## AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES SCHOOLS ENRICHMENT CENTRE (AIMSSEC) <br> AIMING HIGH

AIMSSEC

## MATHGO

This game is like Bingo but each player makes their own gamecard by drawing a grid and choosing to write on their gamecard 25 numbers between 0 and 100, not repeating any numbers. They cannot change the numbers once they are written on their gamecard.

Each round the caller draws 2 cards at random from a bag, replacing the first before drawing the second, and makes a note of the numbers so the winning card can be checked.

Players try to make some of the numbers on their board by

| MA H-G |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Play $\text { to } \mathrm{wr}$ | ite on |  |  | $\begin{aligned} & \text { ds } \\ & \hline \end{aligned}$ |
| Pick Com | $2 \text { num }$ | $\text { e } 2$ |  | d. <br> by |
| $\begin{array}{r} +, \\ 8 \text { an } \\ \hline \end{array}$ | $14 \mathrm{giv}$ | $\begin{aligned} & \therefore \text { For } \\ & \text { e } 12,4 \end{aligned}$ | $\begin{aligned} & \text { exam } \\ & 4,32 a \end{aligned}$ | $\begin{aligned} & \text { ane } \\ & \text { and } 2 . \end{aligned}$ |
| like | Som 6 and | $d 65 \text { car }$ | bers <br> nnot |  | combining the two numbers called using one of the operations,,$+- \times$ or $\div$. If they succeed, they mark the numbers on their gamecard e.g. 5 and 10 make $15,5,50$ and 2.

The winner is the first player to get 5 numbers in a line and to explain how the numbers were calculated.

| Numbers <br> called | Results: |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Difference | Product | Quotient |  |
|  | 12 | 4 | 32 | 2 |
|  | 19 | 1 | 90 | - |
|  |  |  |  |  |
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## HELP

To make a good choice of numbers for a Mathgo gamecard, find out which numbers cannot be formed by combining two numbers from 1 to 10 and investigate how many ways each of the other numbers between 1 and 100 can arise.

For example, are prime numbers a good choice to write on your card?
Are larger numbers more likely to arise than smaller numbers or less likely?
In this Bingo game it is possible to make a line of 5 numbers on 5 vertical, 5 horizontal and 2 diagonal lines. When you find a number that occurs frequently it's good strategy to put it at the centre when you set up your board.

## NEXT

HOLDING MATHGO: This version of the game can only be played when there is a small number of players.

You can choose to miss a turn and hold a pair of numbers in reserve until the next turn. If you hold numbers in reserve, you must use 3 numbers at the next two turns. You can combine the three numbers in any order using any of the four operations.

## NOTES FOR TEACHERS Why do this activity?

The game will motivate the learners to think mathematically in order to construct their own game card that will give them a good chance of winning. Some older learners may be able to write a computer program to find the frequency of occurrence of the results so that they can decide on the best numbers to put on their gamecard, and the best arrangement of those numbers. Younger learners should be able to work out 4 results that occur when two particular cards are drawn and that, if they put on them on the same line and a number that occurs frequently in the fifth place, then and they do this on five rows, then they have a better chance of winning.

## Learning objectives

- to develop numeracy and problem-solving skills;
- to develop strategic thinking and strategic planning skills;
- to develop teamworking skills.


## Suggestions for teaching

With the youngest learners play a simpler version of Mathgo, for addition only, using 4 by 4 gamecards and the winner being the first player to get a line of 4 numbers. Otherwise use the same rules. Give out blank 4 by 4 grids and ask the leaners to write 16 different numbers from 0 to 20 anywhere, one in each of the 16 squares. Then play the game and keep a record of the winning lines. Repeat this several times over a period of a few weeks. Then look together at all the winning lines of 4 numbers and ask if they think some numbers on the cards win more often than other numbers. Would there be an advantage in using those numbers when they make their own gamecards? Give them the chance to try it.

For upper primary ask the class to tell you all the ways of getting the answer 10 by adding 2 numbers chosen from 1 to 10 . List all these ways and you will arrive at the conclusion that there are 9 ways. Then do the same for the number 20 and they will realise that there is just one way to get 20, namely $10+10$. So 10 is a good number to have on their Mathgo card and 20 is not. Suggest that they work in pairs to find out how many ways there are of getting each total. Share the results and the class will find that they have a total of 100 results altogether. Then suggest that they work in pairs and use this information to make up a gamecard that will have a good chance of winning. Then have a championship tournament to find the champion pair.

For older learners, with the standard Mathgo Game on a 5 by 5 card using the four rules, give out blank 5 by 5 grids and ask the leaners to write 25 different numbers from 0 to 100 anywhere in the 25 squares. Then play the game and keep a record of the winning line. Repeat this several times over a period of a few weeks. Then look together at all the winning lines of 5 numbers and ask if they think some numbers on the cards are in the winning lines more often than other numbers. Would there be an advantage in using those numbers when they make their own gamecards? Give them the chance to try it.

You can suggest to secondary learners that they will have a better chance of winning if they use numbers on their gamecards that come up most often, so it's worth trying to discover the number of ways each result from 0 to 20 occurs for the sum, the difference and the quotient and the number of ways that the results from 1 to 100 occur for the products. Perhaps divide the class into 4 sections and share the results to share the work involved by working as a team. One section should list the sums, another the differences, another the products and the fourth section the quotients. This provides the opportunity for differentiation and inclusion as you can assign the 4 distinct tasks accordingly. Put the 4 lists on the board so that the learners can copy the results. Suggest that they work in pairs and use this information to make up a gamecard that will have a good chance of winning. Then have a championship tournament to find the champion pair.

## Key questions

1. For Mathgo can you make your own gamecard that gives you the best chance of winning every time you play the game?
2. To make a good choice of numbers for a Mathgo gamecard, can you explore how many ways each number from 1 to 100 can be formed by combining two numbers by,,$+- \times$ or $\div$.
3. Are prime numbers a good choice or are some prime numbers a good choice and others a bad choice? Why or why not?

## SOLUTIONS

A code can be written in the language with which you are familiar to calculate the frequencies of the events that each of the numbers from 0 to 100 can arise when 2 cards are drawn at random in this game.

## Pseudo-code for listing the sums:

$\%$ To find sums $a+b$
for $m=(2: 20)$
$c(m)=0 ;$ Note: This sets a counter endfor
for $a=(1: 10)$
for $b=(1: 10)$
$\mathrm{x}=\mathrm{a}+\mathrm{b}$;
$\mathrm{c}(\mathrm{x})=\mathrm{c}(\mathrm{x})+1$;
end end
for $\mathrm{k}=(2: 20)$
$\mathrm{y}=[\mathrm{k}, \mathrm{c}(\mathrm{k})]$;
$\operatorname{disp}(y)$
endfor

## Pseudo-code for listing the differences:

To determine the frequency of POSITIVE numbers $a-b$. Note that $a-b=0$ if and only if $a=b$, it occurs ten times. Also $a-b>0$ if and only if $a>$ $b$ which occurs $9+8+\ldots+2+1=45$ times, and likewise, $a-b$ occurs 45 times.
================================
for $m=(1: 9)$
$\mathrm{P}(\mathrm{m})=0$; counts frequencies of $a-b=m>0$
$\mathrm{N}(\mathrm{m})=0$; counts frequencies if $a-b=-m<0$
endfor
===============================
for $\mathrm{a}=(1: 10)$
for $b=(1: 10)$
$\mathrm{x}=\mathrm{a}-\mathrm{b}$;
This program is written in Octave. It demonstrates the process needed to write the program in other languages.

```
if a-b < 0
    N(-x) = N(-x)+1;
    endif
    endfor endfor
=================================
    disp("The following gives (N,frequency of N) for positive N")
    for k=(1:9)
    y =[k,P(k)];
    disp(y)
    endfor
disp("The following gives (N,frequency of N) for negative N")
for k=(1:9)
y =[-k,N(k)];
disp(y)
endfor
```


## Pseudo-code for listing the products:

To find products $a^{*} b$
for $\mathrm{m}=(1: 100)$
$\mathrm{c}(\mathrm{m})=0$;
endfor
for $\mathrm{a}=(1: 10)$
for $b=(1: 10)$
$\mathrm{x}=\mathrm{a}$ *;
$\mathrm{c}(\mathrm{x})=\mathrm{c}(\mathrm{x})+1$;
endfor endfor
for $\mathrm{k}=(1: 100)$
$\mathrm{y}=[\mathrm{k}, \mathrm{c}(\mathrm{k})]$;
if $c(k)>0$
disp(y)
endif
endfor
for k=[1:10)
y =[k,c(k)];
disp(y)
endfor

```

\section*{Pseudo-code for listing the quotients:}

To find quotients \(a / b\) when an integer
for \(m=(1: 10)\)
\(\mathrm{c}(\mathrm{m})=0\);
endfor
```

```
for \(\mathrm{a}=(1: 10)\)
```

```
for \(\mathrm{a}=(1: 10)\)
for \(b=(1: 10)\)
for \(b=(1: 10)\)
\(\mathrm{x}=\mathrm{a} / \mathrm{b}\);
\(\mathrm{x}=\mathrm{a} / \mathrm{b}\);
if \(\mathrm{x}==\mathrm{floor}(\mathrm{x}) \quad\) Note: floor \((x)=\) integer part of \(x\)
if \(\mathrm{x}==\mathrm{floor}(\mathrm{x}) \quad\) Note: floor \((x)=\) integer part of \(x\)
\(\mathrm{c}(\mathrm{x})=\mathrm{c}(\mathrm{x})+1\);
\(\mathrm{c}(\mathrm{x})=\mathrm{c}(\mathrm{x})+1\);
endif
endif
endfor endfor
```

endfor endfor

```

Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and curriculum


\section*{links: http://aiminghigh.aimssec.ac.za}

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Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South \\
Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa. \\
New material will be added for Secondary 6. \\
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339 \\
Mathematics taught in Year 13 (UK) \& Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12
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