



FIFTEEN GAME COLLECTION Inclusion and Home Learning Guide is part of a Learning Pack downloadable from the AIMING HIGH website <https://aiminghigh.aimssec.ac.za> on the common theme of **MATHEMATICAL DISGUISES or ISOMORPHISM**. It provides the related games of Pat the Pig, Siege, Magic Squares and Ultimate Tic Tac Toe for inclusion in school lessons and home learning for all ages and learning stages from pre-school to school-leaving, together with guidance for supporting the learning.

Choose what seems suitable for the age or attainment level of your learners.

FIFTEEN GAME

This is a game for two players (or a group split into two teams).

Take it in turns to choose one of the numbers 1 to 9. It is now your number and your opponent cannot choose it. Each number can be chosen only once.

To win, be the first to pick 3 numbers that add up to 15.

The game may end in a draw if all the numbers have been chosen and nobody makes 15.



HELP

- Write down all the combinations of three of the numbers that add up to 15.
- Enter your triples of numbers in the rows, columns and diagonals of a 3 by 3 grid.
- Have you met anything like this before?
- Remember you can't use 2 numbers to make 15, you can't use 4 numbers, it must be 3 numbers.

NEXT

- Is it an advantage to go first (or second?)
- Can you work out how to play this game so that nobody can beat you?
- When you write down all the different combinations of 3 numbers that sum to 15:
 - How can you be sure that you have found all the combinations?
 - How many combinations of 3 numbers summing to 15 are there?
 - Have you met anything like this before?
 - Which numbers appear in 2 of these combinations?
 - Which numbers appear in 3 of these combinations?
 - Which numbers appear in 4 of these combinations?
- Can you think of any other games you know like this one?
- Make up a number game of your own.

LARGER CARDS FOR FIFTEEN GAME

1

2

3

4

5

6

7

8

9

GOT

AI

GAME

OIE

PET

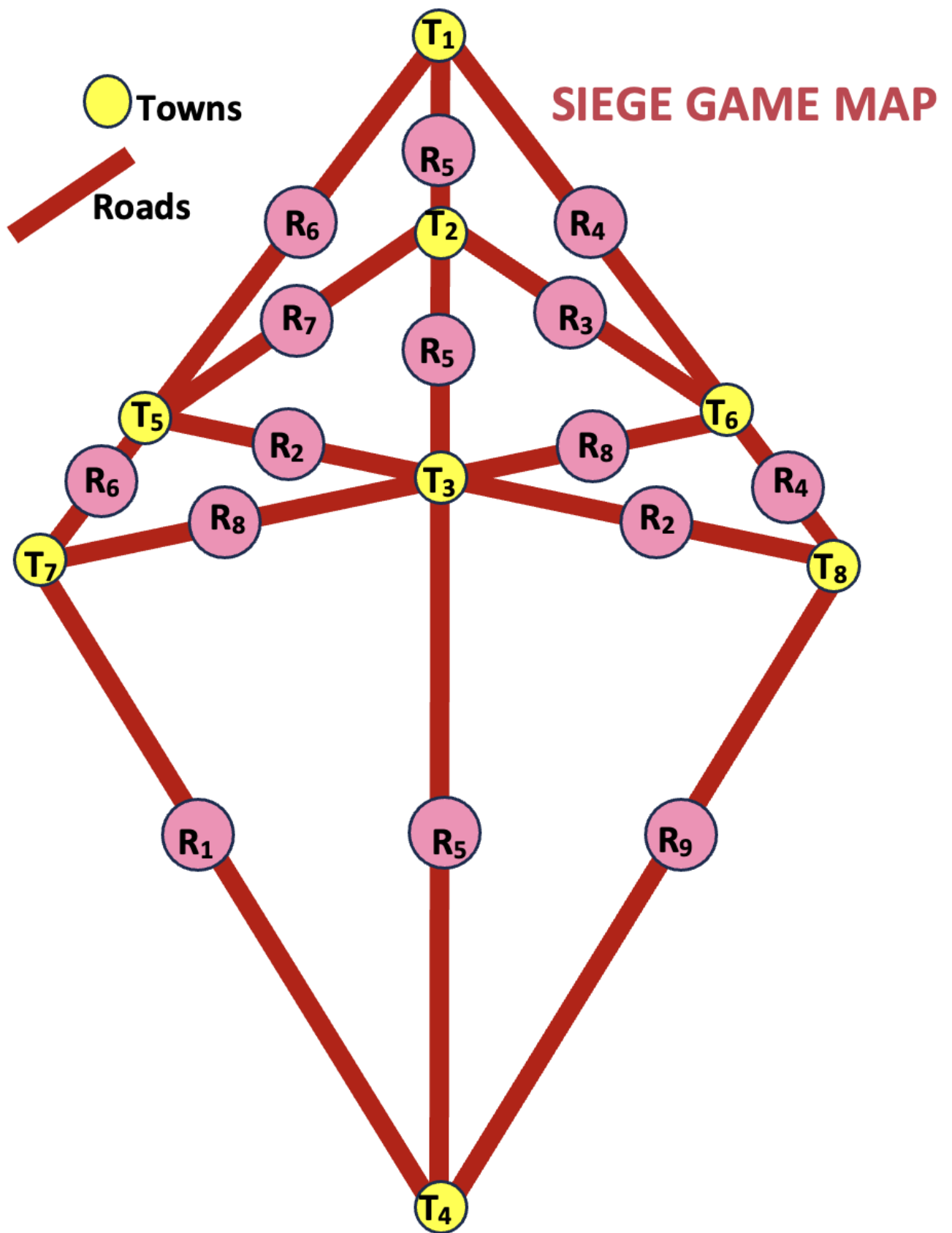
PIG

PM

AT

MO

**CARDS FOR
PET PIG
GAME**



INCLUSION AND HOME LEARNING GUIDE

THEME: MATHEMATICAL DISGUISES

Early Years and Lower Primary

Make a set of cards for 1 to 9 with **spots on instead of numbers**. Play the Fifteen Game using these cards so that little children can count the dots.

When the children can recognise the written numbers, use number cards.

MAKE TEN GAME

This game for two is suitable for very young players, and fun for everyone. You need one die and a coloured pen for each player.

It reinforces recall of the bonds for 10 and it is a good preparation for playing the Fifteen Game. Draw a frame as for Noughts and Crosses

(Tic, Tac, Toe). Take turns to throw the die and write the number in one of the squares. The first player to complete a line of 3 numbers adding up to 10, horizontally vertically or diagonally, wins the game. In this example green has won. The game can be played where the line of 10 can be in any colour, or where it must be in the winner's colour.

5		4
	3	
3	6	2

When introduced to the Fifteen Game, some 7 and 8-year-olds will be able to write down the winning combinations of 3 numbers that add up to 15. Don't rush this. If they don't find all 8 combinations one day then encourage them to keep looking and come back to it another day. Eventually they will find all 8 combinations. Help the learners to arrange the list systematically so that they can check that they have found all the combinations.

$$1 + 9 + 5; 1 + 8 + 6;$$

$$2 + 9 + 4; 2 + 8 + 5; 2 + 7 + 6;$$

$$3 + 8 + 4; 3 + 7 + 5;$$

$$4 + 6 + 5.$$

Upper Primary All the above.

Now the learners should play Tic-Tac-Toe (Noughts and Crosses) and think about what is similar, and what is different, about Tic tac Toe and The Fifteen Game.

Next play the **PET PIG GAME**. Use the cards on page 4 or make your own cards showing the words given below.

To play the game, players take a word alternately.

You win if you get all three occurrences of the same letter. For example PET, OIE and GAME contain all occurrences of the letter E.

Can you devise a strategy so that you never lose?

Can you explain your strategy?

GOT	PET	PIG	AI
GAME	AT	OIE	MO
PM			

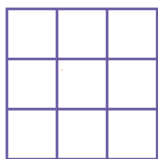
Game for 2.
Take words alternately.
You win if you get all the
occurrences of the same letter.

KEY QUESTIONS:

- What is similar and what is different about the games Tic Tac Toe, Fifteen and Pet Pig?
- What occur in the games in sets of 9?
- What occur in the games in sets of 8?
- What is similar about how to win the games?
- What are the correspondences between the games?
- Have you found a winning strategy?
- Is it possible to avoid losing this game?

Lower Secondary

All the above.



Without telling the learners that there is a connection to the Fifteen Game, ask them to write the numbers 1 to 9 into a 3-row and 3-column array so that all the rows, columns and diagonals have the same total. But don't tell them what the total is.

This **Magic Square** activity is suitable for a mixed age group that includes primary age learners because it only involves simple addition. Problem solving is involved and some younger learners may succeed in this task before the older ones.

If you have a larger group then you could ask them to do this individually, then in pairs, then in fours, and to exchange and share ideas for methods and solutions.

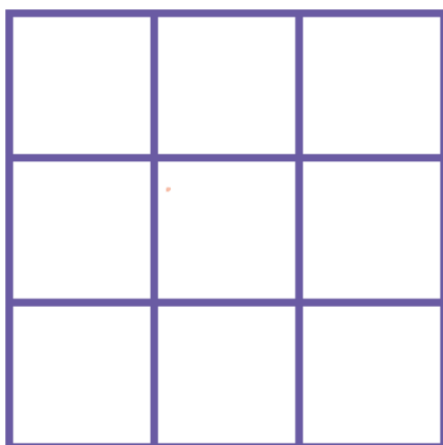
You could give the hint that it helps if you find the total of $1 + 2 + 3 + \dots + 9$. Suggest that, to understand better how to find solutions, learners could write down all the sets of 3 digits from the 1, 2, 3, ...9 that add up to 15 without repeating any digit.

The 8 solutions correspond to the 8 rows, columns and diagonals in a 3 by 3 array.

Learners may find what appear to be different solutions. Ask 'Are they different?' Discuss the fact that **there is only one solution** as all the solutions are rotations or reflections of each other.

Also try the **UnMagic Square** where all the totals of rows, columns and diagonals are **different**. Here is an example. Can you find other solutions with different numbers in the centre?

3	4	5
6	9	2
1	8	7



MAGIC SQUARE GAME

The game is played with a set of cards numbered 1 to 9 and a 3-by-3 gameboard. Take turns either to put one of the 9 cards into an empty square, or to move to an adjacent empty square one of the cards that has already been placed. The winner is the first player to make a line of 3 numbers adding up to 15 (across, up and down or diagonally).



Years 9 and 10

All the above.

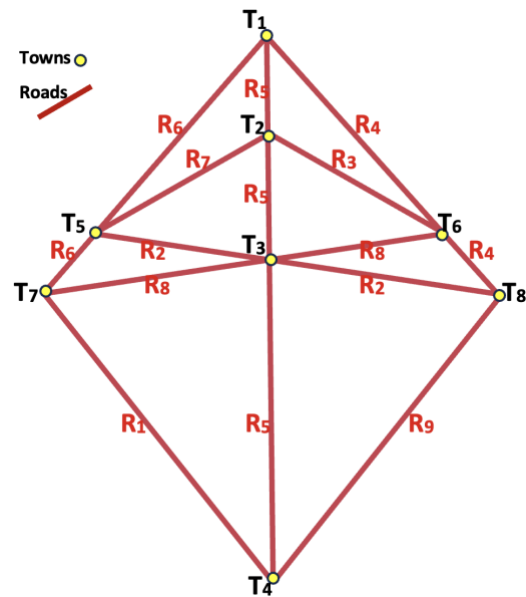
Then play the **SIEGE GAME**.

This is a game for two players. You need the game board shown on page 5, which is easy to copy so you can play this on paper. You will need counters to put on the roads on the map when you claim them, or two different colour pencils (highlighters) to mark the roads claimed.

Notice that there are 8 dots representing towns on the map, and each town lies on exactly 3 roads.

Notice also that there are 9 straight lines representing roads going to or through the towns. Some roads connect 2 towns, some connect 3 towns and some connect 4 towns. Each player takes a turn to colour over a whole straight road (even though it may pass through several towns).

To win be the first to block 3 roads going to, or through, the same town.



Key questions

- Have you found a winning strategy?
- Is it possible to avoid losing this game?
- Can you write down all combinations of three roads through the same town?
- Which roads appear in 2 of these combinations?
- Which roads appear in 3 of these combinations?
- Which roads appear in 4 of these combinations?
- Can you become an expert and never lose this game, although the game may sometimes end in a draw?
- Is this game like any other game that you have played before?

Years 11, 12 and 13

All the above. The challenge is to explain the equivalence of:

FIFTEEN GAME

3 BY 3 MAGIC SQUARES

NOUGHTS AND CROSSES

PET PIG GAME

SIEGE GAME

When you investigate the equivalence between one game and another, explain how the properties (features) in one game correspond to properties of the other game.

Discuss the concept of isomorphism which means 'changing in the same way'. Splitting this Greek word, we interpret 'iso' to mean 'same' and 'morphism' to mean 'something changing'. Changes in one game match equivalent changes in the other games. Isomorphism is tremendously important in mathematics. It carries with it the crucial information about the object while discarding irrelevant information. In this case the five contexts are to some extent irrelevant. The relationships and structure are the same in all five. Isomorphism is very useful because, if you recognise the structure and have seen it in another form, you already know a lot about it so it saves work. Some of this can be discovered by learners if they meet all of these games.

Key questions

- Have you found a winning strategy?
- Is it possible to avoid losing this game?
- Can you write down all the combinations of three objects (numbers, words, roads) that form winning combinations?
- Which numbers or words or roads appear in 2 of these combinations?
- Which numbers or words or roads appear in 3 of these combinations?
- Which numbers or words or roads appear in 4 of these combinations?
- What sets in these games have 8 elements?
- What sets in these games have 9 elements?
- Can you play and never lose, although the game may sometimes end in a draw?

Why do this activity?

The FIFTEEN and MAGIC SQUARE games give learners a lot of practice in using number bonds and, in so doing, they build ‘number sense’ and numeracy. All the games develop logical thinking because the incentive to win leads to concentration and thinking several moves ahead “What if...?”, “Then what...?”, “How would it affect my play if my opponent made that move”.

Learning objectives

In doing this activity students will have an opportunity to:

- practise addition;
- think strategically;
- make connections between different mathematical ideas.

Generic competences

- **think mathematically** and give explanations;
- **visualize** and interpret images to represent concepts and situations;
- **work systematically** to investigate all possible cases;
- **develop the life skill of** winning and losing with consideration for others.

SOLUTIONS

Combinations that add up to 15: $1 + 5 + 9$ $1 + 6 + 8$
 $2 + 4 + 9$ $2 + 5 + 8$ $2 + 6 + 7$
 $3 + 4 + 8$ $3 + 5 + 7$
 $4 + 5 + 6$

Note that 5 appears in 4 of the winning triples, more often than any other number, so the player going first should choose 5 because it gives an advantage.

With skilful play by both players nobody should lose this game. It is important to let people discover this for themselves. It may take a long time and playing lots of games.

Number	1	2	3	4	5	6	7	8	9
In how many triples?	2	3	2	3	4	3	2	3	2

8 R₈	1 R₁	6 R₆	In the 3 by 3 Magic Square all rows, columns and diagonals add up to 15. The number 5 occurs in 4 combinations so it must go at the centre. The even numbers 2, 4, 6 and 8 occur in 3 combinations so they go at the corners. The odd numbers 1, 3, 7 and 9 occur in 2 combinations and go at the mid-edges. The roads in the Siege Game are numbered to show the correspondence of 3 numbers adding to 15 and 3 roads blocking a town.	PET	PM	PIG
3 R₃	5 R₅	7 R₇		AT	GAME	AI
4 R₄	9 R₉	2 R₂		GOT	MO	OIE

The connection to the 3 by 3 array also links the Fifteen Game to Tic Tac To or Noughts and Crosses the Pet Pig Game and the Siege Game.

Tables are important for organising and recording information. Putting information into a table often makes patterns clear, shows up gaps in the information, or shows some important feature of the problem being worked on. Sometimes learners can't

solve a problem because they can't sort out the necessary information and a table helps them to sort it out.

The Theme of 'Mathematical Disguises' is just a simpler name for the correct mathematical term - **Isomorphism** (Greek iso-same and morph-change). Isomorphism is tremendously important in mathematics. It carries with it the crucial information about the object while discarding irrelevant information. In this case the three contexts are to some extent irrelevant. The relationships and structure are the same in all three. Isomorphism is very useful because, if you recognise the structure and have seen it in another form, you already know a lot about it so it saves work. Some of this can be discovered by learners if they meet all the games.

Follow up

4 by 4 and 5 by 5 Magic Squares: <https://aiminghigh.aimssec.ac.za/picture-puzzler/>

Symmetries in Magic Squares:

<https://aiminghigh.aimssec.ac.za/durers-magic-square/>



Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: <http://aiminghigh.aimssec.ac.za>

Subscribe to the **MATHS TOYS YouTube Channel**

<https://www.youtube.com/c/mathstoys>

Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see <https://aimssec.app> Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.

New material will be added for Secondary 6.

For resources for teaching A level mathematics (Years 12 and 13) see <https://nrich.maths.org/12339>

Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13