CHANGING THE SUBJECT - MAKING SENSE OF EASY ALGEBRA

The formulas below have been rearranged, some are correct and others are wrong. Decide which are right and which are wrong and be prepared to explain why.
The first one has been done for you.

1. Distance time formula: $\boldsymbol{s = ut + \frac{1}{2}at^2}$
   (s = distance, u = initial velocity, a = acceleration, t =time).

(A) $\boldsymbol{u = \frac{s}{t} - \frac{1}{2}at}$ ✓ (B) $\boldsymbol{a = \frac{(s-ut)}{t^2}}$ X

2. Velocity time formula: $\boldsymbol{v = u + at}$
   (v = velocity at time t, u = initial velocity, a = acceleration, t = time)

(A) $\boldsymbol{u = v - at}$ (B) $\boldsymbol{t = a(v + u)}$ (C) $\boldsymbol{a = \frac{v-u}{t}}$

3. Ohm's Law: $\boldsymbol{V = IR}$
   (V= voltage, I = current, R = resistance)

(A) $\boldsymbol{I = VR}$ (B) $\boldsymbol{R = \frac{V}{I}}$

4. Boyle's Law: $\boldsymbol{PV = C}$ where C is constant. At constant temperature, the volume of a fixed amount of a gas is inversely proportional to its pressure.

(A) $\boldsymbol{V = \frac{P}{C}}$ (B) $\boldsymbol{P = VC}$

5. Newton's 2nd law of motion: $\boldsymbol{F = ma}$ (F = Force, m = mass, a = acceleration).

(A) $\boldsymbol{a = Fm}$ (B) $\boldsymbol{m = \frac{F}{a}}$

6. Einstein's Law: $\boldsymbol{E = mc^2}$ (E = energy, m = mass, c = the speed of light).

The equation revealed that mass and energy are different forms of the same thing.

(A) $\boldsymbol{c = \frac{E}{m}}$ (B) $\boldsymbol{c = \sqrt{\frac{E}{m}}}$ (C) $\boldsymbol{m = Ec^2}$

HELP

Remember the Golden Rules.

Golden Rule 1 What is on either side of an equals sign will still be equal if you do the same to both sides like weighing with old fashioned scales.

Golden Rule 2 For all numbers, the operations of adding and subtracting reverse each other e.g. 7 + 5 – 5 = 7, multiplying and dividing reverse each other e.g $\mathbf{7 \times 3 \div 3 = \frac{7 \times 3}{3} = 7}$ and squaring and taking square roots reverse each other $\sqrt{3^2} = 3$ and $(\sqrt{3})^2 = 3$. **These are pairs of inverse operations**.

NEXT

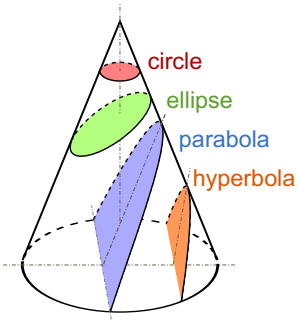
Make up your own similar questions based on these 4 formulas:

Parabola: $y^2 = cx$ for constant c.

Rectangular hyperbola: $xy = c$ for constant c.

Circle (radius r): $x^2 + y^2 = r^2$

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Some Diagnostic Quiz Questions

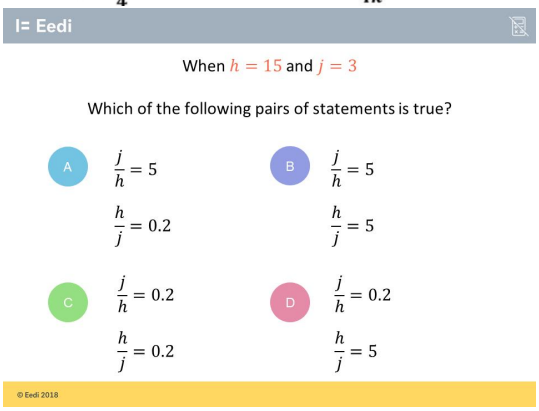
Rearrange the following to make k the subject p = 4k + 3

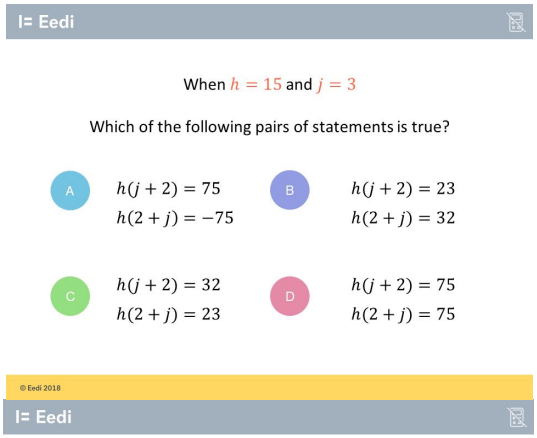
A $k = \frac{p-3}{4}$

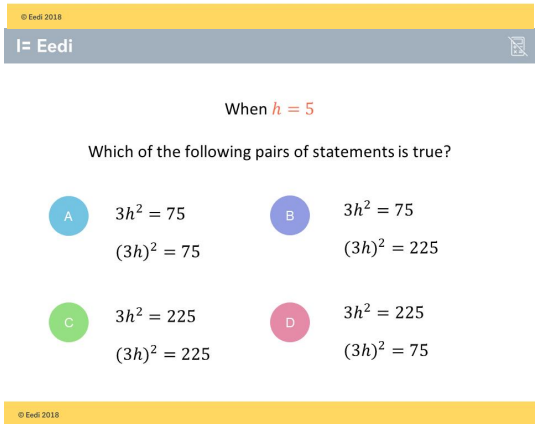
C $k = \frac{p-4}{3}$

B $k = \frac{p+3}{4}$

D $k = \frac{p-3}{4k}$







NOTES FOR TEACHERS

SOLUTION

1. Velocity time formula: $\boldsymbol{v = u + at}$

   (v = velocity at time t, u = initial velocity, a = acceleration, t = time)

(B) $\boldsymbol{u = v - at}$ ✓ (B) $\boldsymbol{t = a(v + u)}$ X $t = \frac{(v-u)}{a}$ (C) $\boldsymbol{a = \frac{v-u}{t}}$ ✓

2. Ohm's Law: $\boldsymbol{V = IR}$

   (V= voltage, I = current, R = resistance)

(B) $\boldsymbol{I = VR}$ X $I = \frac{V}{R}$ (B) $\boldsymbol{R = \frac{V}{I}}$ ✓

3. Boyle's Law: $\boldsymbol{PV = C}$ where C is constant. At constant temperature, the volume of a fixed amount of a gas is inversely proportional to its pressure.

(B) $\boldsymbol{V = \frac{I}{C}}$ X $\boldsymbol{V = \frac{C}{P}}$ (B) $\boldsymbol{P = VC}$ X $\boldsymbol{P = \frac{C}{V}}$

4. Newton's 2nd law of motion: $\boldsymbol{F = ma}$ (F = Force, m = mass, a = acceleration).

(B) $\boldsymbol{a = Fm}$ X $\boldsymbol{a = \frac{F}{m}}$ (B) $\boldsymbol{m = \frac{F}{a}}$ ✓

5. Einstein's Law: $\boldsymbol{E = mc^2}$ (E = energy, m = mass, c = the speed of light).

The equation revealed that mass and energy are different forms of the same thing.

(B) $\boldsymbol{c = \frac{E}{m}}$ X $\boldsymbol{c = \sqrt{\frac{E}{m}}}$ (B) $\boldsymbol{c = \sqrt{\frac{E}{m}}}$ ✓ (C) $\boldsymbol{m = Ec^2}$ X $\boldsymbol{m = \frac{E}{c^2}}$

Why do this activity?

This is an ideal activity for drill and practice or for revision. It requires learners to understand the process of rearranging formulas and, by explaining the process and identifying errors, to recognise common misconceptions and develop their own communication skills. The examples relate to well-known laws of Physics and help learners to appreciate some of the ways in which mathematics is used and applied.

Learning objectives

In doing this activity students will have an opportunity to:

- practice changing the subject of simple formulas
- recognise and apply inverse operations.

Generic skills and competences

In doing this activity students will have an opportunity to:

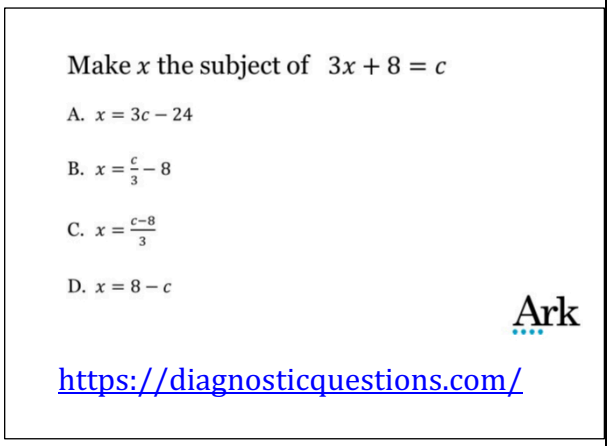
- work in pairs or small groups to help each other to recall concepts and deepen understanding

- explain their reasoning to the class and develop logical thinking and communication skills
- to appreciate a range of applications of mathematics to science and engineering.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.



4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is C: $x = \frac{c-8}{3}$

Possible misconceptions:
A: Multiplying by 3 instead of dividing
B: Dividing only one term on the right-hand side by 3
D: Appears to be a wild guess

Suggestions for teaching

Arrange for learners to work in pairs and give out the worksheet on page 1, one between 2, or write the questions on the board. Give out the HELP sheet to learners who struggle, or write the Golden Rules on the board.

Go around the class and ask Key Questions when learners need guidance. You could stop the class and have everyone doing the actions of standing up then sitting down etc to illustrate inverse operations. When they finish the worksheet g up ask learners to check their answers. When they have done that give out the NEXT task. When more than half the class has finished the main worksheet go through the answers one by one asking for thumbs up 👍 for correct answers and thumbs down 👎 for wrong answers.
Make sure that all learners respond. Ask pairs of learners who correctly identified right answers to come to the board and explain their reasoning and working to the class. Give encouragement and praise and treat mistakes as learning opportunities.

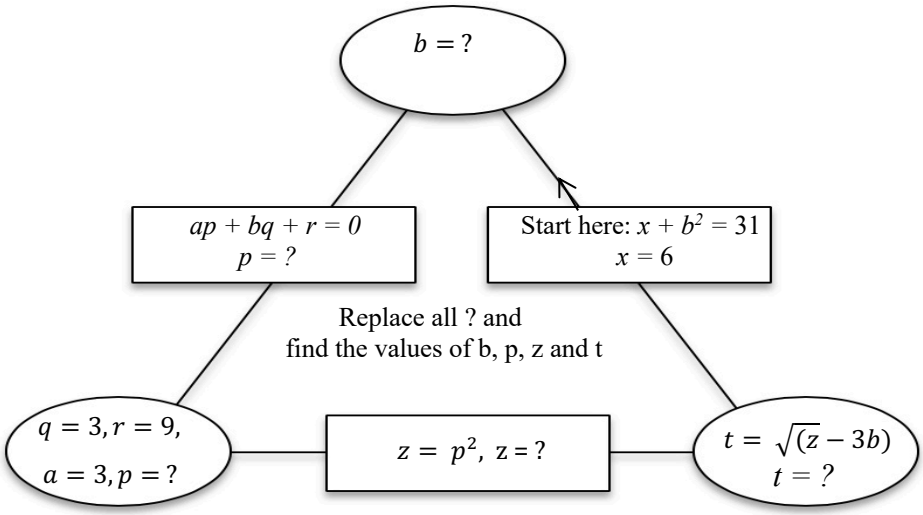
Homework or follow up work could be the task in the NEXT box or answering the Diagnostic Quiz questions on page 2.

Key questions

1. What changes after you untie your shoelaces then tie them up again?
2. Tying and untying your shoelaces are inverse processes. What is similar about adding and subtracting the same term to an algebraic expression? What happens to formula 1. When you subtract *at* from the right-hand side? Can you use that idea for other formulas?
3. Standing up and sitting down are inverse processes. What is similar about multiplying and dividing an algebraic expression by the same amount? What happens to formula 1 when you divide *at* by *t* ? Can you use that idea for other formulas?
4. Raising your arms then putting them down by your sides are inverse processes. What is similar squaring and taking the square root? What is 5^2? What is $\sqrt{5^2}$? Can *you use that idea for formula 5?*

Follow up

Inverse operations: MD Puzzle https://aiminghigh.aimssec.ac.za/md-puzzle/
Solving equations: Matchless https://aiminghigh.aimssec.ac.za/matchless/
Changing the subject of formulas and solving equations: Checkit Puzzle
https://aiminghigh.aimssec.ac.za/checkit-game/



Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/MathsToys/videos
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.