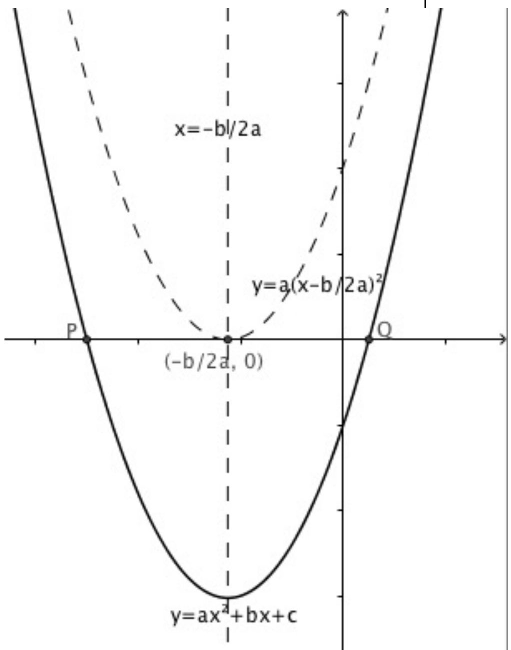
Solving Quadratic Equations and Transforming Parabolas

Thinking at the same time about the algebra and about transformations of the graph of the quadratic function $y = x^2$, we see how the formula for the solutions of a quadratic equation $ax^2 + bx + c = 0$ arise from the coordinates of the point where the graph of $y = ax^2 + bx + c$ cuts the x-axis. Replace the red question marks below.

To solve the quadratic equation $ax^2 + bx + c = 0$ we rearrange it to complete the square.	Think about where the graph of $y = ax^2 + bx + c$ cuts the y axis.
$a(x + ?)^2 + c - \frac{b^2}{4a} = 0$	Rearranging the equation.
$a(x + ?)^2 - \frac{b^2 - ?}{4a} = 0$	The graph is a translation of y=x² by $-b/2a$ in the x direction and $\frac{b^2-4ac}{4a}$ in the negative y direction so it is symmetrical about the line x= ?
$(x + ?)^2 = \frac{b^2 - ?}{4a^2}$	
Next take the square roots of both sides $(x + ?) = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	The roots of the equation are given by the x coordinates of the points P and Q which are symmetric on either side of the line x = ?
The roots of the equation are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	P = ? Q = ?



Example

To solve $2x^2 + 9x - 3 = 0$ by completing the square the equation can be rearranged to give

$$2\left[(x + ?)^2 - \frac{3}{2} - \frac{81}{16}\right] = 0$$

$$2[(x + ?)^2 - ?] = 0$$

$$(x + ?)^2 = ?$$

So the solutions are:

$x = -\frac{9}{4} \pm \frac{\sqrt{105}}{4} = \frac{-9 \pm 10.247}{4}$ = ? and ? (to 3 decimal places)

PROOF SORTER: Give a proof of the formula for the solutions of a quadratic equation by putting these statements in order:

$(x + \dfrac{b}{2a})^2 = \dfrac{b^2 - 4ac}{4a^2}$

Divide both sides by a: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Rewrite the right-hand side with common denominator $4a^2$

The solutions are $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

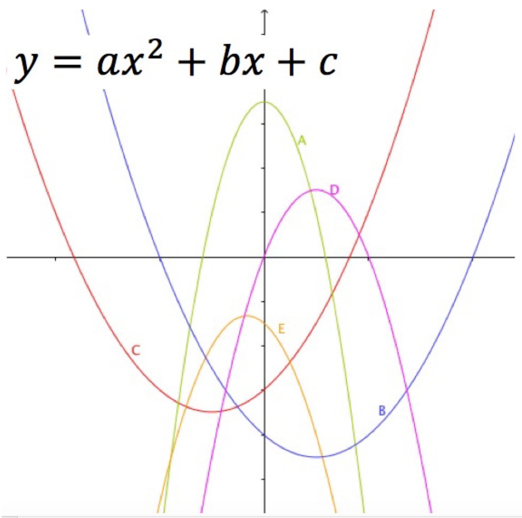
Add $(\frac{b}{2a})^2$ to both sides: $(x + \frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2a})^2$

Subtract $\frac{c}{a}$ from both sides: $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Subtract $\frac{b}{2a}$ from both sides

$x + \dfrac{b}{2a} = \pm\dfrac{\sqrt{b^2 - 4ac}}{2a}$

Complete the square: $(x + \frac{b}{2a})^2 - (\frac{b}{2a})^2 = -\frac{c}{a}$



HELP How do the two points where the graph cuts the x-axis relate to the axis of symmetry of the graph and the solutions of the quadratic equation $ax^2 + bx + c = 0$?

Match the graphs in the diagram to the following descriptions and give reasons for your decisions.

1. $y = ax^2 + bx + c$ if $a > 0, b > 0$ and $c < 0$
2. $y = ax^2 + bx + c$ if $a < 0, b = 0$ and $c > 0$
3. $y = ax^2 + bx + c$ if $a < 0, b < 0$, and $b^2 - 4ac < 0$
4. $y = a(x + p)^2 + q$ if $p < 0, q < 0$ and the x-intercepts have different signs.
5. $y = a(x + p)^2 + q$ if $a < 0, p < 0, q > 0$ and one root is zero.

https://aiminghigh.aimssec.ac.za/quadratic-functions/

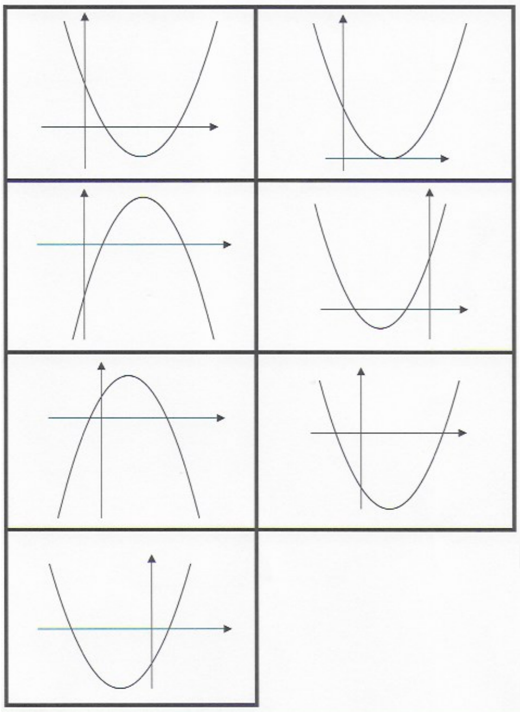
NEXT

A. **What if $b^2 - 4ac < 0$?**

What if the graph of the function does not intersect the x-axis?

See the video 'What are Numbers' https://youtu.be/s7uffFGhFLM This takes you beyond school mathematics and it explains that, all quadratic equations have solutions because, in addition to one dimensional real numbers (represented by points on the real number line) there are also two dimensional complex numbers (represented by points in the plane).

B. Match these equations to their graphs.



$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$

https://aiminghigh.aimssec.ac.za/quadratic-matching-1/

https://aiminghigh.aimssec.ac.za/quadratic-matching-2/