AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

The SQUARES GAME Inclusion and Home Learning Guide is part of a Learning Pack downloadable from the AIMING HIGH website http://aiminghigh.aimssec.za
It provides ideas for activities for inclusion and differentiation in school lessons, guidance for home-learning, and related learning activities for all ages and learning stages from pre-school to school-leaving, all on the Common Theme **Play for Fun – Think to Win – Play to Learn.** Guidance for school lessons is given in the separate Notes for Teachers documents.
Choose what seems suitable for the attainment levels of your learners.

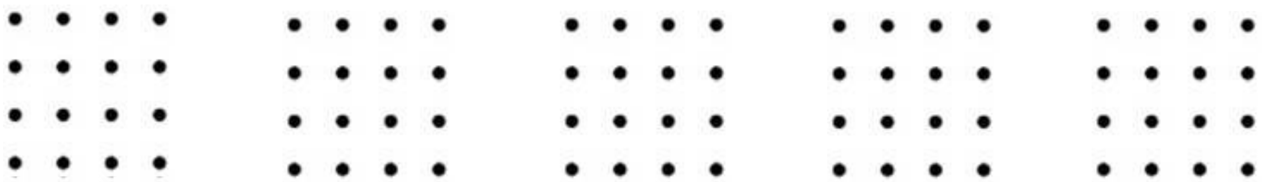
SQUARES GAME

TEAM GAME: Draw a dotty grid on a board or a piece of scrap card where everyone can see it. This game can be played by two teams. Take it in turn for a player from each team to mark a point with the team colour. The winning team is the first to have 4 points in their colour at the vertices of a square. The teams must try to stop their opponents from making squares.

GAME FOR 2 PLAYERS Play exactly as in the team game. Score a point for each win. Try to find winning strategies that will help you to win.

HELP

Start by drawing squares on these dotty grids to use later during the games. Look for tilted squares. Compare the squares that you have drawn with other people.
Are your quadrilaterals all squares? Have they found squares that you did not see?



NEXT

If you can play against a computer at http://nrich.maths.org/2526 then try to learn how to win by thinking about the strategies used by the computer. For games like Chess and Go people become world champions by studying past games. If you can't play against a computer you can still go back over the game and see what difference an alternative move could have made. Study your moves and your opponent's and try to decide what are winning moves and what are losing moves.

The computer follows an algorithm (which may or may not be random) to place its pieces. By studying the moves over a series of games can you work out the computer's strategy?

Do you think that it is random or deterministic (i.e. the computer will always play in a certain position given a certain configuration of pieces)?

HOME LEARNING AND INCLUSION GUIDE

THEME: PLAY FOR FUN – THINK AND WIN – PLAY TO LEARN

Think about the future that lies ahead for young people. They need a GROWTH MINDSET which means believing that they can succeed in what they want to do ***if they persevere and work hard enough.*** *Share this idea with your children in an age-appropriate way. They may not win and they may lose interest. Champions are the ones who persevere, practise and think about how best to play the game.*

Suggestions for Home Learning

Young children

The Squares Game is suitable even for 5 year-olds. Start by playing it on a 9-dot grid. Encourage the children to look at the moves their opponent make and to try to stop their opponent getting a square.

When they are interested in working out how to get better so that they can win more often, you might together draw all 6 squares that are possible on 6 different 9-dot grids.

Upper Primary

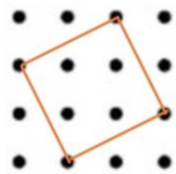
This game can be used in different forms for short periods on many occasions. Like many games Squares can be played by learners of different ages who can return to it again and again and learn more each time. Depending on the age of the learners, the topic for the session and the learning objective, start with the whole group playing one of the versions of this game in two teams. Learners are often surprised when the winning square isn't aligned with the grid. This leads to discussions about what makes a square a square.

After a demonstration of the game, learners could play for a while in pairs, either on a paper grid or on a computer. Give learners the option of reducing the size of the board if they seem overwhelmed or increasing from a 16-dot grid to 25-dot (or even to 36-dot) if they would benefit from a greater challenge.

Have a discussion of their thoughts on the game. Did anyone consistently win or lose? Can anyone think of any good strategies which might help them win?

Once ideas have been shared they can return to playing in pairs, or they can play a game together against the computer, trying, to decide on the best move at each stage. Ask the learner to explain the reasoning behind the moves they choose.

The Maths Underlying the Game

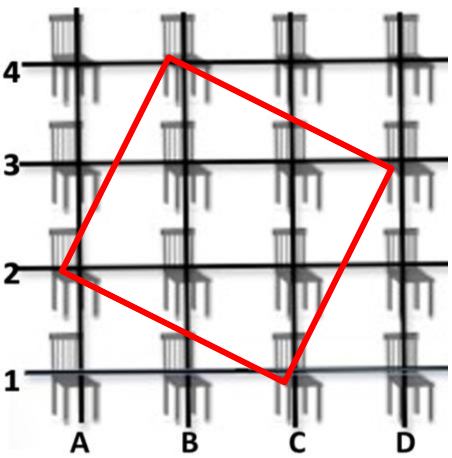


The game can be used as a basis for learning about the properties of squares.
How are we sure that shape is a square? How do we check?
The 4 edges must be equal in length – are they?
The 4 angles must be 90° – are they?
From one vertex to another is it the same distances across and up (or down)?
In this diagram we see: 1 across-2 down; 1 down-2 across; 1 across-2 up; 1 up-2 across.

Key questions

- How do you know that is a square?
- Is your move a good one? Why did you make it?
- Why do you think your opponent made that move? Was it a good one?

Whole class Squares rope activity *(string can be used for this activity)*



Mark 16 positions in a square array. The easiest way to do this may be to set out chairs but you could also mark lines on tarmac in the playground using playground chalk. Labelling, to help learners to name the positions, is optional depending on the class. The labelling shown often appears on maps. Alternatively, you could use Cartesian coordinates.

Ask 4 students to sit on 4 chairs (or stand on 4 grid-points) so they make a square. Have a class discussion about how they can be sure that the quadrilateral with the 4 students at the vertices is a square. To show this, get the 4 students to **stretch** a rope between them, holding it in 4 places without any slack, with one of the students holding one end and another point of the rope. For example, the square shown in red joins four points C1, D3, B4, A2. Ask questions like: "Is it a square?" "How do you know it is a square?"... Notice that:

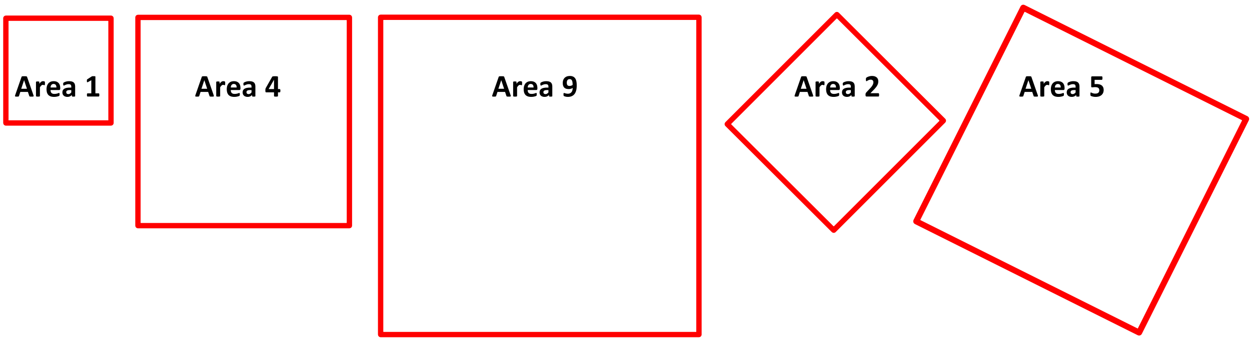
(1) The points C1, D3, B4, A2 on the outside edge of the board, leave **4 congruent right-angled triangles** outside the marked quadrilateral. Discuss the reasons why these triangles must be right-angled, and congruent, and why this shows that the quadrilateral marked by the rope is a square.

(2) Moving around the square from vertex to vertex is equivalent to moving:

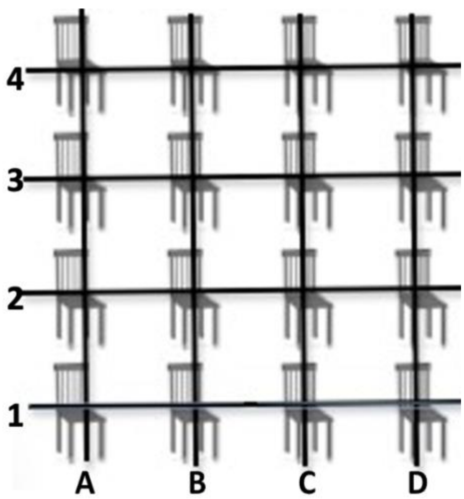
one square across and two up, then two squares across in the negative direction and one up, then one square across in the negative direction and two down, and then two squares across in the positive direction and one down.
The mathematical way to record moves from point to point is to write them as 2-dimensional vectors (not to be confused with coordinates, namely ($^{+}1$, $^{+}2$), ($^{-}2$, $^{+}1$), ($^{-}1$, $^{-}2$), ($^{+}2$, $^{-}1$). Teachers should know this, but they may not choose to talk about vectors to their students.

Then ask for a **different** 4 learners to show a **different** square. Repeat to involve all the learners and to show as many squares as possible. Make sure that all the learners understand that there can be tilted squares as well as straight squares. To end this 'People Maths' activity the students could record, on squared paper, all the squares that can be made.

There are 5 different types of squares on a 4 by 4 grid (as illustrated below). Can you see 9 congruent squares with area 1 square unit, 4 congruent squares with area 4 square units, 1 square with area 9 square units, 4 congruent squares with area 2 square units and 2 congruent squares with area 5 square units, making 20 different squares altogether?



SQUARES TEAM GAME



This game can be played by two teams and can be played by a whole class. Take it in turn for a player from each team to sit on one of the chairs. The winning team is the first to have 4 team members sitting at the vertices of a square. The teams must try to stop their opponents from making squares.

This works well if the learners stand together in 2 groups so that they can consult each other and if they are given time to decide where their next player should sit.

If there is doubt use the rope to check whether the 4 points form a square.

Lower Secondary

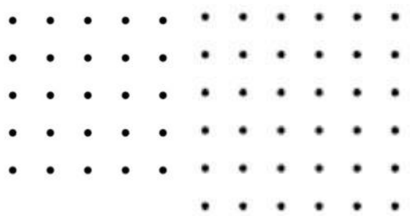
The number of distinctly different starting points (6 on a 5×5 board because of symmetries) is one aspect of developing a winning strategy that you could discuss and also the number of different squares that can be drawn that include each of those points. Ask the question: "Is there a good place to start and why?".

This is an open-ended investigation, involving many different mathematical topics. It can be generalised by changing the number of dots in the starting grid, and the mathematics leads back to the game itself. The investigation can be built up gradually from a 9-dot grid to a 16-dot grid, to a 25-dot grid, and to a 36-dot grid. A mixed-age group could be asked to draw examples of the different squares on their specific grid size, and to compare notes to check for wrong or omitted solutions.

Learners could experiment making different squares using the interactivity in Square Coordinates

Some learners might find 'believing' in the tilted squares difficult. On paper they could use the corner of a piece of paper (or a set-square) to convince themselves that the angles in a shape are 90^o. They could be encouraged to cut the shapes out and move them around to see if the cut-out really looks square.

UPGRADE TO A 5×5 GRID OR 6×6 GRID

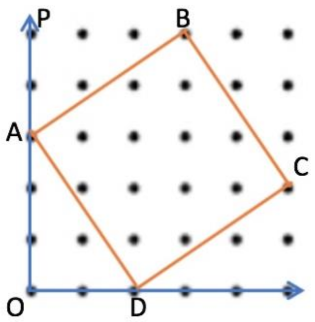


This is a game for 2 players. The game is more challenging with a 5×5 or 6×6 grid. Take it in turn to mark an intersection point with your mark. The winner is the first to mark 4 points that are vertices of a square.

PLAY THE GAME AGAINST A COMPUTER http://nrich.maths.org/2526

The Maths Underlying the Game

The game can be used as a basis for learning about how positive and negative numbers correspond to movements on number lines.



Working on the properties of a square offers an opportunity to lay the foundations for understanding gradients.

Here the lines of dots across and up are perpendicular so $\angle APB = \angle DOA = 90^o$

To draw the square we count steps across and up between points so that AP = DO and PB = OA.

This makes triangles $\triangle PAB$ and $\triangle ODA$ congruent right angled triangles.

The lines with arrows show the positive directions across to the right and up, and the opposite directions are the negative directions.

Write +1 across for one step to the right; -1 for one step to the left;

+1 up for one step up; -1 for one step down.

Record moves point to point as pairs of numbers: (number of steps across ; number of steps up).

Going around the square clockwise starting from the point on the *y*-axis the moves are:

(+3 ; +2) (+2 ; -3) (-3 ; -2) (-2 ; +3)

Years 9 and 10

This version builds familiarity with coordinates to prepare for learning Analytic Geometry.

SQUARES COORDINATE GAME

This is a game for 2 players. Take it in turn to give the coordinates of your chosen points. Both players must write down the coordinates and mark them on their charts. The winner is the first to capture 4 points that are vertices of a square.

				y	5				
					4				
					3				
					2				
					1				
-5	-4	-3	-2	-1	0	1	2	3	4
					-1				
					-2				
					-3				
					-4				

Player 1 coordinates:	**Player 2 coordinates:**

Years 11, 12 and 13

PLAY THE SQUARES GAME AGAINST A COMPUTER http://nrich.maths.org/2526

If you can't play against a computer you can still go back over any game to see what difference an alternative move could have made. Study your moves and your opponent's and try to decide what are winning moves and what are losing moves.

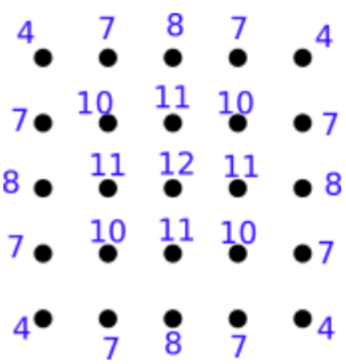
Try to learn how to win by recording all the moves so that, after the game, you can think about the strategies used by the computer or by your opponent.

For games like Chess and Go people become world champions by studying past games.

The computer follows an algorithm (which may or may not be random) to place its pieces. By studying the moves over a series of games can you work out the computer's strategy? Do you think that it is random or deterministic (i.e. the computer will always play in a certain position given a certain configuration of pieces)?

a winning move and to avoid Thinking about the best move to take in a game involves realising that there are consequences of decisions that you take and considering these consequences in advance so as to choose losing.

Strategies for Winning and The Maths Underlying the Game



1.This diagram shows the number of squares that can be drawn from each point on a 5 by 5 grid. Can you find all these squares and develop your winning strategy?

Notice that you only need to find the squares for six different points because of symmetry.

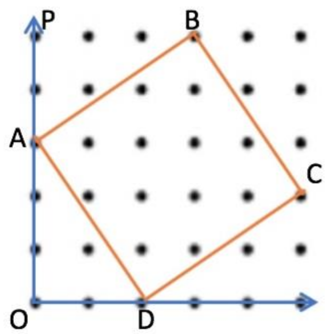
2.If the product of the gradients of is -1 then the lines are perpendicular.

Define gradient as $\frac{\text{Directed Distance Up}}{\text{Directed Distance Across}}$

In general squares are constructed by counting distances across and up on the grid. For example, in the diagram:

Gradient AB $= \frac{2}{3}$ Gradient AD $= \frac{-3}{+2} = \frac{-3}{2}$

Notice the product of these gradients of is -1.



We can prove that, in general, if the product of the gradients is -1 this means that AB = AD and also AB and AD are perpendicular so ∠BAD = 90°.

Proof: The grids are constructed so that ∠APB = ∠DOA = 90°.

By construction PB = OA and PA = OD

So triangles △PAB and △ODA are congruent right angled triangles (SAS).

It follows that AB = AD (hypotenuses of congruent right angled triangles) and ∠ODA = ∠PAB (1).

Also ∠ODA + ∠OAD = 90° (2) supplementary angles in △ODA.

From (1) and (2) it follows that ∠PAB + ∠OAD = 90°

Hence ∠BAD=90° as angles ∠BAD, ∠PAB and ∠OAD are on a straight line.

The converse, that if lines are perpendicular then the product of the gradients is -1, can also be proved.

Key questions

Strategic thinking

- Is your move a good one? Why did you make it?
- Why do you think the computer made that move? Was it a good one?

Gradients

- How do you know that is a square?
- What is the gradient of that line segment?
- How do you know that those lines are parallel?
- How do you know that those lines are perpendicular?

Why play this game?

This game is a good starting point for learning school maths from Year 5 to year 10. It This offers an excellent opportunity to practise visualising squares, angles and gradients on grids and also it encourages students to look at strategies using systematic approaches. Describing strategies to others is always a good way to focus and clarify mathematical thought.

Working with tilted squares provides an opportunity to examine the properties of gradients and to think about the gradients of parallel and perpendicular lines.

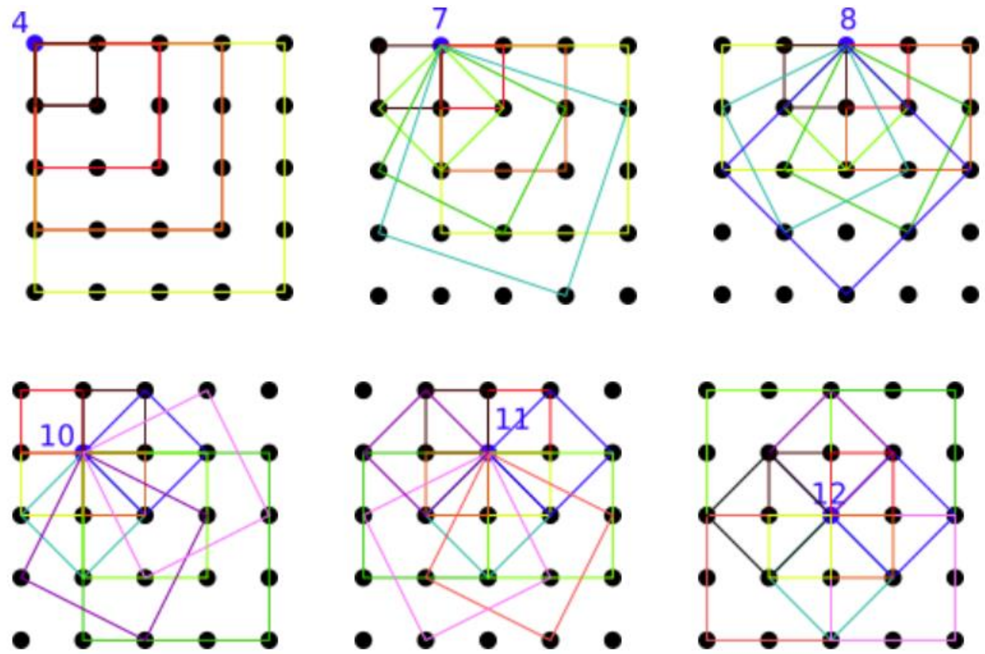
This can lead on to further work on coordinates (see Square Coordinates on the NRICH website) and to proving Pythagoras Theorem (see Make Squares Jigsaw and Pythagoras Jigsaw on the AIMING HIGH website).

Learning objectives

Lessons can be planned to use this game to work towards one or more of the following:

- to develop problem solving and visualisation skills and experience of working systematically;
- to develop concepts of area;
- to introduce the ideas of distance, gradient and perpendicularity in analytic geometry;
- to lead up to work on Pythagoras Theorem.

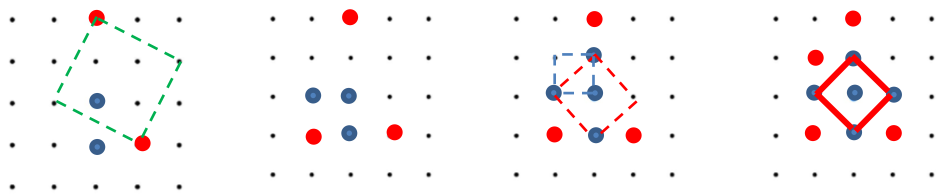
SOLUTIONS



If you go first, you can be in control of the game, because you can always make 3 corners of a square so that the computer is forced to choose the 4th point. Start in the middle because that dot can make the most squares.

Choose points that make squares with as many of your other points as possible. Once you can make two different squares you know you will win. For example, this configuration:

Example 1



Step 1: I put my first mark (blue) on the centre point.

Step 2: After 2 moves each, to stop the computer getting 3 vertices of the green square, I chose one of them that gave me 3 vertices of a square forcing the computer to block.

Step 3: My next move gave me chances to complete either the red or the blue square as the computer could not stop both.

Step 4: I won!   See https://nrich.maths.org/squareit/solution for more winning strategies.

Generic competences

By playing this game students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop visualization** and the skill to interpret or create images to represent concepts and situations;
- interpret and **solve problems**;
- **work in a team to** collaborate and work with a partner or group.

Follow up

How Many Squares https://aiminghigh.aimssec.ac.za/how-many-squares/

Square Coordinates http://nrich.maths.org/2667

Pythagoras Theorem; https://aiminghigh.aimssec.ac.za/make-squares-jigsaw/
and https://aiminghigh.aimssec.ac.za/pythagoras-jigsaw/

Subscribe to MATHS TOYS if you have not already done so.
https://www.youtube.com/mathstoys
Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions, home learning guides and curriculum links: https://aiminghigh.aimssec.ac.za
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App https://aimssec.app available to download from Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.

For resources for teaching A level mathematics see https://nrich.maths.org/12339

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6