-code below to whatever language you know to find different ways of writing 2025 as sums of squares:

Pseudo-code to find sums of 4 squares that give the answer 2025 and the results

```
count = 0
for a = (1:45)
for b=(a:45)
for c=(b:45)
for d = (c:45)
if 2025 == a^2+b^2+c^2+d^2
count = count + 1;
y=[count,a,b,c,d];
disp(y)
end end end end end
```

count	a	b	c	d	count	a	b	c	d	count	a	b	c	d	count	a	b	c	d
1	1	2	16	42	21	2	17	24	34	41	6	12	18	39	61	10	15	16	38
2	1	2	24	38	22	2	22	24	31	42	6	16	17	38	62	10	15	26	32
3	1	8	14	42	23	3	4	8	44	43	6	17	26	32	63	10	20	25	30
4	1	10	18	40	24	3	4	20	40	44	6	18	24	33	64	12	12	21	36
5	1	10	30	32	25	3	12	24	36	45	6	23	26	28	65	12	14	23	34
6	1	16	18	38	26	4	4	12	43	46	7	12	26	34	66	12	16	16	37
7	1	18	26	32	27	4	6	23	38	47	7	14	22	36	67	12	16	20	35
8	2	2	9	44	28	4	7	14	42	48	7	20	26	30	68	12	16	28	29
9	2	4	18	41	29	4	8	24	37	49	8	10	30	31	69	12	23	26	29
10	2	4	22	39	30	4	9	22	38	50	8	14	26	33	70	12	24	24	27
11	2	6	7	44	31	4	12	29	32	51	8	18	26	31	71	13	16	24	32
12	2	6	31	32	32	4	16	27	32	52	8	19	24	32	72	14	16	22	33
13	2	7	26	36	33	4	18	23	34	53	9	10	20	38	73	14	20	23	30
14	2	9	28	34	34	4	21	28	28	54	9	12	30	30	74	15	18	24	30
15	2	10	20	39	35	4	22	25	30	55	9	18	18	36	75	16	16	27	28
16	2	10	25	36	36	5	12	16	40	56	9	22	2	28	76	16	17	18	34
17	2	12	14	41	37	5	20	24	32	57	10	10	12	41	77	16	18	22	31
18	2	14	15	40	38	6	7	28	34	58	10	10	15	40	78	17	22	24	26
19	2	14	23	36	39	6	9	12	42	59	10	10	23	36	79	18	20	25	26
20	2	16	26	33	40	6	10	17	40	60	10	12	25	34	80	20	20	21	28

3. A number is said to be ***polite*** if it can be expressed as the sum of at least two consecutive natural numbers and 2025 can be written in 14 ways as a sum of consecutive natural numbers. Can you write a code to find these 14 sums?

1012 + 1013

674 + 675 + 676

403 + 202 + 405 + 406 + 407

335 + 336 + 337 + 338 + 339 + 340

221 + 222 + 223 + 224 + 225 + 226 + 227 + 228 + 229

198 + 199 + 200 +201 + 202 + 203 + 204 + 205 + 206 + 207

128 + 129 + 130 + … + 142

104 + 105 + 106 + … + 121

69 + 70 + 71 + … + 93

62 + 63 + 64 + … + 88

53 + 54 + 55 + … + 82

23 + 24 + 25 + … + 67

16 + 17 + 18 + … + 65

11 + 12 + 13 + … + 64

SOLUTIONS

Although commonly used 'twenty twenty-five' is not correct and we should say 'two thousand and twenty-five' given by: (2 × 1000) + (2 × 10) + 5 = 2025.

People do say 'twenty twenty' and that means 20 hundreds and 20 units. We can write this: (20 × 100) + (20 × 1) + 5 = 2024 but this is not the usual format for our place value system which should be:

(2 × 1000) + (0 × 100) + (2 × 10) + 5 = 2025.

For younger learners any sum that gives the answer 2025 is a solution. Learners should find many different ways to get 2025 and share their ideas with each other. For example 1010 + 1010 + 5 = 202%.

2 × 1010 + 5 = 2025 or

4 × 505 + 5 = 2025 or

5 × 404 + 5 = 2025 or …

Clearly there are many simple solutions.

Older learners may find more complicated solutions. Some solutions are more accessible and more interesting than others. What is interesting for 10 year-olds may not be interesting for 18 year-olds and vice versa.

Here are some interesting properties of 2025:

1. If you reverse the digits of 2025 and add the two numbers together:
   2025 + 5202 = 7227, a palindromic number.
2. 81 × 25 = 2025. The factors of 2025 are: 1, 3, 5, 9, 25, 27, 45, 75, 81, 225, 405, 675, 2025
3. As a product of prime factors $2025 = 3^4 \times 5^2 = 45^2$ so 2025 is a square number. The last year that was a square number was $1936 = 44^2$ and the next one will be $2116 = 46^2$.
4. The sum of the first n odd numbers is $S_n = n^2$ so 2025 is the sum of the first 45 odd numbers:
   $1 + 3 + 5 + 7 + 9 + \ldots + 89 = 2025$.
5. As a product of prime factors of $2025 = 3^4 \times 5^2$. As the product of 2 squares and 3 squares
   $2025 = 9^2 \times 5^2 = 3^2 \times 3^2 \times 5^2$.
6. 2025 is the sum of two squares $27^2 + 36^2$.
7. 2025 is the sum of 3 squares $40^2 + 20^2 + 5^2$
8. The sum of the natural numbers from 1 to 9 (one of the triangle numbers) is (9 × 10)/2 = 45
   so $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)^2 = 2025$.
9. The sum of cubes of natural numbers is given by the formula $S = [n^2 (n + 1)^2]/4$, where S is the sum and n is the number of natural numbers taken.
   $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3 = (9^2 \times 10^2)/4 = 2025$
10. A number is said to be ***polite*** if it can be expressed as the sum of at least two consecutive natural numbers. The number 2025 is a polite number, it can be written in 14 ways as a sum of consecutive natural numbers, for example, 403 + 404 + 405 + 406 + 407 = 2025 (see page 5)

11. The number 2025 written as a binary number is 11111101001.

$2^{10} = 1024$ $\quad 2025 = 1024 + 1001$

$2^9 = 512$ $\quad = 1024 + 512 + 489$

$2^8 = 256$ $\quad = 1024 + 512 + 256 + 233$

$2^7 = 128$ $\quad = 1024 + 512 + 256 + 128 + 105$

$2^6 = 64$ $\quad = 1024 + 512 + 256 + 128 + 64 + 41$

$2^5 = 32$ $\quad = 1024 + 512 + 256 + 128 + 64 + 32 + 9$

$2^3 = 8$ $\quad = 1024 + 512 + 256 + 128 + 64 + 32 + 8 + 1$

$2^0 = 1$

Base	Representation
bin	11111101001
3	2210000
4	133221
5	31100
6	13213
7	5622
oct	3751
9	2700
10	2025
11	1581
12	1209
13	bca
14	a49
15	900
hex	7e9

Why do these activities?

This collection of activities gives open questions suitable for learners of all ages on number, sequences and series, factors and prime factorization, exponents and simple coding.

The question "2025 is the answer, what is the question?" is suitable for younger and less confident learners, but older learners can use their greater mathematical knowledge. Use this activity to encourage learners to think for themselves and to use everything they know about numbers to create their own solutions to invent questions. This activity allows learners to be creative, to explore and play with numbers and to come up with their own solutions. It is also an example of an inverse problem, one that gives the answer so that the learner has to think of where to start to get to that answer.

Learning objectives

In doing these activities students will have opportunities to:

Primary:

- develop vocabulary about measures and cycles in the passage of time
- practise calculations involving operations that they know

Lower Secondary, all the above and:

- learn about factors and prime factorization.

Upper Secondary, all the above and:

- use some simple coding

Generic competences

In doing this activity students will have an opportunity to **think flexibly**, be creative and innovative and apply knowledge and skills.

Follow up

Calendar Patterns
https://aiminghigh.aimssec.ac.za/calendar-patterns/

Factors and Multiples Game
https://aiminghigh.aimssec.ac.za/factors-and-multiples-game/

Triangle Number Picture
https://aiminghigh.aimssec.ac.za/triangle-number-picture/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

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