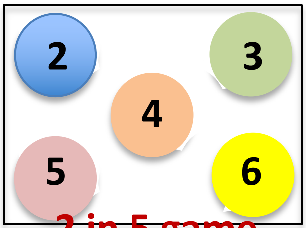
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

ODDS GAME

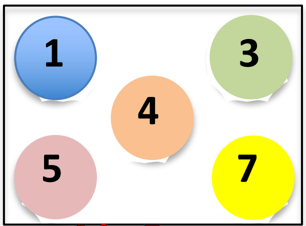


2 in 5 game

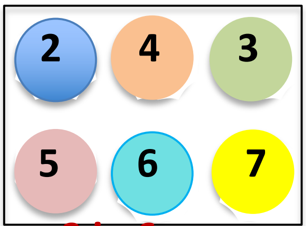
In the 2 Odds in 5 game you pick out two balls at random from a bag containing 5 balls numbered 2, 3, 4, 5 and 6.

If the total is odd you win. If it is even you lose.
Are you equally likely to win or to lose (a fair game)?
In the 3 games shown below, the rules are the same.
Which set of balls would you choose to give yourself the best chance of winning?

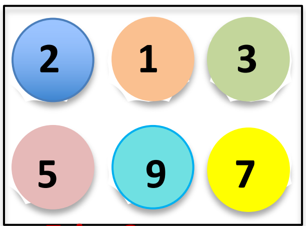
Do your own experiments. Make number cards and put them in an envelope or bag.
For interactive computer simulations of the games see http://nrich.maths.org/4308



4 in 5 game



3 in 6 game



5 in 6 game

To do interactive computer simulations of these games see
http://nrich.maths.org/4308

HELP

2 in 5 Game		FIRST CHOICE				
		2	4	6	3	5
SECOND CHOICE	2	X			5	
	4	6	X			
	6		10	X		
	3				X	8
	5					X

Play the 2 odds in 5 game several times and record the results. You are conducting trials in a probability experiment.

Fill in a table colouring the odd and even totals in contrasting colours. Put crosses to show you can't choose the same ball twice.

This is called **a sample space diagram**.

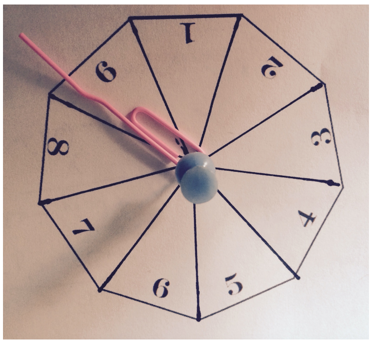
The sample space diagram shows that, when you choose 2 cards at random there are exactly 20 possible outcomes, that is 20 elements in the sample space.

NEXT

None of the sets looked at so far gives a fair game.
Can you find a set of numbers that would give a fair game?

1	2	3
4	5	6
7	8	9

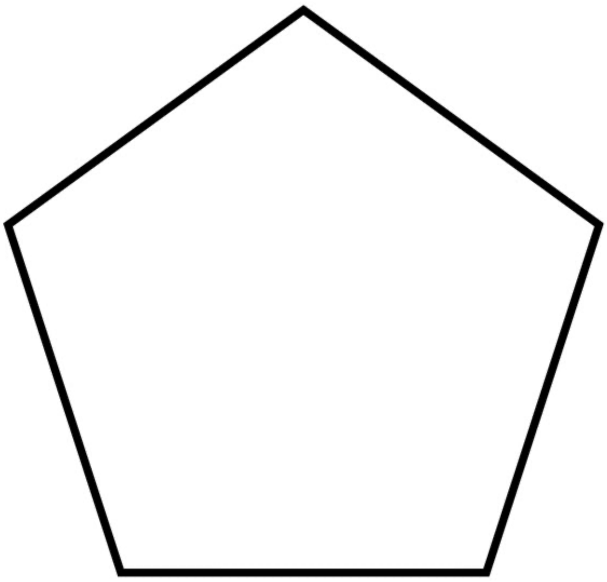
MAKING SPINNERS FOR THESE GAMES

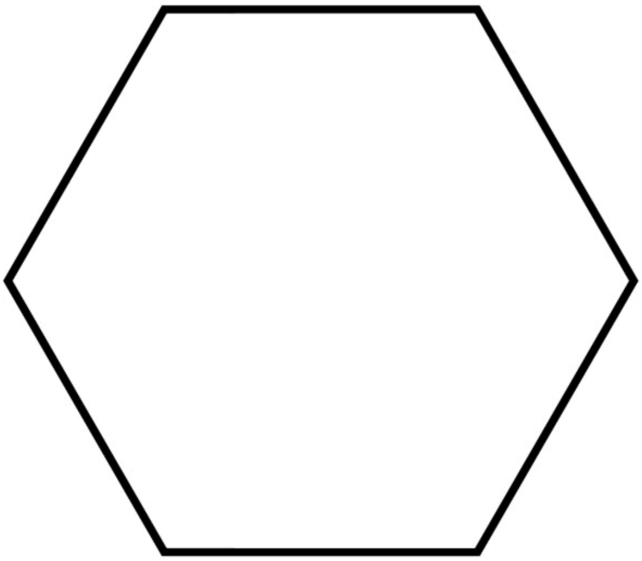


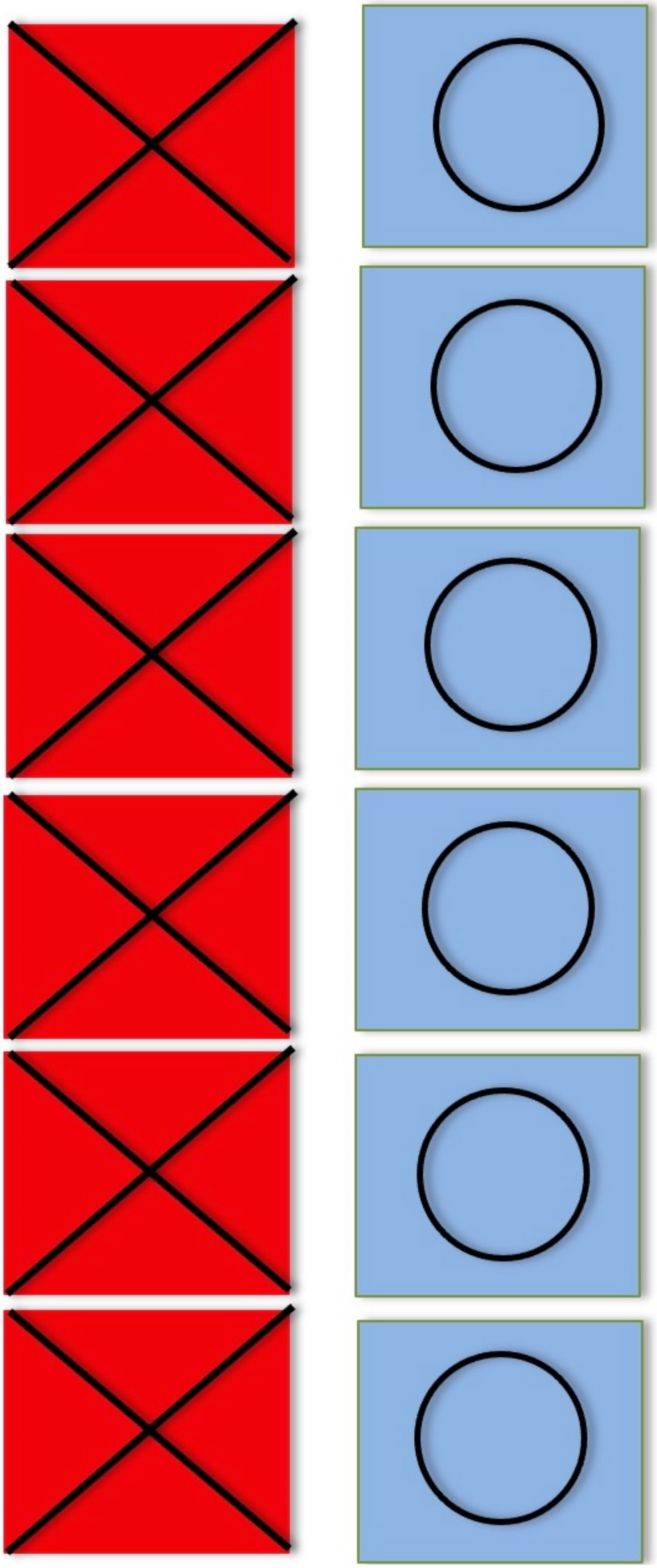
You can simulate the games using spinners made from the templates below, a pin and a paperclip.

Mark equal sectors on the spinners for all the numbers for the game or for the different colours.

Cut out and pin on a flat surface through the paper clip and the template (as in the picture) making sure that the paper clip spins freely.







PROOF SORTING EXERCISE FOR YEAR 12 AND 13 STUDENTS.

This is the 1st statement. Cut out the strips and re-construct the proof by arranging the remaining statements in the correct order.

Odds and Colours Games

Imagine we play the game with a even numbered balls and b odd numbered balls.

If a and b are any two consecutive triangle numbers, the reverse argument holds.
We have proved that a necessary and sufficient condition for a game of this type to be a fair game is that the total number of balls ($a + b$) is a square number n^2, moreover a and b turn out to be consecutive triangle numbers.

For a fair game we must have the probability of an even sum equal to the probability of an odd sum; that is:

$(a^2 - a) + (b^2 - b) = ab + ab$

$\Leftrightarrow \quad a^2 - 2ab + b^2 = a + b$

$\Leftrightarrow \quad (a - b)^2 = a + b$

This suggests that a and b are consecutive triangle numbers n because the rule for generating the sequence of triangle numbers is that the difference between the n^{th} and $(n-1)^{th}$ number is equal to n.

Proof:

For a even and b odd numbers this is the sample space diagram, with blue for even totals and yellow for odd totals:

	a even	b odd
a even	$a^2 - a$	ab
b odd	ab	$b^2 - b$

We know that

$a + b = n^2$

and $a - b = n$ $\quad$ $(\text{or } b - a = n \text{ if } b > a)$.

We have proved that: the game will be a fair game if and only if $(a - b)^2 = a + b$

This leads us to think that:

- the total number of balls (i.e. $a + b$) has to be a square number, say n^2;
- the difference between the number of odd and even balls is n.

Adding the two equations:

$2a = n^2 + n$

$\Rightarrow a = \frac{1}{2}(n^2 + n)$

$\Rightarrow a = \frac{1}{2}n(n+1)$

This formula gives a as the n^{th} triangular number.

Similarly, b is the $(n - 1)^{th}$ triangular number $\frac{1}{2}n(n - 1)$.

Conjecture: The game will be a fair game if and only if the number of balls is a square number, in which case the number of odd, and even balls, a and b, are two consecutive triangle numbers.