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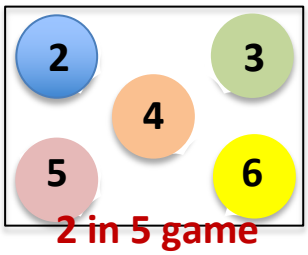
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

The ODDS Game Inclusion and Home Learning Guide provides related activities for lessons in school and home learning for all ages and learning stages from pre-school to school-leaving, on the Common Theme PROBABILITY SAMPLE SPACES
Choose what seems suitable for the age or attainment level of your learners.
Watch the videos: GTEN workshop https://youtu.be/aOoIdvJMUi8
AIMING HIGH LEARNING PACK https://aiminghigh.aimssec.ac.za/odds-and-evens/

ODDS GAME



2 in 5 game

In the 2 Odds in 5 game you pick out two balls at random from a bag containing 5 balls numbered 2, 3, 4, 5 and 6.

If the total is odd you win. If it is even you lose.
Are you equally likely to win or to lose (a fair game)?
In the 3 games shown below, the rules are the same.
Which set of balls would you choose to give yourself the best chance of winning?

Do your own experiments. Make number cards and put them in an envelope or bag.
For interactive computer simulations of the games see http://nrich.maths.org/4308

4 in 5 game

3 in 6 game

5 in 6 game

HELP

2 in 5 Game		FIRST CHOICE				
		2	4	6	3	5
SECOND CHOICE	2	X			5	
	4	6	X			
	6		10	X		
	3				X	8
	5					X

Play the 2 odds in 5 game several times and record the results. You are conducting trials in a probability experiment.

Fill in a table colouring the odd and even totals in contrasting colours. Put crosses to show you can't choose the same ball twice.

This is called **a sample space diagram**.

The sample space diagram shows that, when you choose 2 cards at random there are exactly 20 possible outcomes, that is 20 elements in the sample space.

NEXT

None of the sets looked at so far gives a fair game.
Can you find a set of numbers that would give a fair game?

1	2	3
4	5	6
7	8	9

INCLUSION AND HOME LEARNING GUIDE

THEME: Probability Sample Spaces – Counting all possible outcomes

Lesson starter for a mixed-age group pre-school to 18 or 19 years

Game for Early Years: Pre School and Years 1 & 2

RED AND BLUE GAMES – any number of players

Resources: You need small objects of two different colours that are the same in all respects other than colour, such as counters or buttons or cards, and a bag, box, hat or envelope.

Put 3 red cards and 6 blue cards in a bag or envelope and take turns to pick 2 cards at random from the bag.

Each player must decide to be a 'same' player who wins when the objects drawn are the same colour or a 'different' player who wins when the two objects are different colours.

Pick 2 cards at random from the bag one after the other without replacing the first one. Score a point (or not) according to whether the cards are the same or different colours, and put the cards back in the bag.

Shake the bag before the next player takes a turn.

Repeat until one of the players has won 5 points.

Record the results as an experiment to discover if this is a fair game. After playing the game many times do your results seem to show that the players have an equal chance of drawing cards of the same colour as of drawing cards of different colours?

Older learners can be given the task of explaining why they believe that there is an equal chance of drawing the same colour or different colours (or otherwise).

Lower Primary Years 3 to 5

RED AND BLUE GAMES – any number of players

Resources: You need small objects of two different colours that are the same in all respects other than colour, such as counters or buttons or the cards cut out from page 5, and a bag or envelope.

Put 3 red cards and 6 blue cards in a bag or envelope and take turns to pick

2 cards at random from the bag.

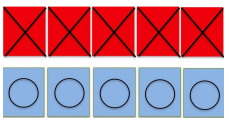
Each player must decide to be a ‘same’ player who wins when the cards are the same colour or a ‘different’ player who wins when the two cards are different colours.

Pick 2 cards at random from the bag, give a point to the winner, according to whether the cards are the same or different colours, and put the cards back in the bag.

Shake the bag before the next player takes a turn.

Repeat until one of the players has won 5 points.

To discover if it is a fair game, note which team wins each time the class plays the game. Keep a record of the total number of times the cards chosen randomly are the same colour, and the number of times they are different colours. If it is a fair game then, the more often your class plays the game the closer these two totals will get.

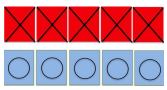


Play the game with 5 red and 5 blue objects and investigate what happens. Record the results for each draw as ‘same colour’ or ‘different colour’.

Upper Primary Years 6 and 7

Resources: You need small objects of two different colours that are the same in all respects other than colour, such as counters or buttons or the cards cut out from page 5, and a bag or envelope.

IN YEAR 6 CHANGE THE GAME – PUT 5 RED CARDS AND 5 BLUE CARDS IN THE BAG.



Play this game and investigate what happens. Record the results for each draw as 'same colour' or 'different colour'.

Many children will discover that this is NOT a fair game. They are more likely to get 2 cards of different colours than to get 2 cards of the same colour. The teacher should not give away this 'secret winning strategy' so that more and more members of the class will realize **for themselves** that they should choose to be 'different'. The teacher should encourage them to want to find out why.

	R_1	R_2	R_3	R_4	R_5	B_1	B_2	B_3	B_4	B_5
R_1	X									
R_2		X								
R_3			X							
R_4				X						
R_5					X					
B_1						X				
B_2							X			
B_3								X		
B_4									X	
B_5										X

To understand why players are more likely to draw two cards of different colours than two of the same colour, learners need to understand how to fill in a 2-way table showing all possible outcomes.

Ask the learners to write S for 'same' and D for 'different' in the empty squares in the diagram.

Ask the learners What do the red crosses on the diagonal line tell you?

By Year 7 learners should be able to fill in a 2-way table like the one shown and to understand why it happens that players are more likely to draw two cards of different colours than two of the same colour.

RED BLUE GAME INVESTIGATION

What do you notice about the small squares (cells) covered by the yellow triangles?
What do the covered cells represent?
How many are there?

	R_1	R_2	R_3	R_4	R_5	B_1	B_2	B_3	B_4	B_5
R_1	X									
R_2		X								
R_3			X							
R_4				X						
R_5					X					
B_1						X				
B_2							X			
B_3								X		
B_4									X	
B_5										X

What do you notice about the small squares (cells) covered by the grey squares?
What do the covered cells represent?
How many are there?

	R_1	R_2	R_3	R_4	R_5	B_1	B_2	B_3	B_4	B_5
R_1	X									
R_2		X								
R_3			X							
R_4				X						
R_5					X					
B_1						X				
B_2							X			
B_3								X		
B_4									X	
B_5										X

RED BLUE GAME SAMPLE SPACE DIAGRAM SUMMARY

The SAMPLE SPACE is the set of all possible outcomes. In this case it is the set the 90 possible results when you draw 2 cards from a bag with 5 red and 5 blue cards.

	R_1	R_2	R_3	R_4	R_5	B_1	B_2	B_3	B_4	B_5
R_1	X	S	S	S	S	D	D	D	D	D
R_2	S	X	S	S	S	D	D	D	D	D
R_3	S	S	X	S	S	D	D	D	D	D
R_4	S	S	S	X	S	D	D	D	D	D
R_5	S	S	S	S	X	D	D	D	D	D
B_1	D	D	D	D	D	X	S	S	S	S
B_2	D	D	D	D	D	S	X	S	S	S
B_3	D	D	D	D	D	S	S	X	S	S
B_4	D	D	D	D	D	S	S	S	X	S
B_5	D	D	D	D	D	S	S	S	S	X

Each cell shows the result for picking cards at random, first one, then another, from a bag containing 5 red cards and 5 blue cards. 2

The crosses show you can't pick the same card twice.

There are 90 possibilities: 40 with the same-coloured cards, 50 with different coloured cards. Picking different coloured cards is more likely than the same colour.

The probability of 'same' is $\frac{40}{90} = \frac{4}{9}$.

The probability of 'different' is $\frac{50}{90} = \frac{5}{9}$

It is NOT A FAIR GAME.

Introduce the Odds Games as on page 1 and in the worksheet. The 4 games are all played with the same rules.

Are any of the games fair?

Find a set of numbers that gives a fair game.

How do you know which sets of numbers give fair games?

Explore some of the properties of odd and even numbers. Tell the class to close their eyes and keep them closed until you tell them to open their eyes. Tell them:
ODD + ODD
Think of any two odd numbers. Add them. Is the sum odd or even?
Think of another two odd numbers. Add them. Is the sum odd or even?
Do this a few more times. Ask: What do you notice?
EVEN + EVEN
Think of any two even numbers. Add them. Is the sum odd or even?
Think of another two even numbers. Add them. Is the sum odd or even?
Do this a few more times. Ask: What do you notice?
ODD + EVEN
Think of any two numbers, one odd one even. Add them. Is the sum odd or even?
Think of another two numbers, one odd one even. Add them. Is the sum odd or even?
Do this a few more times. Ask: What do you notice?

Why do this activity?

In this activity learners can explore and discuss two types of probability: experimental and theoretical. They can experience teamwork in trying to find an example of a fair game of this type and trying to formulate conjectures about what conditions might lead to a fair game. Older learners can work on the challenge of proving the conjectures.

Learning objectives

In doing this activity students will have an opportunity to:

- conduct probability experiments and record results;
- experience and discuss random sampling;
- reflect on the differences between experimental probability and theoretical probability;
- learn to use a sample space diagram to show all the elements in a sample space and to analyse the theoretical probabilities of different events;
- find and prove conjectures about conditions for a fair game.

Generic competences

In doing this activity students will have an opportunity to:

- conduct probability experiments, reflect on the differences between experimental probability and theoretical probability and discuss the types of applications that require the use of one or the other;
- work as a team towards a common goal;
- **communicate** in writing, speaking and listening:
    - exchange ideas, criticize, and present information and ideas to others;
    - analyze, reason and record ideas effectively.

SOLUTION

2 in 5 game

Probability of an even sum is $\frac{8}{20} = \frac{2}{5}$.
Probability of an odd sum is $\frac{12}{20} = \frac{3}{5}$.
So it is not a fair game.

	2	4	6	3	5
2	X	6	8	5	7
4	6	X	10	7	9
6	8	10	X	9	11
3	5	7	9	X	8
5	7	9	11	8	X

4 in 5 game

Probability of an even sum is $\frac{12}{20} = \frac{3}{5}$.
Probability of an odd sum is $\frac{8}{20} = \frac{2}{5}$.
So it is not a fair game.

	1	3	5	7	6
1	X	4	6	8	7
3	4	X	8	10	9
5	6	8	X	12	11
7	8	10	12	X	13
6	7	9	11	13	X

2 in 6 game

Probability of an even sum is $\frac{14}{30} = \frac{7}{15}$.
Probability of an odd sum is $\frac{16}{30} = \frac{8}{15}$.
So it is not a fair game.

	2	4	6	8	3	5
2	X	6	8	10	5	7
4	6	X	10	12	7	9
6	8	10	X	14	9	11
8	10	12	14	X	11	13
3	5	7	9	11	X	8
5	7	9	11	13	8	X

5 in 6 game

Probability of an even sum is $\frac{21}{30} = \frac{7}{10}$.
Probability of an odd sum is $\frac{9}{30} = \frac{3}{10}$.
So it is not a fair game.

	1	3	5	7	9	4
1	X	4	6	8	10	5
3	4	X	8	10	12	7
5	6	8	X	12	14	9
7	8	10	12	X	16	11
9	10	12	13	16	X	13
4	5	7	9	11	13	X

Follow up

The activity In a Box https://aiminghigh.aimssec.ac.za/in-a-box/ offers another context for exploring exactly the same game and underlying mathematical structure. Use it as a follow-up a few weeks after working on Odds & Colours Games

Also see Special Sums https://aiminghigh.aimssec.ac.za/special-sums/

Red or Black: https://aiminghigh.aimssec.ac.za/red-or-black-game/

Nines or Tens https://aiminghigh.aimssec.ac.za/nines-and-tens/

Twos Company https://aiminghigh.aimssec.ac.za/twos-company/

Same Sweets https://aiminghigh.aimssec.ac.za/same-sweets