AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

The Answer Is 2024, but WHAT IS THE QUESTION?

This **Inclusion and Home Learning Guide** is part of a Learning Pack downloadable from the AIMING HIGH website https://aiminghigh.aimssec.ac.za

It provides related activities for all ages and learning stages from pre-school to school-leaving, together with guidance for inclusion in school lessons and for home-learning, all on the Common Theme 'Working Backwards from 2024'

Choose what seems suitable for the age or attainment level of your learners.

THE ANSWER IS 2024, BUT WHAT IS THE QUESTION?

You can be a creative mathematician; you can be someone who has your own mathematical ideas. Find your own interesting facts about 2024 and calculations that have the answer 2024.

1. **What different questions can you find with 2024 as the answer?** Perhaps you can make up an easy question, a harder one and one that is very hard. Compare your questions with other people's. For example you might ask: 'What is (20 + 24) + (20+24)(20+24) + (20 + 24)?'

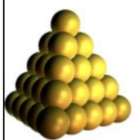
You might like to investigate powers of whole numbers. For example you might ask:
'What is: $2^3 + 3^3 + 4^3 +5^3 + 6^3 + 7^3 + + 8^3 + 9^3$?'

What interesting facts can you find about the year that you were born?

Is it correct to say "twenty twenty-four" or should we say "two thousand and twenty-four" or are both correct? Why?

People say "twenty twenty four" so do the 2 twenties mean – 20 thousands, 20 hundreds, 20 tens or 20 units?

2. The number **2024 is a tetrahedral number**, that is a number that can be represented by a tetrahedral stack like the one shown for the tetrahedral number 35 which has 35 spheres in 5 triangular layers.



The numbers of spheres in each layer are triangle numbers. For a total of 2024 spheres 22 layers are needed. In the picture, the bottom layer has 15 spheres in it (1 + 2 + 3 + 4 + 5), the second layer has 10 spheres, the 3rd has 6, the 4th has 3 and the top layer has only 1. Why is it useful to know how to stack objects like this?

Write down the first 22 triangle numbers that give the number in each layer and the first 22 tetrahedral numbers giving the total number of s in each stack.

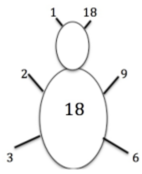
The number 2024 is very special because it is over 250 years since the year was a tetrahedral number and it will be over 300 years until the next one.

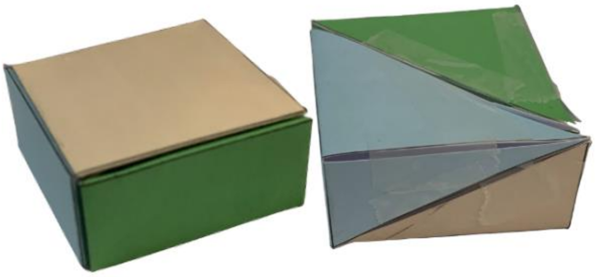
3. Find the prime factors of 2024 and write 2024 as a product of its prime factors. Draw the factor bug for 2024. How many legs does it have?

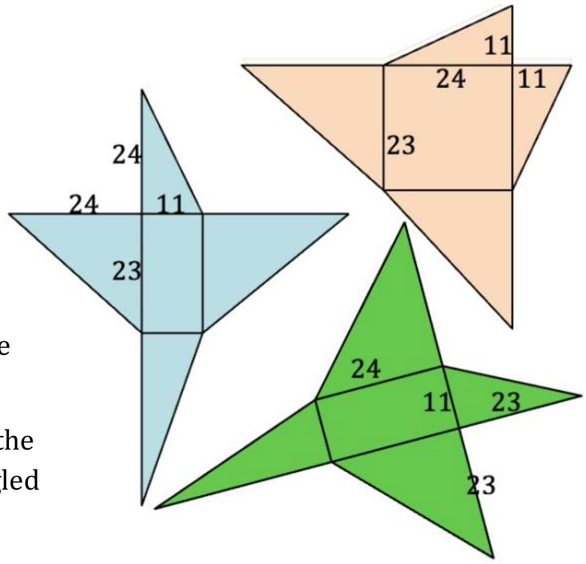
Here is the factor bug for 18. The antennae show 1 × 18 = 18.

The pairs of legs show and 2 × 9 = 18 and 3 × 6 = 18.

Factor bugs for other numbers can have more legs.







4. Download the worksheet and use the nets like those shown to make 3 pyramids.

Fit them together to make a cuboid. Notice that the triangular faces of the pyramids are all right angled triangles.

What does this tell you about the volume of each pyramid?

What does it tell you about the number 2024?

HELP

How old are you? If you are 9 years old then write down some interesting calculations that have the answer 9 (or whatever your age is, do the same for your age).

For example all these have the answer 9:

3 × 3 ; half of 18 ; 10 – 1 ; 20 – 11; 16 -7 ; 3^2 ; square root of 81 etc.

See the problem 'I'm Eight'

https://aiminghigh.aimssec.ac.za/years-3-10-i-am-eight/

NEXT

5. How many ways can you arrange the digits 2, 2, 0 and 4 to get different numbers? What is the sum of those numbers?

6. Explore Wild and Wonderful Number Patterns, see http://nrich.maths.org/33

Make up some of your own number patterns.
You've probably come across number patterns before, ones like :-
2 4 6 8 10 12 . . .
512 256 128 64 32 . . .
220 210 200 190 180 170 . . .
11 14 17 20 23 26 . . .
Work out the rules that produced each of the patterns.
What is the reason for the series of dots appearing after each one?
Now make up some of your own number patterns.

INCLUSION AND HOME LEARNING GUIDE
THEME: WORKING BACKWARDS from 2024

Early Years and Lower Primary
Understanding time and related vocabulary.

Use a calendar for 2024. If your calendar has pictures, look at them and ask the children to tell you what they see in the pictures. Do they like the pictures? Why?

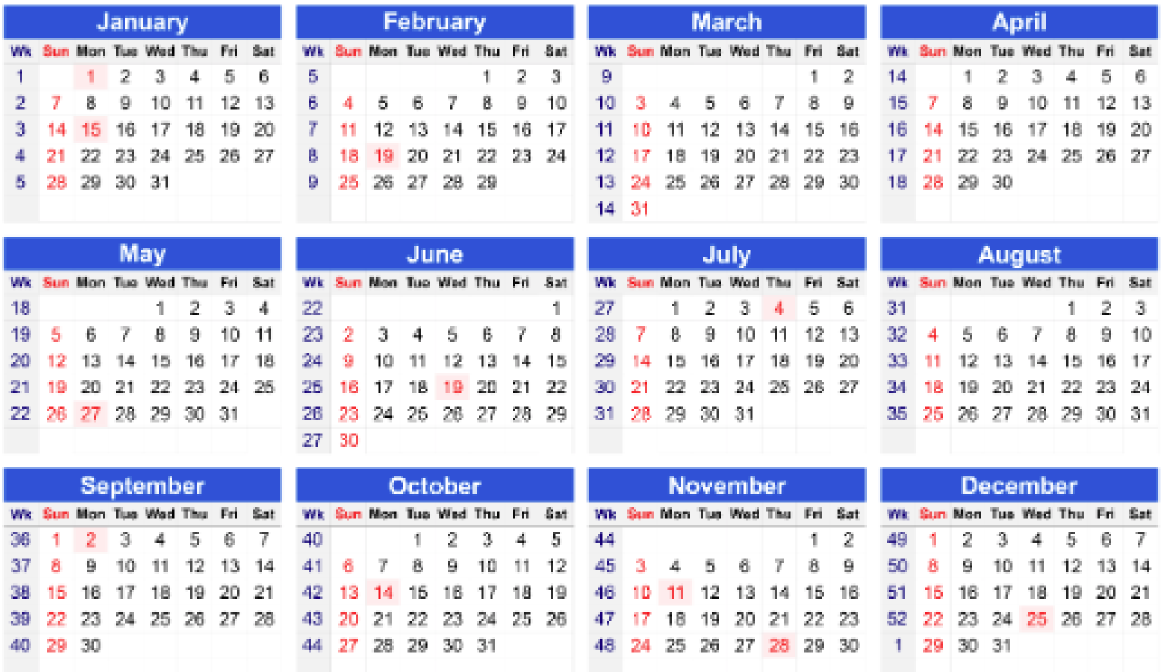
Ask about their birthdays. Find their birthdays on the calendar.

Talk about the weeks, count the 7 the days of the week and talk about their names.

On what days of the week will their birthdays occur in 2024?

Talk about the months, count the 12 months of the year and talk about the names of the months.

In what months will their birthdays occur in 2024?



January

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1		1	2	3	4	5	6
2	7	8	9	10	11	12	13
3	14	15	16	17	18	19	20
4	21	22	23	24	25	26	27
5	28	29	30	31			

February

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
5					1	2	3
6	4	5	6	7	8	9	10
7	11	12	13	14	15	16	17
8	18	19	20	21	22	23	24
9	25	26	27	28	29		

March

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
9						1	2
10	3	4	5	6	7	8	9
11	10	11	12	13	14	15	16
12	17	18	19	20	21	22	23
13	24	25	26	27	28	29	30
14	31						

April

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
14		1	2	3	4	5	6
15	7	8	9	10	11	12	13
16	14	15	16	17	18	19	20
17	21	22	23	24	25	26	27
18	28	29	30				

May

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
18				1	2	3	4
19	5	6	7	8	9	10	11
20	12	13	14	15	16	17	18
21	19	20	21	22	23	24	25
22	26	27	28	29	30	31	

June

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
22							1
23	2	3	4	5	6	7	8
24	9	10	11	12	13	14	15
25	16	17	18	19	20	21	22
26	23	24	25	26	27	28	29
27	30						

July

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
27		1	2	3	4	5	6
28	7	8	9	10	11	12	13
29	14	15	16	17	18	19	20
30	21	22	23	24	25	26	27
31	28	29	30	31			

August

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
31					1	2	3
32	4	5	6	7	8	9	10
33	11	12	13	14	15	16	17
34	18	19	20	21	22	23	24
35	25	26	27	28	29	30	31

September

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
36	1	2	3	4	5	6	7
37	8	9	10	11	12	13	14
38	15	16	17	18	19	20	21
39	22	23	24	25	26	27	28
40	29	30					

October

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
40			1	2	3	4	5
41	6	7	8	9	10	11	12
42	13	14	15	16	17	18	19
43	20	21	22	23	24	25	26
44	27	28	29	30	31		

November

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
44						1	2
45	3	4	5	6	7	8	9
46	10	11	12	13	14	15	16
47	17	18	19	20	21	22	23
48	24	25	26	27	28	29	30

December

Wk	Sun	Mon	Tue	Wed	Thu	Fri	Sat
49	1	2	3	4	5	6	7
50	8	9	10	11	12	13	14
51	15	16	17	18	19	20	21
52	22	23	24	25	26	27	28
1	29	30	31				

Look for patterns in the numbers on the calendar.

How old will they be on their birthday in 2024?

In which year were they born? Count from the year of their birth up to 2024: for example 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024. So someone born in 2016 will be 8 years old in 2024.

Upper Primary and Lower Secondary Years 4 – 10

Developing independent learning and understanding of factors.

THE ANSWER IS 2024, BUT WHAT IS THE QUESTION?

Copy this question on the board.

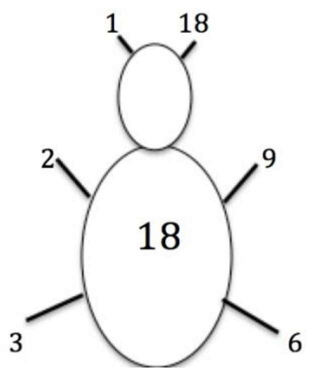
It is worthwhile to have a preliminary discussion about the number 2024, what words we use to talk about it and place value.

There is no need for assessment of prior knowledge for this activity, but teachers can use the activity to assess what understanding of number operations and what number sense and creativity the individual learners have. It is a good activity for the start of the school year. It can be adapted year by year to give 'The answer is 2025…' or 'The answer is 2026…' etc.

Explain to learners that they can and should be able to think for themselves like mathematicians and not just follow instructions and copy other people. Explain that computers now do all the routine jobs and, in this century, higher skills are needed. Tell them that sometimes you are going to give them problems without telling them what to do to solve them but that you will teach them a lot of new mathematics so that they will get better and better at problem solving.

You might like to use the 'One-Two-Four-More' strategy getting the learners to work individually until each learner has a calculation that gives the answer 2024. Then tell the learners to work in pairs to check that their partner's calculation does have the answer 2024. Then ask the learners to work in fours. Perhaps each group could make a poster showing some calculations that have the answer 2024. You could have a class discussion as to what would make one of these 'made-up questions' interesting.

Ask each group to contribute one of their 'calculations' and the class could vote on which is the 'most interesting'. For example you might think that $673 + 674 + 675 + 2 = 2024$ is interesting because it is the sum of 3 consecutive integers plus 2, but not as the sum of 3 consecutive integers. Which years can be written **exactly** as the sum of 3 consecutive integers and which cannot be written like that?



Here is the factor bug for 18. The antennae show $1 \times 18 = 18$. The pairs of legs show and $2 \times 9 = 18$ and $3 \times 6 = 18$. Factor bugs for other numbers can have more legs.
Draw the factor bug for 2024. How many legs does it have?

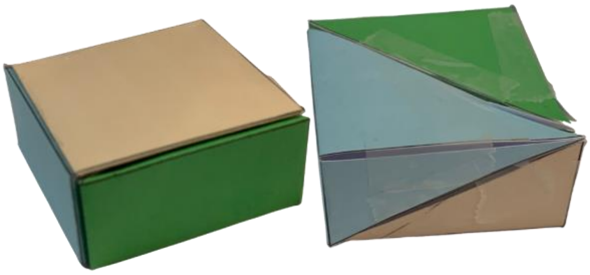
Key questions

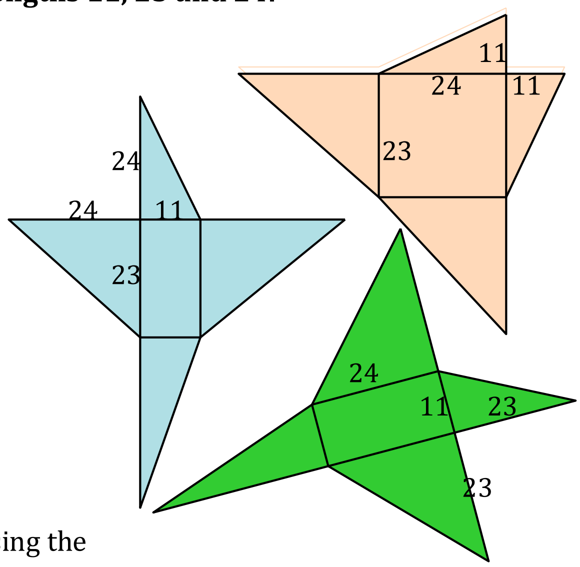
- What calculations can you do to get an answer of 2024? Write them down.
- Can you explain your method to me and how it works?
- What answer does your calculation give when you work it out?
- What could you add to your answer to get 2024?
- Can you use other operations (such as subtraction, multiplication, division, squaring etc)?

Secondary Years 9 to 12 – Properties of 3D shapes.

The diagrams show 2 views of a cuboid with edges 11, 23 and 24, made up of 3 pyramids, and nets for the pyramids.

Each pyramid has 3 perpendicular edges of lengths 11, 23 and 24.





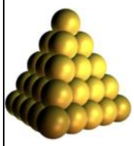
Copy Question 5 from the worksheet onto the board or give out copies of the worksheet. Ideally print the nets on card or stick the paper nets onto scrap cardboard. Give them copies of the nets, scissors and sticky tape or glue, organising the class so that the learners work in groups of six to make the 3 different pyramids. Guide the learners to cut out the nets and to make the pyramids, then to work in groups to fit 3 differently coloured pyramids together to make a cuboid.

Notice that the triangular faces of the pyramids are all right angled triangles. Learners might be asked to work out all the lengths of the edges using Pythagoras Theorem.

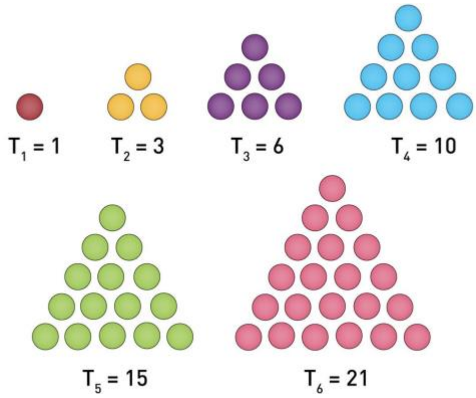
Ask learners to work out the volume of the cuboid that has edges 11, 23 and 24 and to find the volumes of all three pyramids.

Ask the class what this tells them about the volume of each pyramid and the number 2024.

Secondary Years 9 to 12 - Number and Geometric Patterns.



The number **2024 is a tetrahedral number**, that is a number that can be represented by a tetrahedral stack like the one shown for the tetrahedral number 35 which has 35 spheres in 5 triangular layers. The numbers of spheres in each layer are triangle numbers as shown in different colours in the diagram below.



Copy these diagrams on the board and ask the learners what they notice about them. Have a class discussion about stacking objects like fruit and cans of food in supermarkets. Demonstrate the process if you have suitable objects available.

Ask the learners to work out

a. the next triangle number in the pattern T_7

b. the first 6 tetrahedral numbers.

Ask the learners to give their answers and check with the list below.

Triangle Numbers: $T_1=1$, $T_2=3$, $T_3=6$, $T_4=10$, $T_5=15$, $T_6=21$, ...

Tetrahedral Numbers: $P_1=1$, $P_2=1+3=4$, $P_3=4+6=10$, $P_4=10+10=20$, $P_5=20+15=35$, $P_6=35+21=56$...

Tell the class that you want them to find out whether or not 2024 is a tetrahedral number and that **they will share the work involved.** Check that everyone understands the problem and that, between them, they will write down a list of triangle numbers that give the number in each layer and a list of the tetrahedral numbers that giv the total number of in each stack.

Split the class into two groups and give them the following tasks. Then check the answers and discuss the findings.

Group 1

$T_7=1+2+3+4+5+6+7=28$ and $P_6=56$.

Write down the next 8 Triangle Numbers and the next 8 Tetrahedral Numbers.

Answer: Triangle numbers 28, 36, 45, 55, 66, 78, 91, 105, 120

Tetrahedral numbers 56, 84, 120, 165, 220, 286, 364, 455, 560

Group 2

$T_{15} =1+2+3+4+5+6 \ldots +13+14+15=120$ and $P_{14}=560$.

Write down the next 8 Triangle Numbers and the next 8 Tetrahedral Numbers.

Answer: Triangle numbers 120, 136, 153, 171, 190, 210, 231, 253, 276

Tetrahedral numbers 560, 680, 816, 969, 1140, 1330, 1540, 1771, 2024

Ask the learners: 'Why is it useful to know how to stack objects like this?'

The number 2024 is very special because it is over 250 years since the year was a tetrahedral number and it will be nearly 300 years until the next one.

Older learners could find the formulae for the n^{th} terms of the sequences.

See triangle number Picture https://aiminghigh.aimssec.ac.za/triangle-number-picture/

Upper Secondary Years 12 and 13 – addressed directly to students

1. Do the tasks on page 1.

2. If you understand proof by Mathematical Induction then, using the formula for the n^{th} triangle number: $T_n = \frac{1}{2}n(n+1)$ prove that the nth tetrahedral number is

$$P_n = \frac{1}{6}n(n+1)(n+2)$$

3. Create some calculations using powers of whole numbers that give the answer 2024.

4. The number 2024 cannot be written as the sum of 2 squares, but it can be written as the sum of 3 squares in 7 different ways.

   All natural numbers can be written as the sum of 4 squares and 2024 can be written as the sum of 4 squares in 16 different ways. For example

   $2^2 + 18^2 + 20^2 + 36^2 = 2024$

If you know how to do simple coding then adapt this pseudo-code to whatever language you know to find different ways of writing 2024 as sums of squares:

Pseudo-code to find sums of 4 squares that give the answer 2024

```
for a = 1 to 44
for b = a to 44
for c = b to 44
for d = c to 44
if a² + b² + c² + d² = 2024
x = [a, b, c, d]
display x
end end end end
```

SOLUTIONS

Question *1* Although commonly used 'twenty twenty-four' is not correct and we should say 'two thousand and twenty-four' given by: 2 × 1000 + 2 × 10 + 4 = 2024.

People do say 'twenty twenty' and that means 20 hundreds and 20 units. We can write this: (20 × 100) + (20 × 1) + 4 = 2024 but this is not the usual format for our place value system which should be:

(2 × 1000) + (0 × 100) + (2 × 10) + 4 = 2024.

For younger learners any sum that gives the answer 2024 is a solution. They should find many different ways to get 2024 and share their ideas with each other.

For them 1010 + 1010 + 4 = 2024 or

2 × 1010 + 4 = 2024 or

4 × 505 + 4 = 2024 or 5 × 404 + 4 = 2024 or ...

Clearly there are many simple solutions.

Older learners may find more complicated solutions. Some solutions are more accessible and more interesting than others. What is interesting for 10 year-olds may not be interesting for 18 year-olds and vice versa.

Here are some interesting properties of 2024:

1. Reversing the digits gives 4202 and adding 2024 + 4202 = 6226 which is a palindromic number.
2. (20 + 24) + (20+24)(20+24) + (20 + 24) = 44 × 6 = 2024
3. $2024 = 2^3 + 3^3 + 4^3 + \ldots 9^3$
4. $2024 = 2^2 + 16^2 + 42^2$
5. 2024 = (22 × 23 × 24)/6 is the 22nd tetrahedral number.

Question 2

The nth triangle number $T_n = 1 + 2 + 3 + \ldots + (n - 1) + n$

is given by the formula $T_n = \frac{n(n+1)}{2}$

Triangle Numbers: {1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120,136, 153, 171, 190, 210, 231, **T_{22}=253,** 276, 300, 325, 351, 378, 406, 435, 465, 496, 528, 561, 595, 630, 666, 703, 741, 780, 820, 861, 903, 946, 990, 1035, 1081, 1128, 1176, 1225, 1275, 1326, 1378, 1431,...} There are infinitely many Triangle numbers.

The nth tetrahedral number $P_n = T_1 + T_2 + T_3 + \ldots + T_{(n-1)} + T_n$

is given by the formula $P_n = \frac{n(n+1)(n+2)}{6}$

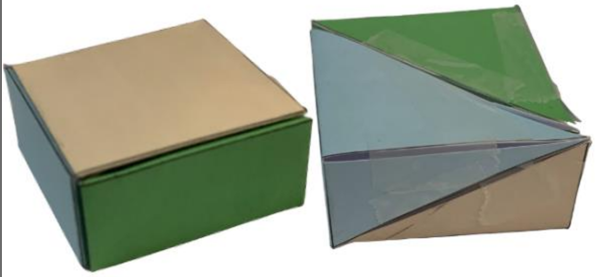
Tetrahedral Numbers: {1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286, 364, 455, 560, 680, 816, 969, 1140, 1330, 1540, 1771, **P_{22}=2024**, 2300, 2600, 2925, 3276, 3654, 4060, 4495, 4960, 5456, 5984, 6545, 7140, ...} There are infinitely many Tetrahedral numbers.

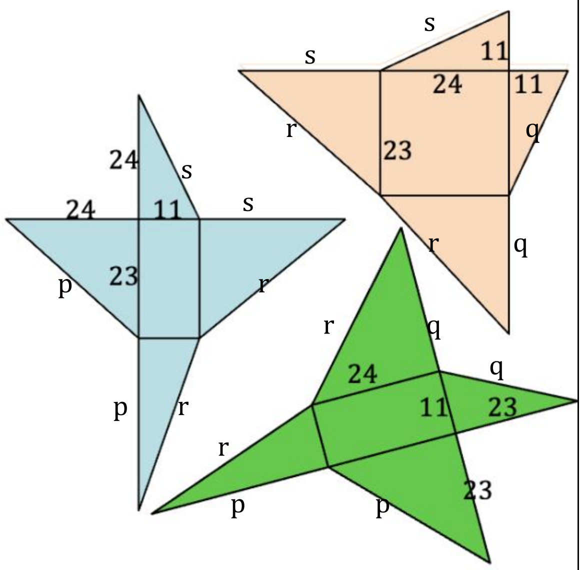
Question 3 The prime factors of 2024 are , 2, 11 and 23 and, written as a product of prime factors **2024 =2³ × 11 × 23**.

The factors of 2024 are: 1, 2, 4, 8,11, 22, 23, 44, 46, 88, 92,184, 253, 506,1012 and 2024.

The factor bug has 1 × 2024 on its antennae and 7 pairs of legs for: 2 × 1012, 4 ×506, 8 ×253, 11 ×184, 22 ×92, 23 × 88, 44 × 46

Question 4 **YEARS 9 TO 12 – INVESTIGATION OF MAKING A CUBOID FROM 3 PYRAMIDS**.





$p^2 = 23^2 + 24^2$ $\quad p = 33.24$

$q^2 = 11^2 + 23^2$ $\quad q = 25.50$

$s^2 = 11^2 + 24^2$ $\quad s = 26.40$

$r^2 = 11^2 + 23^2 + 24^2$ $\quad r = 35.01$

UPPER SECONDARY - PROOF BY MATHEMATICAL INDUCTION

The following proof uses **the method of Mathematical Induction. F**irst a simple case is proved. Next it is proved that, if the statement S(n) is true for a general value n, then it must always be true for the next case S(n+1) (*the inductive step*).

Mathematical induction proves that we can climb as high as we like on a ladder, by proving that we can climb onto the bottom rung (the simple case or **basis**) and that from each rung we can climb up to the next one (the **inductive step**).

Another analogy uses a line of dominoes. If each domino knocks over the next one in line, and if we show that the first domino knocks the second one over, then they all fall.

Proof of the formula for the *n*th tetrahedral number

$$P_n = \frac{n(n+1)(n+2)}{6}.$$

We prove that, for all natural numbers n, the statement

S(n) : " the *n*th tetrahedral number $P_n = \frac{n(n+1)(n+2)}{6}$ " is true.

This proof uses the fact that the nth triangular number is given by $T_n = \frac{n(n+1)}{2}$

The simple case: The theorem is true for n = 1 and n = 2.

Note that $P_1 = \frac{1\times2\times3}{6} = 1$ and $P_2 = \frac{2\times3\times4}{6} = 4$ where $P_2 = P_1 + T_2 = 1 + 3 = 4$.

The inductive step: If the theorem is true for *n* then it is true for (*n*+1).

If $P_n = \frac{n(n+1)(n+2)}{6}$ is true then

$$P_{n+1} = P_n + T_{n+1} = \frac{n(n+1)(n+2)}{6} + \frac{(n+1)(n+2)}{2}$$

$$= \frac{(n+1)(n+2)(n+3)}{6}$$

giving the required formula for P_{n+1} and proving that the statement S(n+1) is true.

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AIMING HIGH

UPPER SECONDARY - A SIMPLE CODING EXAMPLE

Learners at upper secondary level might investigate sums of squares. There are 7 sums of three squares that add up to 2024. How many can you find?

The following program can be written in different programming languages to find sets of numbers whose squares add up to 2024. If you can write computer code then can you adapt the program to the language you know and you can find how many sets of squares add up to 2024 or adapt the program for cubes instead of squares.

Pseudo-code to find sums of squares that give the answer 2024 The number 2024 cannot be written as the sum of 2 squares. The program below is written so that $c \geq b \geq a$. It checks values up to the square root of 2024, so it checks numbers from 1 to 45.	**The sum of the squares of 7 sets of 3 numbers all add up to 2024.** **The sum of the squares of 16 sets of 4 numbers all add up to 2024.**		
Note, this is a slight variation to the code on page 7 writing the sets in increasing order. for a=(1:45) for b=(a:45) — *as $b \geq a$* for c=(b:45) — *as $c \geq b$* x = a^2+b^2+c^2; — *checks sum of squares* if x == 2024 — *If sum is 2024 prints a,* disp([a,b,c]) — *b and c. If not goes on to* end end end end — *next values.*	2 16 42 2 24 38 8 14 42 10 18 40 10 30 32 16 18 38 18 26 32	2 18 20 36 4 6 6 44 4 6 26 36 4 10 12 42 4 18 28 30 6 8 18 40 6 8 30 32 6 12 20 38 6 16 24 34	8 22 24 30 10 12 22 36 10 18 24 32 12 14 28 30 12 18 20 34 14 24 24 26 18 20 20 30

Why do these activities?

This collection of activities gives open questions suitable for learners of all ages on number, sequences and series, factors and prime factorization, 3D shapes, model making, volume, Pythagoras Theorem, exponents and simple coding, and proof by Mathematical Induction.

The question “2024 is the answer, what is the question?” is suitable for younger and less confident learners, but older learners can use their greater mathematical knowledge. Use this activity to encourage learners to think for themselves and to use everything they know about numbers to create their own solutions to invent questions. This activity allows learners to be creative, to explore and play with numbers and to come up with their own solutions. It is also an example of an inverse problem, one that gives the answer so that the learner has to think of where to start to get to that answer.

Learning objectives

In doing these activities students will have opportunities to:

Primary:

- develop vocabulary about measures and cycles in the passage of time
- practise calculations involving operations that they know

Lower Secondary, all the above and:

- earn about factors and prime factorisation
- learn about number patterns and geometric patterns and sequences
- learn about properties of 3D shapes and volume
- apply knowledge of Pythagoras theorem in a practical model making context.

Upper Secondary, all the above and:

- use some simple coding
- learn about proof by Mathematical Induction

Generic competences

In doing this activity students will have an opportunity to **think flexibly**, be creative and innovative and apply knowledge and skills.

Follow up

Calendar Patterns
https://aiminghigh.aimssec.ac.za/calendar-patterns/

Factors and Multiples Game
https://aiminghigh.aimssec.ac.za/factors-and-multiples-game/

Triangle Number Picture
https://aiminghigh.aimssec.ac.za/triangle-number-picture/

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