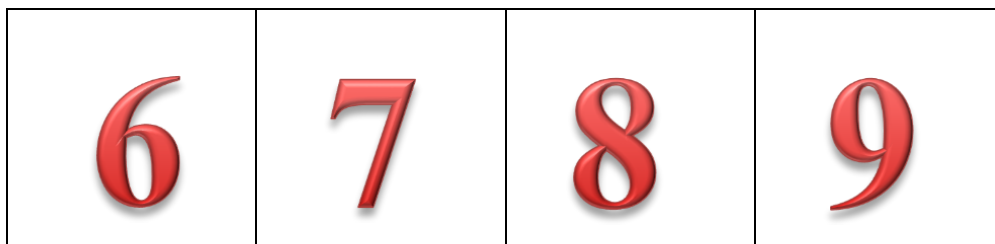## FIFTEEN GAME

This is a game for two players (or a group split into two teams).

Take it in turns to choose one of the numbers 1 to 9. It is now your number and your opponent cannot choose it. Each number can be chosen only once.

To win, be the first to pick 3 numbers that add up to 15.

If all the numbers have been chosen and nobody can make 15, then the game ends in a draw.

HELP
- Write down all the combinations of three of the numbers that add up to 15.
- Have you met anything like this before?
- Remember you can't use 2 numbers to make 15, you can't use 4 numbers, it must be 3 numbers.

NEXT
- Is it an advantage to go first (or second?)
- Can you work out how to play this game so that nobody can beat you?
- When you write down all the different combinations of 3 numbers that sum to 15:
  - How can you be sure that you have found all the combinations?
  - Have you met anything like this before?
  - Which numbers appear in 2 of these combinations?
  - Which numbers appear in 3 of these combinations?
  - Which numbers appear in 4 of these combinations?
- Can you think of any other games you know like this one?
- Make up a number game of your own.
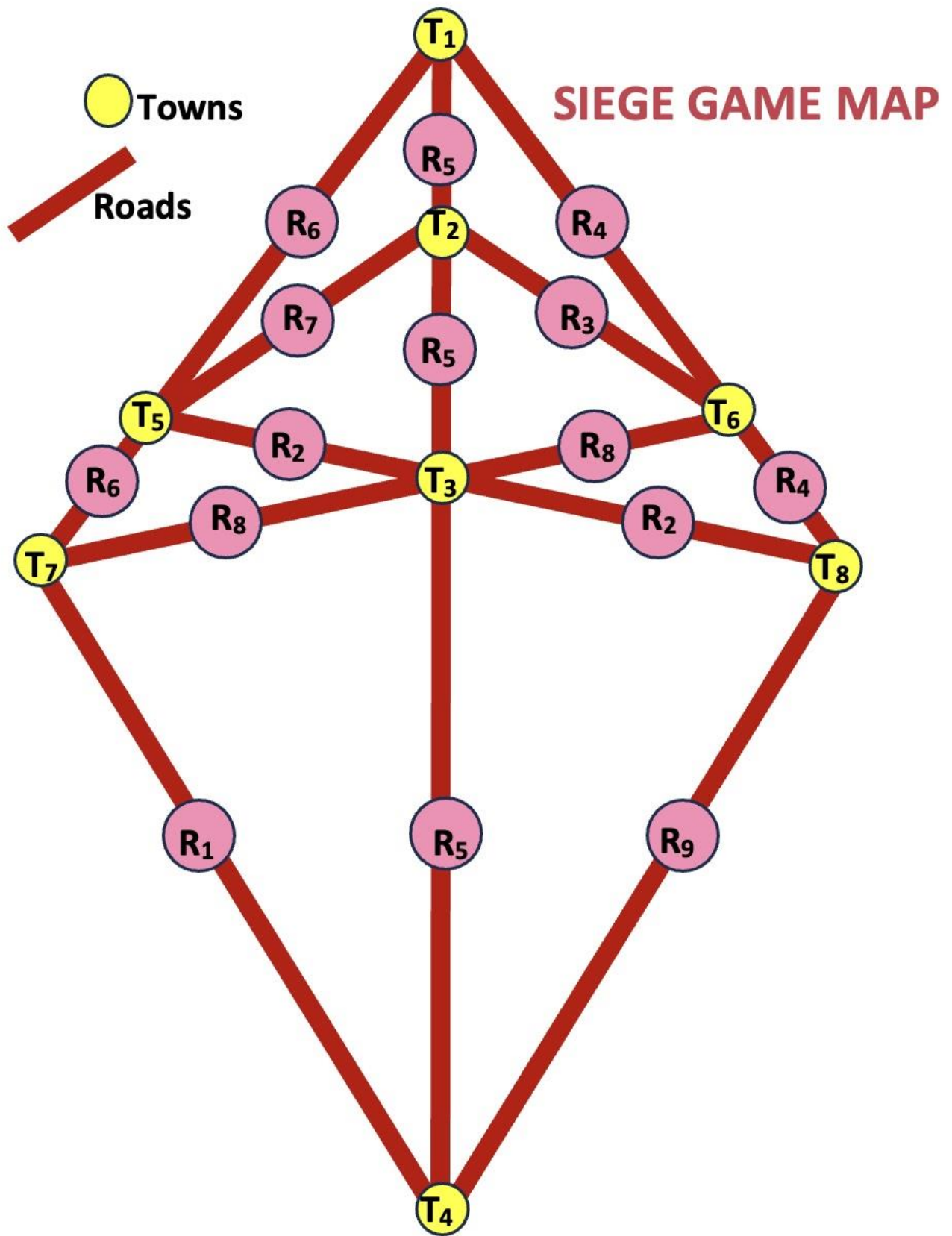
Large cards for the Fifteen Game

1	2
3	4
5	6
7	8
9	**CARDS FOR FIFTEEN GAME**

Cards for the Pat the Pig Game

GOT	AI
GAME	OIE
PET	PIG
PM	AT
MO	**CARDS FOR THE PAT THE PIG GAME**

Towns

Roads

SIEGE GAME MAP

T₁

R₅

R₆   T₂   R₄

R₇   R₃

R₅

T₅   R₂   R₈   T₆

R₆   R₈   T₃   R₄

R₈   R₂

T₇   T₈

R₁   R₅   R₉

T₄

NOTES FOR TEACHERS

Combinations that add up to 15: $1 + 5 + 9$ $1 + 6 + 8$
$2 + 4 + 9$ $2 + 5 + 8$ $2 + 6 + 7$
$3 + 4 + 8$ $3 + 5 + 7$
$4 + 5 + 6$

Note that 5 appears in 4 of the winning triples, more often than any other number, so the player going first should choose 5 because it gives an advantage.

But with skilful play by both players nobody should lose this game. It is important to let people discover this for themselves. It may take a long time and playing lots of games.

Number	1	2	3	4	5	6	7	8	9
In how many triples?	2	3	2	3	4	3	2	3	9

8	**1**	**6**	All rows, columns and diagonals add up to 15 as in the **3 by 3 Magic Square**. The number 5 occurs in 4 combinations so it must go at the centre.	**PET**	**PM**	**PIG**
3	**5**	**7**	The even numbers 2, 4, 6 and 8 occur in 3 combinations so they must go at the corners.	**AT**	**GAME**	**AI**
4	**9**	**2**	The odd numbers 1, 3, 7 and 9 occur in 2 combinations and go in the middle of the edges.	**GOT**	**MO**	**OIE**

The connection to the Magic Square 3 by 3 array links the Fifteen Game to Tic Tac Toe or Noughts and Crosses. The roads in the Siege Game are numbered to show the correspondence with the Fifteen Game and the words for the Pat the Pig game are shown in a 3 by 3 array of winning triples that demonstrate the correspondences to the other games.

Isomorphism (Greek iso-same and morph-change). This is tremendously important in mathematics. It carries with it the crucial information about the object while discarding irrelevant information. In this case the three contexts are to some extent irrelevant. The relationships and structure are the same in all three. Isomorphism is very useful because, if you recognise the structure and have seen it in another form, you already know a lot about it so it saves work. Some of this can be discovered by learners if they meet all the games and work out the correspondences.

Tables

Tables are important for recording information. Putting information into a table often makes patterns clear, shows up gaps in the information, or shows some important feature of the problem being worked on. Sometimes learners can't solve a problem because they can't sort out the necessary information and a table helps them to sort it out.

Why do this activity?

The game gives learners a lot of practice in number bonds and builds number sense. The incentive to win leads to concentration and thinking several moves ahead "What if…?" Then what…?"

Learning objectives

In doing this activity students will have an opportunity to:

- practise addition;
- think strategically.
- make connections with other mathematical ideas.

Generic competences

- think mathematically and give explanations;
- visualize and interpret images to represent concepts and situations;
- work systematically to investigate all possible cases;
- develop the life skill of winning and losing with consideration for others.

Suggestions for Teaching

- Play the game as a whole class first.
- Demonstrate first then divide the class into two teams.
- Use large cards with the numbers on them so the players actually take hold of the cards
- To keep track of what is happening, write the numbers on the board or display the cards
- Then let the learners play the game in pairs. This can be a 'pencil and paper' game but it works best if, at least at first, you give each pair a set of number cards. Ask the students to write down all the combinations of three of the numbers that add up to 15.
- Ask "Have you met anything like this before?"
- Ask
  - "Which numbers appear in 2 of these combinations?'
  - "Which numbers appear in 3 of these combinations?'
  - "Which numbers appear in 4 of these combinations?'
- You might arrange a competition at the end of the session to see who is the champion.
- You might tell the students that this number work should help them in playing the Fifteen Game.
- Then give the class the opportunity to play the game again.
- You might organise a knockout competition in the class.

Key questions

- Have you found a winning strategy?
- Is it possible to avoid losing this game?
- Can you write down all the combinations of three of the numbers that add up to 15?
- Have you met anything like this before?
- Which numbers appear in 2 of these combinations?
- Which numbers appear in 3 of these combinations?
- Which numbers appear in 4 of these combinations?
- Can you become an expert and never lose this game, but the game may sometimes end in a draw?
- Is this game like any other game that you have played before?

Follow up

6 by 4 and 5 by 5 Magic Squares:
https://aiminghigh.aimssec.ac.za/picture-puzzler/

Symmetries in Magic Squares:
https://aiminghigh.aimssec.ac.za/durers-magic-square/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13