Cut out the two $L$ pieces and 2 counters (see page 3). Put them on the board as shown. Rules: From this starting position, the first player chooses an L shape and moves it on the board, then chooses whether or not to move one of the counters. The second player moves the other L shape, then that one of the counters if they wish.
Pieces can be turned over. Players can only move their own L shape but they can move either counter. Pieces cannot overlap and the $L$ shape must be moved before the counter. The winner is the player
 who blocks in their opponent's $L$ shape so the opponent cannot move. The squares are numbered for play blindfolded or on the phone.

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 |

2 players
or 2 teams

## HELP

Play to trap your opponent in a corner. Blue is in a winning position in this example. Red is trapped in a corner and cannot move.


## NEXT - PLAY THINK LEARN WIN

PUZZLE 1: If a player is looking at this starting position, what does the opponent sitting opposite see? Draw all the reflections and rotations of this position on squared paper.
PUZZLE 2: Find all possible positions of the Red L on the board and draw them on squared paper. Here is one position.

PUZZLE 3: With Red in this position, find all the positions for Blue if the counters are not blocking the squares where Blue is to be placed. Do the same for all the other positions of Red.


PUZZLE 4: For each position of red and blue L pieces on the board, find where you can place the counters so that one or other of the players is in a winning position if their opponent has to move next?



Cut out the counters and L pieces and fold the L pieces along the dotted lines. Stick pairs of pieces back-to-back so that they are coloured on both sides ready for play.


## Why play this game as a learning activity?

The L Game helps people to develop their knowledge and understanding of reflections and rotations, and also their powers of visualisation and logical reasoning which are of crucial importance in doing mathematics. The skills and behaviours developed through playing mathematical games of this type are of utmost importance as they help students to develop essential transferable skills. To thrive in a rapidly changing world, school leavers need to use their human potential to the full to make use of Artificial Intelligence rather than be disempowered by it.

The L Game was invented by Edward de Bono. The video, made in 2011 by Edward de Bono himself, gives the background to the invention of this game and a discussion about creative thinking. Rather than being satisfied with a correct answer which is Excellent But Not Enough (EBNE), he advises on how to go further to achieve more ambitious goals. He explains the concept of the 'beautiful mind' and the importance of trying to develop new ideas by thinking of all the possibilities

Learning objectives: understanding of reflection and rotation.
Generic competences: Playing the game helps to develop:

- Independent thinking and learning
- Testing intuition and feelings,
- Creativity,
- Values,
- Critical thinking,
- Systematic planning and organising.

DIAGNOSTIC QUIZ This should take about 5-10 minutes at the end of the lesson.
Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A , 2 fingers for $B$, 3 fingers for C and 4 for $\mathrm{D}^{\prime \prime}$.

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
3. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
4. Ask the class again to vote for the right answer by putting up $1,2,3$ or 4 fingers. Notice if there is a change and who gave right and wrong answers.
The correct answer is: A because Red can move from position 3.
Positions 2 and 6 are both winning positions for Blue but position 3 is not.

## Suggestions for teaching

Give each player copies of pages 1,2 and 3 . Ask them to cut out the game pieces and follow the instructions on page 3 . They can study the rules when they have to wait their turn If they are sharing scissors and glue-sticks.

Explain the rules of the game. Ask the class to say what they notice about the positions shown in the two diagrams. Give the class a few minutes to discuss this in pairs then have a class discussion.

The learners may say that, in both of these two positions, if Red has the next turn, then Red is trapped and cannot make a move, so Blue has won. Help everyone to understand this answer and to understand that, to win the game, they should try to force their opponent to move their L piece
 into a corner so that they can trap it there.

Divide the class into two teams and provide each team with a gameboard and game pieces. If possible, let teams sit in two groups so that they can see their gameboard and communicate with each other and make a joint decision as to what will be their next move. Set an upper limit on the time that teams have to decide on their next move. Use a master board to record the game as the teams make their moves. A representative from each team should call out the team's next move or move the pieces on the master board.


The team should play 3 games then return to their usual seats and draw all the reflections and rotations of this position on squared paper (PUZZLE 1). Ask the question: If a player is looking at this position, what does the opponent sitting opposite see?

The learners should find for themselves and describe the reflections and rotations shown and identify the mirror lines and centres of rotation. Have a class discussion about how these eight positions are essentially equivalent which cuts down the number of possible game positions. Each winning move in one
 layout corresponds to winning moves in the others.

At the end of the lesson carry out the Diagnostic Quiz and afterwards discuss the solution so that learners can recognise examples of winning play. Suggest that learners should play the game for homework. Encourage playing this game in a Maths Club if you have one in your school.

## Key questions

These are questions for players to ask themselves individually or in their team, or for teachers to ask to prompt and help a player to play strategically and win.

1. Where could you put your $L$ piece and would that be a good move?
2. If you put your L piece there, what would be the best move for your opponent and would that be a threat to you?
3. If you move your L piece there, would you move a counter or not? If you move it, where would you put it and why?
4. You know some of the winning positions so can you recognise reflections and rotations of the same layout?
5. You know some of the winning positions so how can you avoid being the losing player in that set-up?

## SOLUTIONS



Here is a solution to Puzzle 1.
Image 2 is the reflection of 1 in the vertical mirror line (dotted) and 4 is a reflection of 3 . Images 3 and 4 are the reflections of 1 and 2 in the horizontal mirror line (dotted).
Note that two reflections give a rotation so that 1 and 4 are images of each other by rotation of order 2 as are images 2 and 3 .

Image 5 is a quarter-turn clockwise of image of 1 (rotation order 4 ) and 6 is a threequarter turn.
Images 1, 2, 3 and 4 together have half turn rotational symmetry (order 2). Images 1, 4, 5 and 6 taken as a single composite have rotational symmetry of order 4.


Check that there are no other positions for red that are not reflections or rotations of one of the six solutions shown, and therefore equivalent?

PUZZLE 3:
With red in the same position here are some of the possible positions for blue (providing the position is not blocked in one of the black pieces). Can you and your class find others?

PUZZLE 4: In each position of red and blue L pieces on the board, where could the counters be placed to put blue in a winning position.
See some examples below. Can you find others?

## STRATEGIES

Can you see the best moves for blue to play from the positions illustrated below? You could try all these positions one by one, and perhaps do this collaboratively in a group.
With red to move next, if blue plays correctly, these are winning positions for blue in $1,2,3$, or 4 moves.


## Follow up

Nine Men's Morris or Morabaraba This is a collection of games that involve blocking your opponent's pieces so that they can't make a move.
https://aiminghigh.aimssec.ac.za/nine-mens-morris-or-morabaraba/
Checkers (also known as Draughts) is a classic board game dating back to about 3000 BCE https://en.wikipedia.org/wiki/Checkers

| MIATHS | Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and <br> curriculum links: http://aiminghigh.aimssec.ac.za <br> Subscribe to the MATHS TOYS YouTube Channel |
| :--- | :--- |
| TOYS | https://www.youtube.com/c/MathsToys/videos <br> Download the whole AIMSSEC collection of resources to use offline with <br> the AIMSSEC App see $\underline{\text { https://aimssec.app or find it on Google Play. }}$ |
|  |  |


| Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa. <br> New material will be added for Secondary 6. <br> For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339 <br> Mathematics taught in Year 13 (UK) \& Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Lower Primary Approx. Age 5 to 8 | Upper Primary Age 8 to 11 | Lower Secondary Age 11 to 15 | Upper Secondary Age 15+ |
| South Africa | Grades R and 1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| East Africa | Nursery and Primary 1 to 3 | Primary 4 to 6 | Secondary 1 to 3 | Secondary 4 to 6 |
| USA | Kindergarten and G1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| UK | Reception and Years 1 to 3 | Years 4 to 6 | Years 7 to 9 | Years 10 to 13 |

