AIMSSEC

WORKHOP GUIDES FOR TEACHERS TO LEARN TOGETHER

DEVELOPING ALGEBRAIC SKILLS

Guide for your own self-help professional development workshop and resources for inquiry-based lessons.

MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they are required to teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.

The workshops are designed to last at least 90 minutes and ideally 2 hours.

EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1 TITLE PAGE

Teaching strategy.
Curriculum content and learning outcomes.
Summary of mathematical topic (FACT BOX.)

PAGES 2 & 3 WORKSHOP ACTIVITIES FOR TEACHERS

Two pages of activities for teachers to work through and discuss with colleagues.
For each activity there: is a list of resources needed  ,
suggestions for organising the class (e.g. individual, pairs, whole class)  ,
suggestions for the time the activity will take  , and when to pause, think and try the activity 
and when to record your work  .

PAGES 4 & 5 CLASSROOM ACTIVITIES FOR LEARNERS

Two pages of help for teachers on lesson planning. Advice on how long to allow for each activity, the resources needed and the key questions to ask to guide learning.

PAGES 6 TO 10 CHANGES IN MY CLASSROOM PRACTICE

Help for teachers on teaching strategies; additional resources and activities for use during or after the workshop; worksheets; solutions; templates; key questions for formative assessment; other key questions to guide learning and follow-up activities.

Developing Algebraic Skills

Teaching strategy: Mathematical Games for Practice, Review and Assessment.

Curriculum content: Rules for combining numbers and algebraic expressions by binary operations and inverses.

Prior knowledge: understanding of the rules for addition, subtraction, multiplication and division of numbers and combining these operations; expressing calculation processes algebraically (but these will all be developed during the activity).

Intended Learning Outcomes At the end of this activity teachers and learners will:

- Know how to use algebra to solve puzzles
- Understand inverse operations
- Be able to create their own puzzles
- *Appreciate that algebra is a generalised form of number*
- Have experienced drill practice through playing games.

Fact box

Useful Techniques used in the puzzles in this workshop

Addition, subtraction, multiplication and division of integers, fractions and decimals.

Prime factorisation and finding Highest Common Factors and Lowest Common Multiples.

Combining and simplifying algebraic expressions.

Addition, subtraction, multiplication and division of algebraic expressions
e.g. $(3a+8) = (2a+3) + (a+5)$ and $(3a+8) - (2a+3) = a+5$
$(3a+8)(2a+3) = 6a^2 + 25a + 24$ and $(6a^2 + 25a + 24)/(3a+8) = (2a+3)$

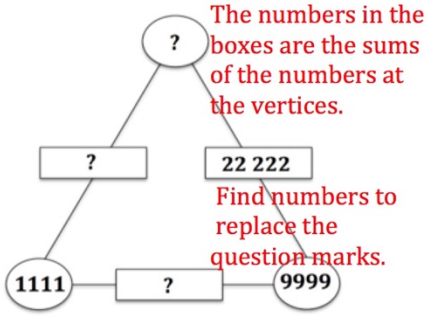
Resources for this workshop: Pencil and paper, Photocopy of worksheets.

Workshop Activities for Teachers

Activity 1: Checkit

Pencil and paper *2s or 3s.* *10 – 15 minutes including discussion*

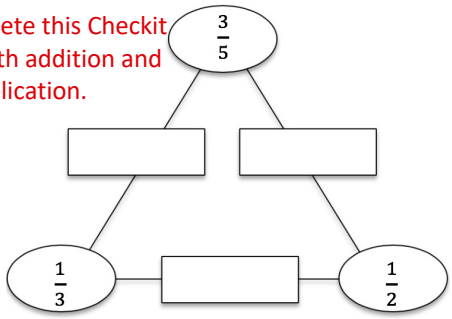
Fill in the empty boxes. Try this now



The numbers in the boxes are the sums of the numbers at the vertices.

Find numbers to replace the question marks.

Complete this Checkit for both addition and multiplication.



Points for discussion. *You might ask these questions in a lesson.*

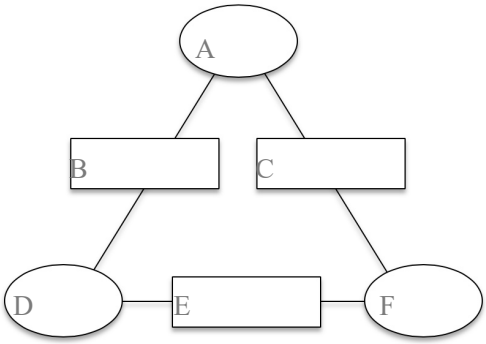
Can you explain how you did that calculation giving reasons for each step.

What calculation would you do to check that answer? Why?

Activity 2: Checkit Games for 2 players or 2 teams.

Pencil and paper *2s or 3s.* *40 – 55 minutes including discussion*

Choose teams. Play this game several times with different types of entries and operations.



Checkit games are suitable for all ages because you can choose the type of numbers or algebraic expressions and the operation that suits your class.

You can play this game with

- addition and subtraction or with
- multiplication and division
- factorisation or any other mathematical

operation you choose.

CHECKIT RULES

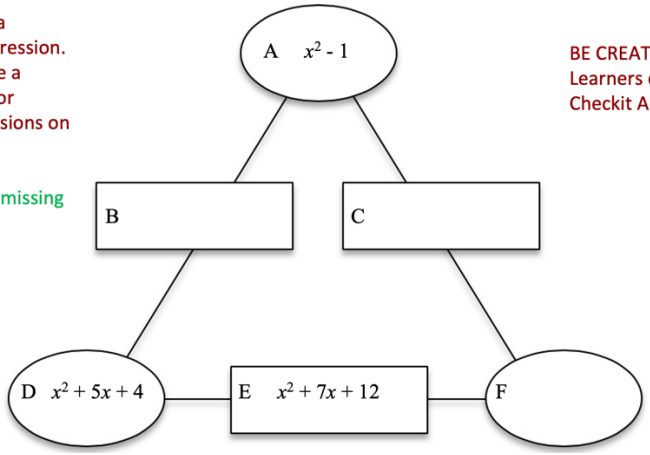
1. Before starting to play, the players must agree on what numbers or types of expression are allowed. For example, whole numbers, or integers, or fractions, or decimals, or powers of 2, 3, 4 and 7 or algebraic expressions. To keep the numbers small for young children, you could just use numbers under 20 or under 50.
2. Start with an empty Checkit frame. Flip a coin to decide who goes first.
3. Players take it in turns to fill in a number in an empty box.
4. Players check if the numbers entered are correct. If a player makes a mistake and is challenged by an opponent before that opponent enters another number, then he loses the game and his opponent wins a point.

5. When they have put numbers in all 6 boxes, both players check. If there are any mistakes then both players lose a point. If everything is correct both players win a point.
6. The first player to score 5 points wins the game.

Example for secondary teachers, the game after 3 moves. Fill in boxes B, F and C.

Each entry is a quadratic expression. There must be a common factor in all 3 expressions on each edge.

What are the missing expressions?



BE CREATIVE
Learners can invent their own Checkit Algebra games

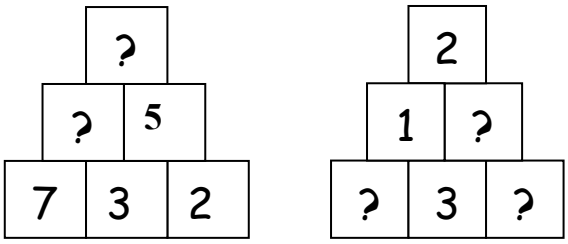
OTHER VERSIONS
Checkit can be played in many algebraic versions. Examples:
-Linear expressions in A, D & F with their sums in B, C & E
- Linear expressions in A, D & F with products in B, C & E

Points for discussion. How would you use this game in your lessons, and in which grades?

Activity 3: Pyramid Puzzle

Pencil and paper *2s or 3s.* *40 - 50 minutes including discussion.*

In a number pyramid, each box is the sum of the 2 boxes it rests on. Can you fill in the gaps?



See the worksheet on page 9 and complete it if you have time.
Notice that this worksheet is not just a list of questions. You are given one or two examples and then asked to **invent** problems for yourself. Inventing problems is the important activity and you should spend most of your time on this. Is there any restriction on the sort of numbers you could use? Invent a pyramid that uses fractions or decimals?

Points for discussion

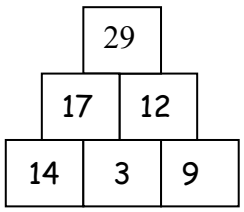
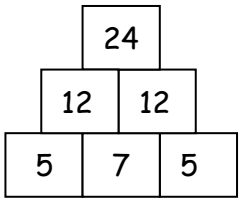
- How can you 'work backwards' to find numbers on the bottom row?
- Does it matter which boxes you are given?
- Is there always only one answer? How do you know?
- How can you predict the box at the top if you are given the bottom row?
- What is the same, and what is different, if you use algebra rather than numbers?
- How will you get your class involved in solving and inventing puzzles?
- How will you use this in making the transition from numbers to algebra seem natural?

Classroom Activity for Learners

Activity 1: A number puzzle *(based on Pyramid Puzzles worksheet)*

Showboards, paper and pencils. At least one copy of the Pyramid Puzzles worksheet

Whole class and pairs or small groups 40 minutes.

Ideas for Teaching	**Solutions to worksheet:**
This activity may be worked on in groups using copies of the worksheet or the worksheet on a board; or the *ideas* may be used to structure whole-class teaching, using 'show-me' boards to give answers so everyone has to take part. First introduce number pyramids with bottom row given, then different boxes given. When learners have written some of their own and tested them on each other, move to using algebra – first one variable, then two for more confident learners. *Always* expect learners to check that their solutions work, and when they find solutions in algebra, to see what they look like for a particular value of the variable: does the pyramid still work? If they are making up their own pyramids using algebra, suggest they use the first letter of their name as a variable – it gives them 'ownership'. Only take learners as far as they can go with reasonable effort: it's better that they make up lots of examples of their own at a level they are happy with, and build up their confidence. You can return to do more, and perhaps harder, later. The difficulty of the ones they are happy to make up for themselves gives you a good idea of how confident they are feeling.	1a) 20 at top b) 14,1 in bottom row Top number = $a+2b+c$ if the bottom row is a, b, c: accept any words which explain this, and perhaps encourage learners to give examples, and then to express it in algebra. 2i) $4a+4$ ii) $a+2$ 3i) $2a+5b+6$ (can learners make the link with their answer to question1? If not, support them in doing so). 4.  5. If $4a+4=24$ then $a=5$ so the pyramid is  If $3a+8 = 29$ then $a = 7$ If $3a+8 = 12,5$ then $a = 1,5$ If $3a+8 = -22$ then $a = -10$

Activity 2: Checkit Games as Lesson Starters

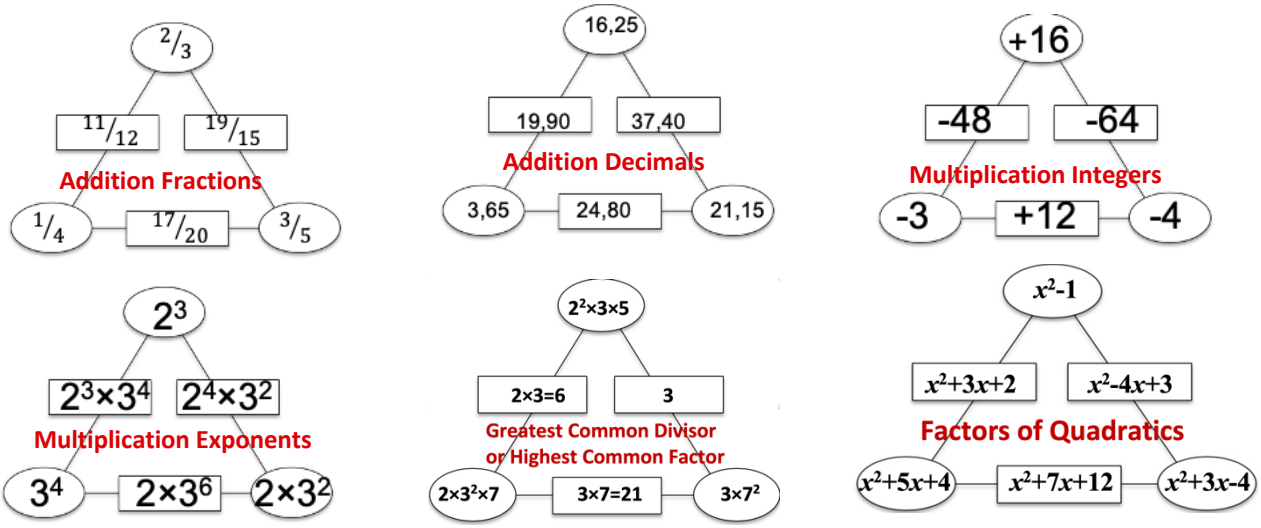
Paper and pencils. Whole class and pairs or small groups 10 minutes.

To introduce Checkit games you could show this video to the class https://youtu.be/QRMZl9H2kqQ (or part of it). Checkit games can be used frequently for short periods of time as lesson starters to give practice in doing different calculations. Once the class know the rules it takes only a few minutes to play the game with the class in 2 teams holding up answers on showboards. The game can be used as part of a review of addition of fractions and in a similar way for other types of calculations or algebraic work.

As a review, Checkit can be played at first with the whole class split into 2 teams and the teacher giving some explanations. Then pairs of players could play just 3 times. Because it may be difficult for the

learners to check their own calculations, pairs could then come to the board and write up the 6 numbers with which their game ended, and these could be checked by the whole class.

Teachers can choose to give different levels of challenge to different learners to cater for inclusion and individual needs, including offering more challenge to highflyers. Creating their own puzzles is a good challenge for learners.



Activity 3: Checkit Challenge

Showboards, paper and pencils. At least one copy of the triangle puzzles worksheet page.

Whole class or small groups 30 minutes.

Ideas for Teaching	**Solutions to worksheet:**
This activity may be done in groups using the worksheet (either by making copies of the worksheet or by writing it on the board); or the *ideas* may be used to structure whole-class teaching, using showboards to give answers so everyone has to take part. A different structure from that used for Pyramid Puzzles gives variety. This activity could be used in the lesson after Number Pyramids, or at a later stage. First introduce the idea of adding the numbers in the rings to get the number in the rectangle. When learners show they can follow the idea with easy numbers in all rings they are ready to try harder versions. As above, the more learners do for themselves in pairs or small groups the more confident they will be, especially if they can make up their own questions. At a later stage, learners could try puzzles with harder algebraic expressions – or to practise fractions, decimals, negative numbers or exponents.	1 a) 13 b) 8 c) 7 d) 11, 12, 15 e) 36, 42, 40 f) $2x + 1$, $3x + 3$, $3x + 4$ g) 4, 7, 7 h) 1.6, 2.1, 0.5 i) $2x$, $3x + y$, $x + y$ 2a) 5, 3, 4 b) 7, 6, 0 c) 1.3, 1.2, 2.8 Learners may be able to 'spot' answers in easy cases, but for example in part a) if the top number is x then the 2 bottom numbers are $8 - x$ and $9 - x$, so $8 - x + 9 - x = 7$.

Changes in my classroom practice

Implementing the teaching strategies

Better than giving lots of exercises for practice, give learners a game, problem or puzzle which they can only solve by using the techniques you want them to practise. So the learners will be mastering the technique and also doing some mathematical thinking. This is usually much more interesting than an exercise so the learners will be more involved and work harder.

Once the idea of Checkit triangle number pyramid puzzles is understood, they can be used to practise number work of any sort, as well as a variety of algebra, so once the idea is familiar it takes very little time to use them again in lessons. More layers can be added if needed. They are easy to check, and to differentiate, and learners enjoy making up their own for friends. Working in pairs or a small group helps clarify ideas and give confidence.

The **Checkit triangle puzzles** are also versatile, and can even be used in different shapes, e.g. squares, and 'working backwards' is probably harder.

If learners are encouraged to find solutions to algebraic problems and then check using any value for the variable(s), they will begin to appreciate the power of algebra to represent general situations: that will be particularly clear if they then have the opportunity to work with the magic square reference given below.

Key questions to develop understanding

Pyramid Puzzles

- How can you 'work backwards' to find numbers on the bottom row of the pyramid?
- Does it matter which boxes you are given? Is there only one answer? How do you know?
- How can you predict the box at the top if you are given the bottom row?
- What is the same, and what is different, if you use algebra rather than numbers?
- If a learner asks '**Is this right?**' do you reply '**Does it work?**' so that they are relying on themselves and not the teacher?

Checkit Puzzles

- How do you find the numbers in the squares?
- How do you find the numbers in the circles?
- Does your answer work? Is it the only answer? How do you know?

Follow up activities

Checkit Game and Challenge **https://aiminghigh.aimssec.ac.za/checkit-challenge/**

Triangle puzzle

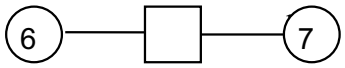
http://nrich.maths.org/public/viewer.php?obj_id=2670 has an interactive version of the triangle puzzle called arithmagons where learners (or teachers!) can make the puzzle as hard as they choose.

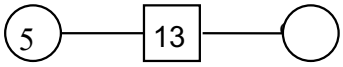
Magic Sums and Products **http://nrich.maths.org/public/viewer.php?obj_id=1376** shows you how to make as many magic squares as you like, by substituting in an algebraic magic square: try it! (Magic squares have both diagonals and every row and column with the same total, which has to be 3 times the middle number – why?) Can your class make a different magic square each?

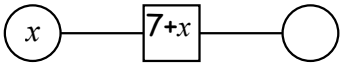
Checkit Triangle Puzzles

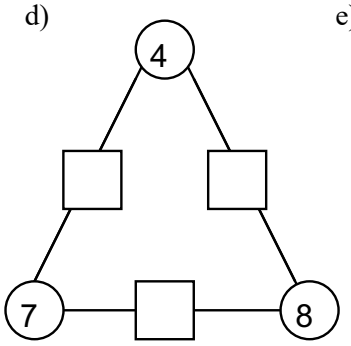
(Questions with a * are harder)

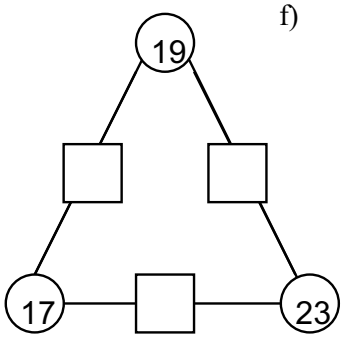
1. In these puzzles the number in a square is the sum of the numbers in the 2 circles next to it. Can you complete these? ‘x’ and ‘y’ could be any numbers – try it and see!

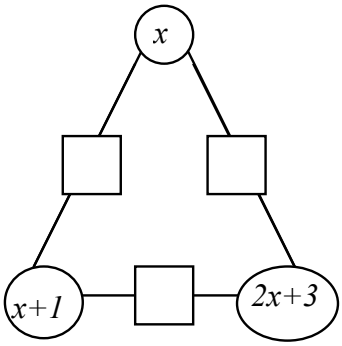
a) 

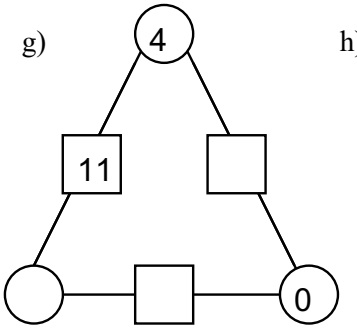
b) 

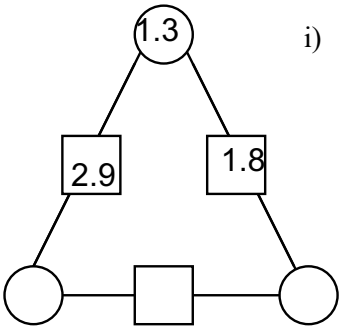
c) 

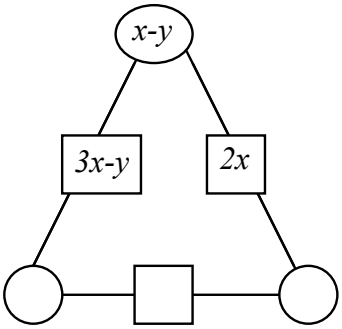
d) 

e) 

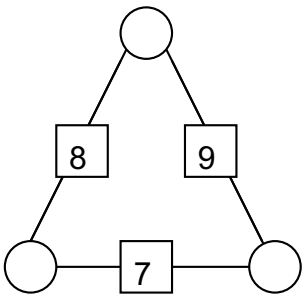
f) 

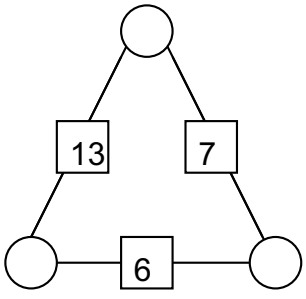
g) 

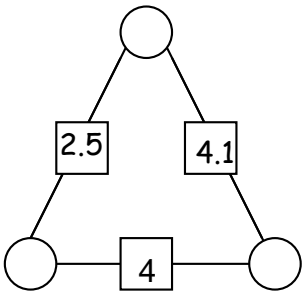
h) 

i) 

2*. These are harder, so it might be easier to work in pairs or in a group. If you get stuck, you might want to call one of the numbers in circle ‘*x*’ and see if you can work out what the other numbers have to be from that. If you can find all the missing numbers, try making up some for the rest of your group. You can use any sort of numbers, positive or negative, or algebra.

a) 

b) 

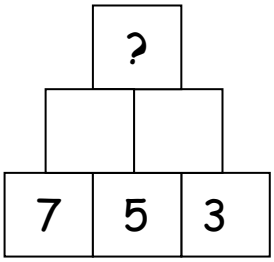
c) 

Number Pyramid Puzzles

(Questions with a * are harder)

1. In a number pyramid each number is the sum of the numbers in the 2 boxes below it. Can you complete these two pyramids?

a)



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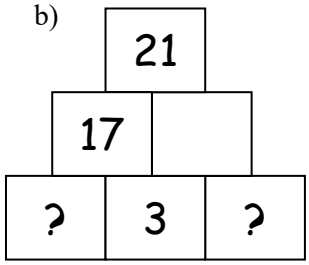
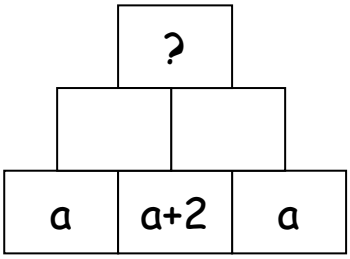


In pairs, make up one for each other to try. Can you find a way of predicting what the top number is going to be if you are given the bottom 3 numbers?

2. We can use number pyramids with algebra also. What do you think the missing box should be in these two pyramids? In each case, make up a similar one of your own and ask a friend to try it. You might like to use the first letter of your own name instead of *'a'*.

a)



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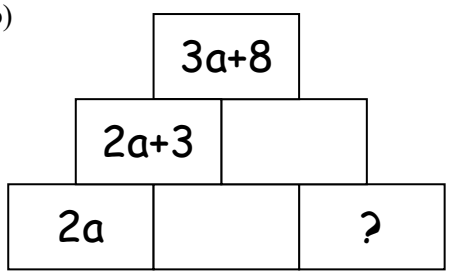
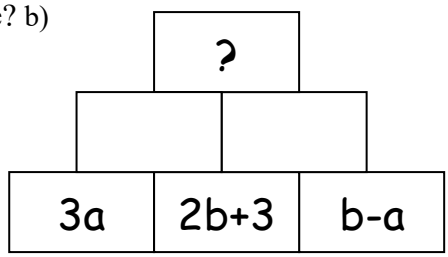
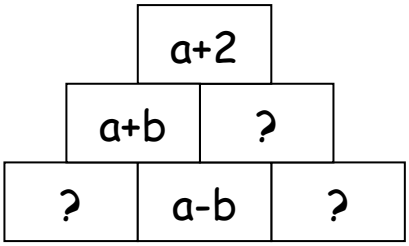


3*. What about these? b)





4. If we know what a letter is worth, we can turn a number pyramid made of algebra into one made of numbers. Re-draw the pyramid in question 2b) for when *a* is worth 7. Does it still work? Now choose your own number instead of 7. (It doesn't have to be a whole number!)

5*. Sometimes we have a clue to what a number is worth. In question 2a, if the top box is worth 24, can you work out what the pyramid must look like in numbers? In question 2b, what if the top box is worth 29? 12.5? -22? Can you make up your own similar questions?