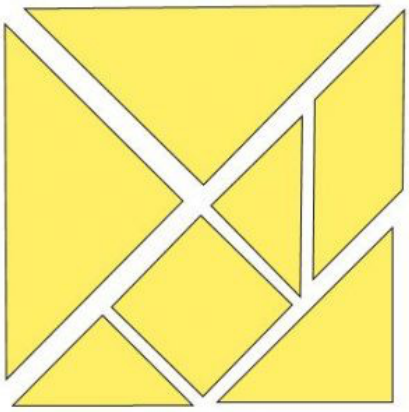
TANGRAM FRACTIONS

STARTER ACTIVITY FOR ALL AGES



In a group make the puzzle by folding a square (See the HELP section for instructions).

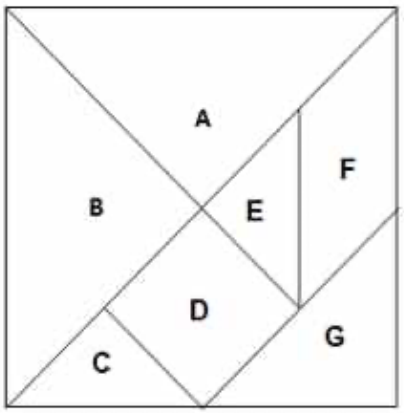
Play with the tangram pieces.

Create your own shapes.

This is a classic Chinese Tangram.

Each of the people illustrated, and many other shapes, can be made with the 7 pieces. There are hundreds of other puzzles based on this tangram.

Can you find a way to fold a sheet of paper so that you can cut out the tangram pieces accurately without any measuring?



FRACTIONS, AREAS AND PERIMETERS

Take the edge length of the square as 1 unit and the area as 1 square unit.

In pairs or groups talk about the areas of the 7 pieces. What fraction of the square does each piece make? Arrange the 7 pieces in order of their areas.

For secondary students: What are the perimeters of the 7 pieces?
Arrange the 7 pieces in order of their perimeters.
Are these two orders the same?

Make a square from the pieces A, G, C and E. What is its area? What is its edge length?

How many different polygons can you make using all 7 pieces of the tangram?

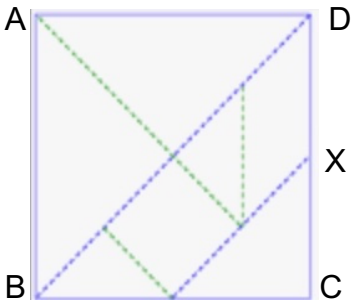
TANGRAM FRACTION GAME

You will need 7 tangram pieces. Draw a square frame into which the pieces fit.

Make 8 identical fraction cards. Write $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$ on 7 cards and leave one card blank. Put the cards in a bag or in a box and shake to mix the cards, or place the cards face down on the table and mix them.

Take the tangram pieces out of the frame. Each player (or team) in turn picks a fraction card, selects a tangram piece with area corresponding to the fraction on the card, notes the area, and places the piece in position in the frame. The winner is the player with the greatest total area when the 7 pieces are back in the frame and all 8 cards have been taken.

HELP MAKE YOUR OWN TANGRAM



The tangram can be made by folding a square of paper. or thin card.

Bring vertices A and C together and fold to on the blue diagonal fold line.

To find the centre, **very lightly** crease on the green diagonal line without creasing the triangle at the bottom right.

Cut along the diagonal BD.

Cut along a green line to cut the large triangle into two more tangram pieces.

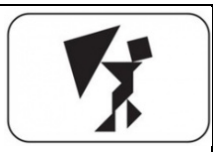
You now have three triangles and a trapezium.

Bring point X to the centre and fold along a green line to make a triangle and a parallelogram.

Bring point B to the centre and fold along a green line to make a triangle and a square.

Cut along these green fold lines and you will have 7 tangram pieces: a parallelogram, a square and 5 triangles.

NEXT

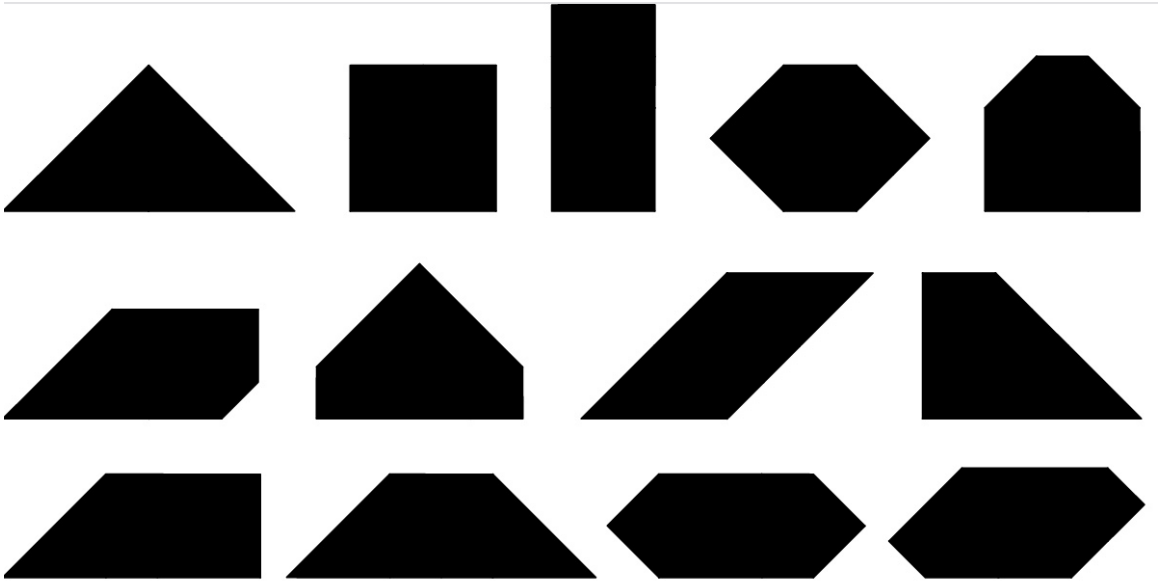


Can you make this little man with the umbrella?

Tangram 2D Shapes https://aiminghigh.aimssec.ac.za/tangram-2d-shapes/

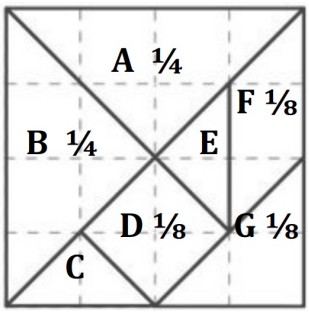
A Bigger Challenge: There are only 13 convex polygons altogether that can be made using all 7 tangram pieces. Can you find them?

A polygon is called convex when all the interior angles are less than 180 degrees.



NOTES FOR TEACHERS

SOLUTION



Draw the tangram on squared paper to help in finding the areas. The 16 small squares each have an area of 1/16.

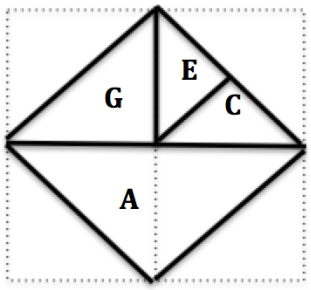
The edge of the square is 1 unit. The area of the big square is 1 square unit.

Pieces C and E are made up of 2 half squares so they have an area of 1/16.

Pieces D, G and F are each made up of 2 small squares and have an area of 2/16 or 1/8.

Area C + Area E + Area D + Area G + Area F + Area A + Area B
= 1/16 + 1/16 + 1/8 + 1/8 + 1/8 + 1/4 + 1/4 = 1 square unit

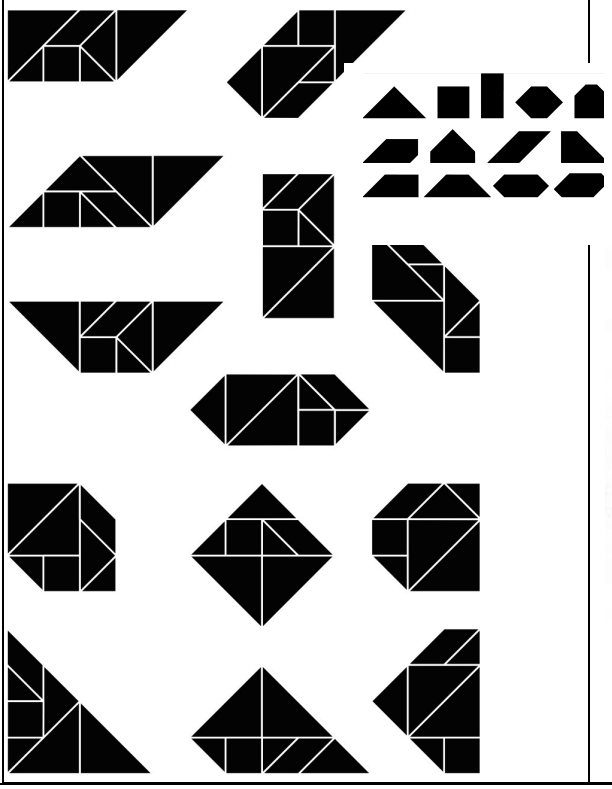
The diagram shows a square made up from pieces A, G, C and E. The area of the square is 1/2 and the edge length is $1/\sqrt{2}$



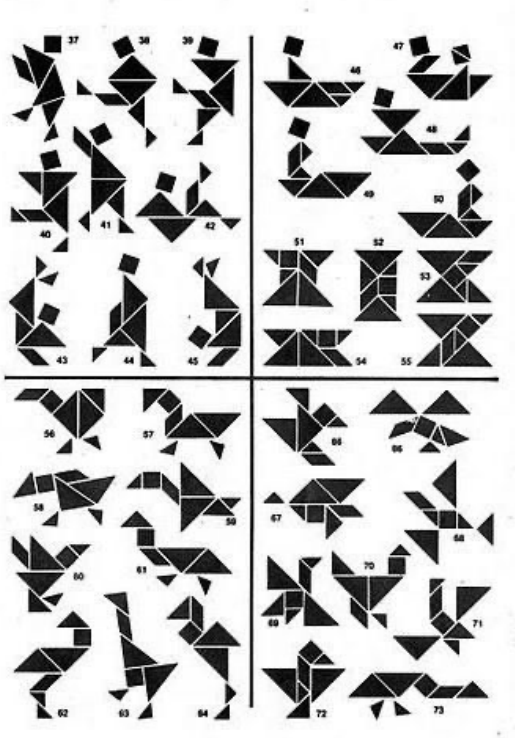
The diagram below on the left shows the 13 convex polygons that can be constructed using the 7 tangram pieces.

The diagram on the right gives solutions to some tangram puzzles. Many more of these puzzles and their solutions can be found by doing a Google search.

Solutions for the 13 convex polygons.



Solutions for a miscellaneous set of tangram pictures.

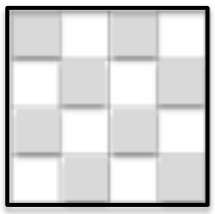


DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The outer square has area 1 square unit.
What is the area of **one** of the grey squares?

A. $\frac{1}{8}$

B. $\frac{1}{16}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

The correct answer is: B

Why do this activity?

This activity can be described a 'low entry point – high ceiling' problem making it suitable for a wide age range and for low and high attaining learners in a class. The area of a triangle is introduced in an informal way as half the area of the square when the diagonal of the square is drawn.

This activity gives good practice in working with areas and fractions and adding fractions to check that the total area is 1 square unit. It also gives practice with angles. Young learners in primary school can find the tile arrangements to make the little people and work out the areas of the 7 pieces. Finding the area of the square made from 4 pieces takes lower secondary learners into working with surds. Perhaps every class can find some other polygons, and maybe between them find all 13 convex polygons, but proving that exactly 13 convex polygons can be made and no more is beyond the scope of school mathematics.

Learning objectives

In doing this activity students will have an opportunity to:

- practise finding areas by splitting squares rather than using formulae;
- make sense of fractions by thinking about dividing up 1 square unit.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically, reason logically and give explanations;
- think flexibly, be creative and innovative and apply knowledge and skills;
- visualize and develop the skill of interpreting and creating visual images to
- represent concepts and situations;
- persevere and work systematically to investigate all possible cases.

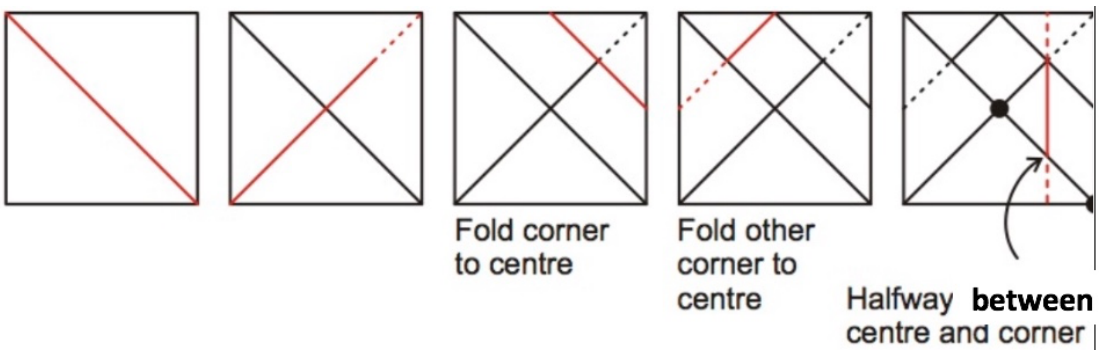
Suggestions for teaching

Start with the diagnostic test. Make sure if possible that all the learners understand that there are 16 small squares making up an area of 1 square unit so the area of each small square is $\frac{1}{16}$. The number in the denominator (16) names the number of equal parts into which the whole unit has been split. Different fractions are made up by combining some of the $\frac{1}{16}$ ths.

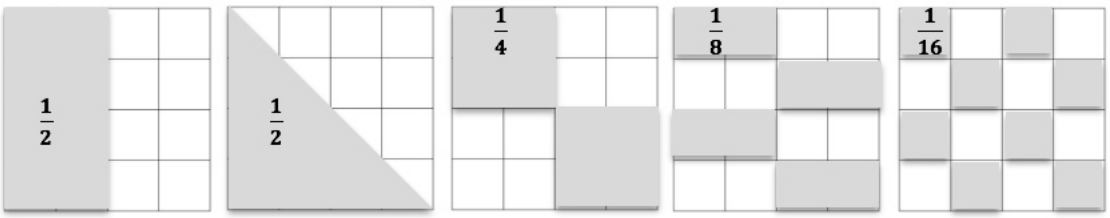
For example, if you have 3 of the small squares you have $\frac{3}{16}$ ths.

Give each learner a square of paper. Explain and demonstrate step by step how to fold the paper to make the 7 tangram pieces without any measuring. Each fold-line gives an edge for one of the pieces. When you make each fold ask the learners "What is being halved by this fold?"

Follow the instructions in the video or the instructions below. In each diagram the red line shows the fold to make. Help the learners to make their own Tangram Puzzle pieces and cut out the 7 pieces.



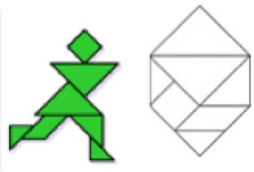
The area of the whole Tangram square is taken to be 1 square unit. **Before you ask** the learners to find the areas of each of the Tangram pieces and fractions of the whole, first discuss the areas and the fractions in the diagram below



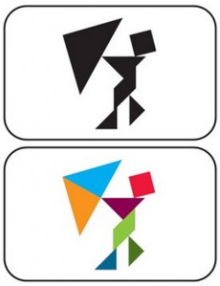
Then give the learners time to work in pairs and decide on the fractions for each of the 7 pieces.

In a whole class discussion ask the learners to tell you the fractions for the 7 pieces and draw the diagram as given in the solution section writing in the aeas when ione o. Ask the learners to name the shapes. Ask about the angles. Perhaps ask which shapes are similar.

Ask the learners to put the 7 pieces together again to make the original square. Then ask them to make a smaller square from pieces A, G, C and E.Ask the learners to name the shapes. Ask about the angles. Perhaps ask which shapes are similar.



Then draw this tangram man on the chalk board and ask the learners to arrange their pieces to give the same picture. Draw this tangram hexagon on the board and ask the learners to arrange their pieces to give the same picture.



The man with an umbrella is another puzzle for learners to try.

Also encourage learners to be creative and make their own tangram people and other pictures.

Key questions

- What are the angles?
- What angle do you need to fit in there?
- When you make a fold what can you say about the shapes on either side of the fold?
- Does the diagonal (fold line) split the square (triangle) into two equal parts?
- What is the same and what is different about these two shapes?

Follow up

Making the complete set of 13 polygons is a good follow up activity. Challenge the learners to make as many different convex polygons as they can using all of the 7 pieces.

The learners could make a poster for the classroom wall and paste up each polygon as one of the learners finds a new one. With encouragement the learners could go on searching for new polygons for a long period. When most have been found, the teacher can give hints about what shapes to look for to find them all.

See also: **Tangram 2D Shapes** https://aiminghigh.aimssec.ac.za/tangram-2d-shapes/
Tangram Pattern https://aiminghigh.aimssec.ac.za/tangram-pattern/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links:
https://aiminghigh.aimssec.ac.za/ http://aiminghigh.aimssec.ac.za
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Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13