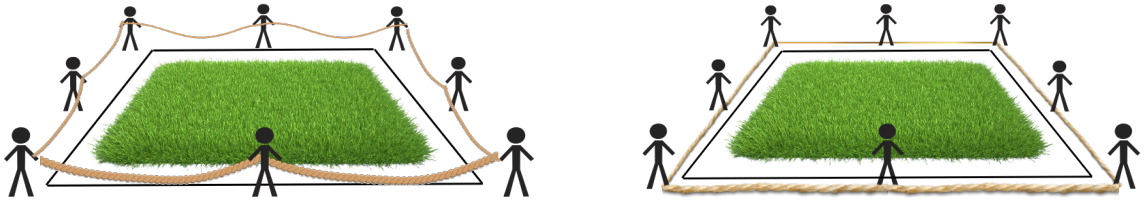
30 Minute Lesson

OUTER RIMS

PEOPLE MATHS ACTIVITY



The lesson in a nutshell:

Students measure the perimeter of a large square on the ground with string and then use that string, with 1 metre of length added to it, to create a slightly larger square surrounding the original square.

The uniform gap size between the two squares is measured.

When students repeat this exercise with several squares of different sizes they are in for a mighty fine surprise! If you have a large class different groups can work on different squares at the same time, using the instructions on page 2, and then compare their results.

Preparation and Supplies:

- Find, or draw on the ground with chalk, several large squares. You might find squares marked in the school quadrangle, or in the floor of the cafeteria, for example. **Alternatively**: If you feel your students are fully capable of drawing large squares on the ground with accurate right angles and common side lengths, then gather the necessary supplies for them to do just this.
- You will **need rulers, measuring tape or metre-sticks, and string** long enough to go around your squares with a few metres of length to spare.
- Have printouts of the worksheets on page 3 (or just squared paper) ready for students.

Planning the lesson for different age groups: *The People Maths activity is sufficient for Lower Primary age students. Upper Primary students could do it and then go into the classroom and do the activity on page 3. If there is not time to complete both activities in 30 minutes, it can be finished afterwards or in a subsequent lesson. Older students who know how to work out the circumference of a circle might do the follow up activities on page 4 as the 30-minute lesson instead of the People Maths and Square Fence activities.*

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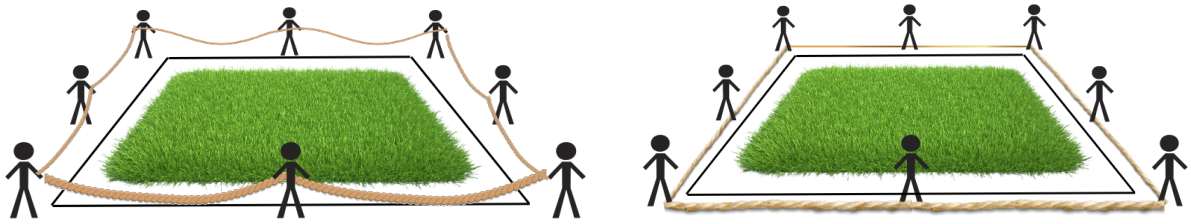


1. Working with a square drawn on the ground in the classroom, or in a bigger space, indoors or outdoors, find the perimeter of the square. The square can be any size. Use string to mark out the line of a fence (string-fence) around the square that is **exactly one metre longer** than the perimeter of the square. The string needs to be a little more than one metre longer than the perimeter of your square to allow for tying a knot to make the string into a loop that is exactly the right length.
2. Mark the string into 4 equal lengths to give the corners of the square 'string-fence' and then hold the 4 corners, stretching the string to make a square. Lower the string close to the ground, adjust the position of the 'string-fence' so that the width of the gap between the square and the string is the same all the way round. Then measure the width of the gap.
3. Talk about what would happen to the gap if the experiment is repeated for a smaller square, and again with the string one metre longer than the perimeter of the square. Vote as to whether the gap between the square and the string will be less, the same, or more. You can think of this as making the path between the square and the fence narrower, the same, or wider. Which will it be? Carry out the experiment with the smaller square to find the answer.
4. If you repeated the experiment for a very large square field making an outer 'string-fence' again exactly one metre longer than the perimeter of the field, would path between the field and the fence be wide enough for a cow to walk along it?
5. Would a mouse be able to run along the path?

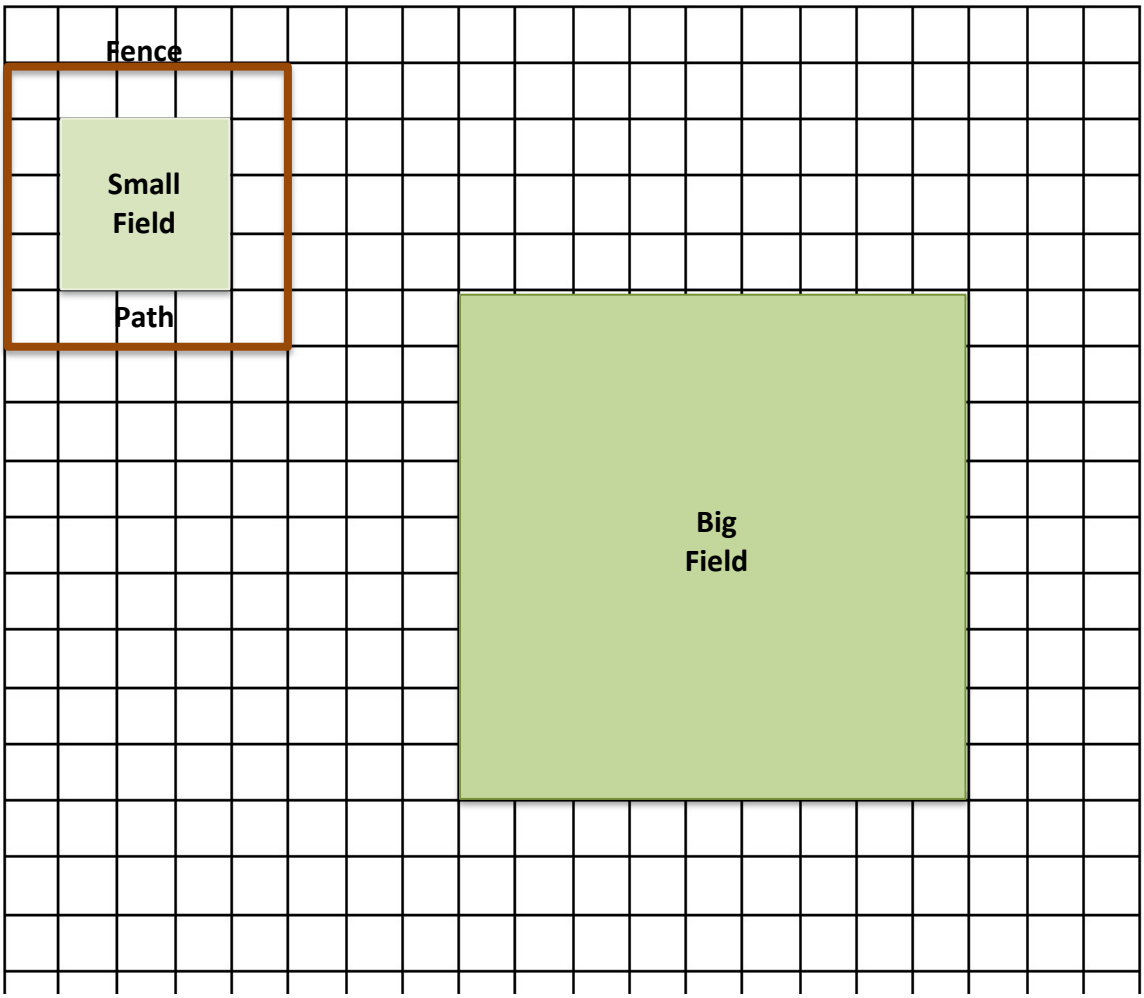
EXTRA CHALLENGE: Suppose you are the farmer.
You want to make a path around the big field four metres wide for the cows to go along. How would you work out how much fencing to buy?

SQUARE FENCE

The diagrams below are drawn to the same scale. The fence around the small field is **exactly one metre longer** than the outer edge or rim of the field (the perimeter).

Draw a fence around the big field that is **exactly one metre longer** than the outer edge of the big field, making a path around the field. (Be sure you can explain how you know that the fence you draw is exactly one metre longer.)

What do you notice? How wide are the two paths between the original square and the fence you make?



HELP

Think about which parts of the outer-rim are equal in length to the edges of the square and where the extra fence goes to make the longer fence. Think about how this extra length of fence is connected to the width of the path.

Follow-up 1

NOT-SO-SQUARE FENCE

A fence is built around a square field. Suppose another fence is built exactly one metre longer, so the path between the two fences is the same width all the way round including at the corners of the field.

How wide would this path be?

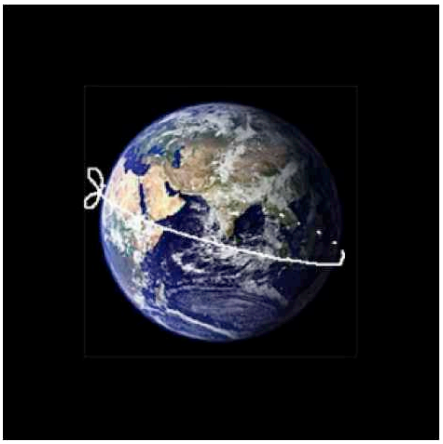
Would a mouse be able to run along it?

Could a farmer drive his cows along the path between the two fences?

See https://aiminghigh.aimssec.ac.za/not-so-square-fence/

Follow-up 2

BELT AROUND THE EARTH



A wire belt is tied tightly around the Earth at the Equator. (Let's assume the Earth is perfectly spherical and we have no worries about oceans and mountains interfering with our ability to conduct this task!)

Now suppose the belt is made exactly one metre longer and held around the Earth at the Equator so that it is the same distance away from the Earth everywhere. Would a mouse be able to crawl under the new belt? How do you know?

Would a cat be able to crawl under?

Would the answer be the same for the moon?

What assumptions do you need to make to answer this question?

See https://aiminghigh.aimssec.ac.za/belt-around-the-earth/

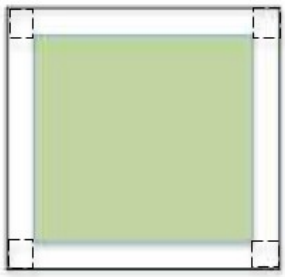
NOTES FOR TEACHERS

SOLUTION

Note: Square Fence, with the People Maths Outer Rims activity, is the first of a sequence of three problems: Square Fence; Not-so-square Fence and Belt Around the Earth. Each one leads to the next and doing them in sequence will help students to solve the harder problems.

Whatever the size of the square field, the width of the path is 12.5 centimetres. This may seem a surprising result. Why does it happen?

This is a question about perimeter. The key to a quick and easy solution here is to understand that segments of the fence parallel to edges of the field will always match the edges of the field, WHATEVER THE SIZE OF THE FIELD, and that the extra fencing is only needed at the corners.



Method 1 If the width of the path is w metres (shown by dotted lines in the diagram) and you think about the extra fence which is only at the corners, this extra fence at each corner forms two straight line segments of length w at right angles. This means that the **extra fence** from the four corners would together make a square of perimeter 8w metres for any sized field.

We know that the extra length of fence is 1 metre so the width of the path w is one eighth of a metre, that is 12.5 centimetres.

The path is too narrow for cows, but a mouse could easily run along it.

Method 2 This algebraic method is equivalent to the mathematical reasoning in Method 1. Let the sides of the square be of length a metres and the width of the path be w metres, then for the outer fence (which is 1 metre longer):

$4(a + 2w) = 4a + 1$

$4a + 8w = 4a + 1$

$w = \frac{1}{8}$ m = 0.125 m = 12.5 cm

Why do this activity?

Many students will have the pleasure of the 'AHA' moment when the light dawns and they understand how to do this problem and why the width of the path is the same whatever the size of the field. The People Maths activity on page 2 helps the students to understand the problem and to visualize and mathematize the problem.

Often the main challenge for students is to read the question, to understand the information given and to understand what has to be found. With a good teacher students will be accustomed to having to think for themselves and most students will not even want a hint.

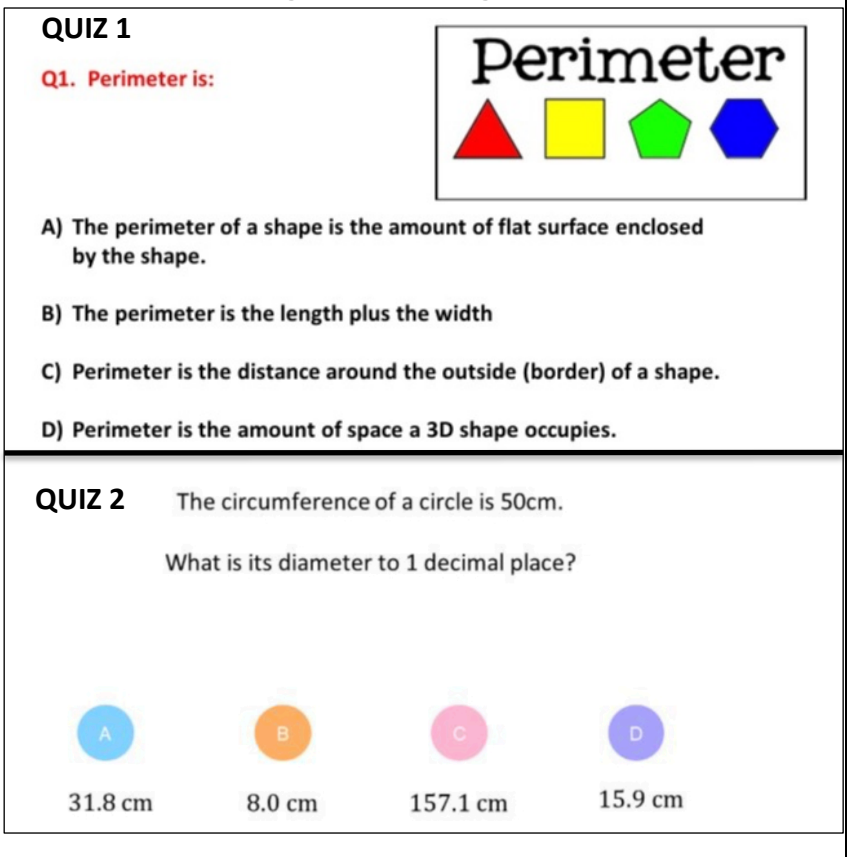
This problem only requires an understanding of the concept of perimeter. Confusion between 'perimeter' and 'area' is very common so the problem has the value that it focuses on the common idea of a fence or boundary. Using the mathematical word pe**rim**eter, with the connection to the term 'rim', will help students to remember the meaning of the word.

Students who know how to work out the circumference of a circle can go on to solve the two further problems Not-so-square Fence and Belt Around the Earth. The surprising fact in all these problems is that the answer is exactly the same whatever the size of the field or the sphere so you do not need to know that size.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the students respond. Ask a student who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the student for giving the answer.
2. It is important for students to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that students listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

QUIZ 1 The correct answer is C.

This Diagnostic Quiz provides an excellent opportunity for students to discuss the correct words to use for answers A), B) and D) which are:

A) Area, B)This ONLY applies to a rectangle and then it refers to the semi-perimeter, D) Volume.

QUIZ 2 The correct answer is D. To answer this quiz and to do the Not-So-Square Fence and Belt Around The Earth problems, students need to know the formula for the circumference of a circle, that the value of pi is a little more than 3, and the relationship between radius and diameter of a circle. https://diagnosticquestions.com/

Learning objectives

In doing this activity students will have an opportunity to:

- Reinforce understanding of perimeter.
- Develop problem solving skills and the ability to use and apply knowledge to a real life context.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **visualize** and to develop the skill of interpreting and creating visual images to represent concepts and situations and apply this to interpreting information and **solving problems.**

Suggestions for teaching

You might introduce the problem by saying that it is about a fence around a field, saying that a fence like that is called a boundary fence and asking if anyone knows what the word boundary means and if anyone knows the mathematical word for boundary (i.e. perimeter). You might like to talk about the fence around the school grounds, also a boundary fence although the shape inside is not a square.

If you decide not to do the People Maths activity on page 1 and 2 then ask the students to read the problem on page 3 sentence by sentence. Then either ask them to draw their own diagrams and to try to find the solution or use the worksheet on page 3. You could have the students work individually, then in pairs, then in fours, then finally have a class discussion (the "One-Two-Four-More" strategy).

For older students the Square Fence activity can be used as a warm-up problem for the problem Not-so-square Fence for which the key idea is similar. Then follow that by the Belt Around the Earth problem.

Key questions

Have you drawn (studied) a diagram?

When you make the fence longer and the path wider, where does the extra fence go?

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13