AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

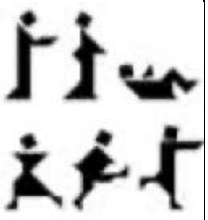
This INCLUSION AND HOME LEARNING GUIDE
suggests related learning activities for all ages from 4 to 18
on the theme of FRACTIONS

Just choose whatever seems suitable for your group of learners

The original TANGRAM FRACTIONS activity was designed for Years 5 to 9

TANGRAM FRACTIONS

This is a classic Chinese Tangram. Each of the people illustrated, and many other shapes, can be made with the 7 pieces. There are hundreds of other puzzles based on this tangram.



Can you find a way to fold a sheet of paper so that you can cut out the tangram pieces accurately without any measuring?

Take the edge length of the square as 1 unit and the area as 1 square unit.

TANGRAM FRACTION GAME

You will need 7 tangram pieces. Draw a square frame into which the pieces fit.

Make 8 identical fraction cards. Write $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$ on 7 cards and leave one card blank. Put the cards in a bag or in a box and shake to mix the cards, or place the cards face down on the table and mix them.

Take the tangram pieces out of the frame. Each player (or team) in turn picks a fraction card, selects a tangram piece with area corresponding to the fraction on the card, notes the area, and places the piece in position in the frame. The winner is the player with the greatest total area when the 7 pieces are back in the frame and all 8 cards have been taken.

FRACTIONS, AREAS AND PERIMETERS

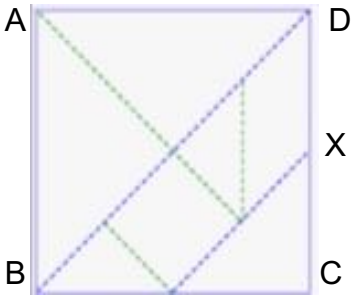
1. The area of the 7 pieces together is one square unit. Find the areas each of the 7 pieces. What fraction of the square does each piece make? What are the perimeters of the 7 pieces?
2. Arrange the 7 pieces **in order of their areas**.

   Arrange the 7 pieces **in order of their perimeters**.

   Are these two orders the same?
3. Make a square from the pieces A, G, C and E. What is its area? What is its edge length
4. How many different polygons can you make using all 7 pieces of the tangram?

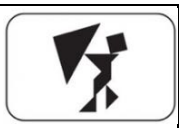
HELP MAKE YOUR OWN TANGRAM

Make a tangram by folding and cutting a square of paper or thin card following the instructions below, or the video https://youtu.be/1l_U31PIfLQ .



1. Bring vertices A and C together and fold on the blue diagonal fold line.
2. To find the centre, **very lightly** crease on the green diagonal line without creasing the triangle at the bottom right.
3. Fold vertex C to the centre and cut along this blue fold line.
4. Cut along the diagonal BD and cut along a green line to cut the large triangle into two triangular pieces. You will now have three triangles and a trapezium.
5. Bring point X to the centre and fold along the green line to make a triangle and a parallelogram.
6. Bring point B to the centre and fold along a green line to make a triangle and a square.
7. Cut along these green fold lines and you will have 7 tangram pieces: a parallelogram, a square and 5 triangles.

NEXT



Can you make this little man with the umbrella?

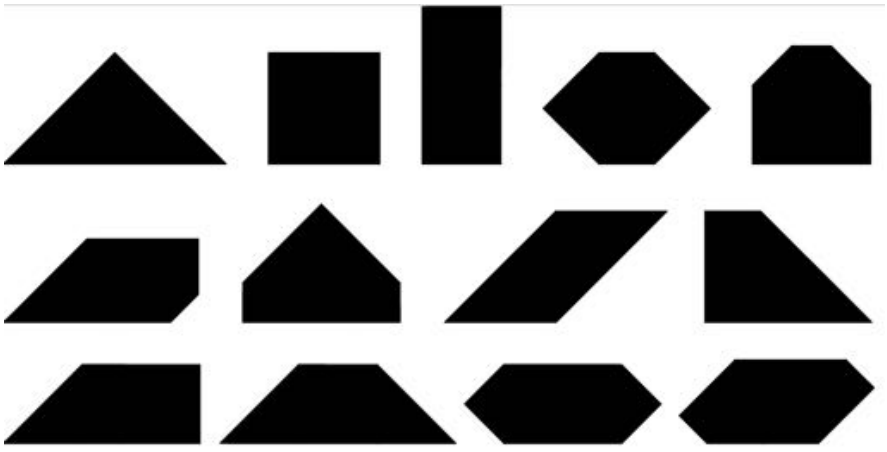
Tangram 2D Shapes

https://aiminghigh.aimssec.ac.za/tangram-2d-shapes/

A Bigger Challenge:

In a convex polygon all the interior angles are less than 180°. Only 13 convex polygons can be made using all 7 tangram pieces.

Can you find them?



STARTER ACTIVITY FOR ALL AGES

In a group make the puzzle by folding a square (See the HELP section for instructions or follow the instructions in this video https://youtu.be/1l_U31PIfLQ).

Play with the tangram pieces.

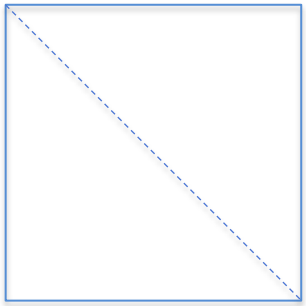
Make a rectangle with the 7 pieces.

Create your own shapes.

INCLUSION AND HOME LEARNING GUIDE

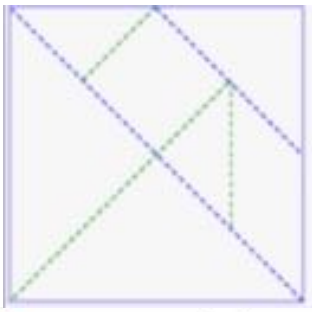
THEME: FRACTIONS

Early Years and Reception Class or Kindergarten



Fold a square of paper along a diagonal and talk to the class about how you are folding it in half. Cut along the diagonal fold-line and put the two pieces one on top of the other to show that they are the same.

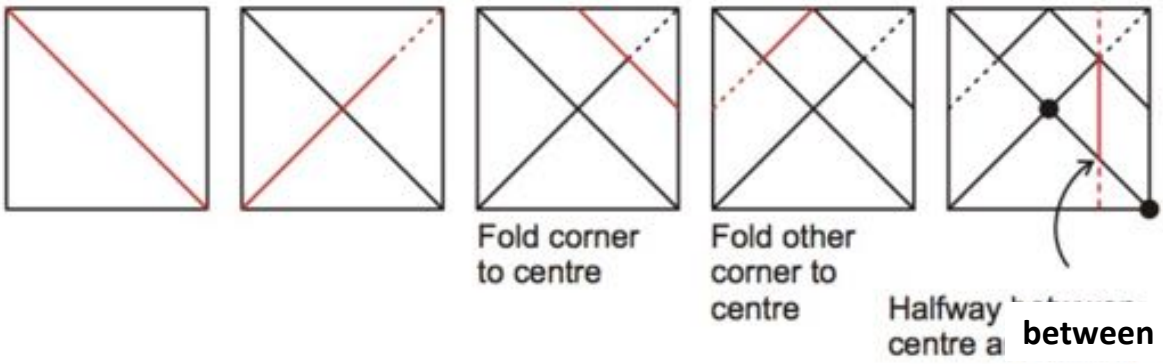
Make sets of tangram pieces before the lesson and give a set of pieces to each group of learners. You might demonstrate to the class how you fold and cut the pieces from a square of paper and talk to them about the different shapes.



To make the 7 tangram pieces by folding a square of card,

follow the instructions in the video or HELP section or follow the instructions below. In each diagram the red line shows the fold to make.

Cut out the 7 pieces.



FITTING THE SHAPES IN A FRAME

Show the learners how to fit the pieces in the square frame. Take one out and ask a learner to put it back. Then take 2 out each time, then 3 or more and let the learners practice putting the pieces back. Talk to the class about the pieces using the names triangle, square and parallelogram.

Give each group of learners a square frame and 7 pieces so that they can take them out and replace them in the frame by themselves.

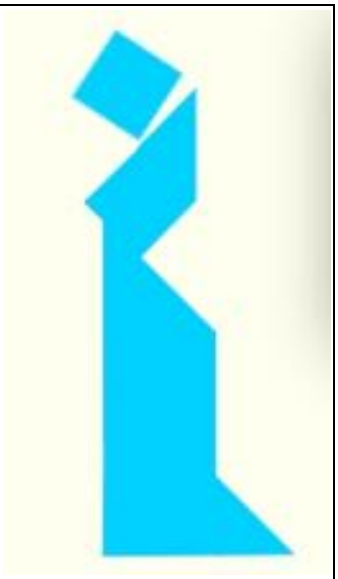
Ask the learners to make the green boy with their tangram pieces.

He is running towards his blue grandmother – Granma T.

Show them how to make Granma T with the tangram pieces.

Visit the World of Tan

You will find a series of stories about GranmaT and her grandchildren. Join the Tan family on their adventures and enjoy creating different pictures from the 7 tangram pieces. https://nrich.maths.org/14074



Years 1 - 4 HALVING A SQUARE

Resources: Squares of scrap paper. Squared paper. Scissors

Start with paper folding and drawing lines that split the square into halves.

PART 1 – paper folding

Show the class one way to fold a square of paper so that the fold-line splits the square in half. Cut along the fold line and place the 2 pieces one on top of the other to show that they are the same. Ask the class if they can suggest a different way to fold the paper in half. Take one of the suggestions and demonstrate it.

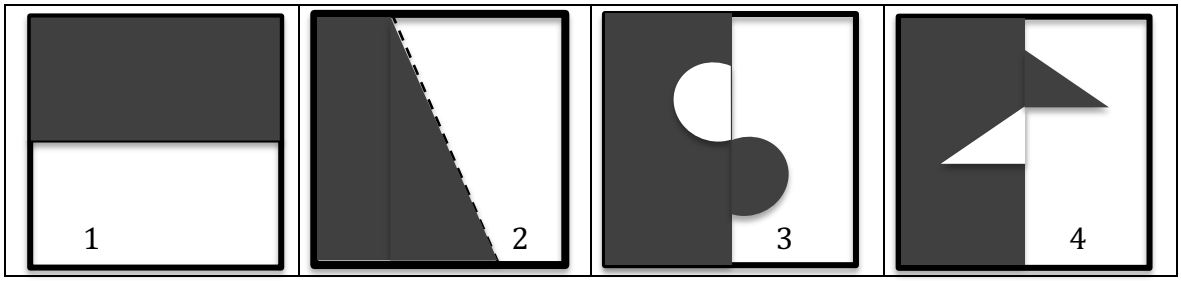
Then give each pair of learners a square of scrap paper and a pair of scissors. Ask them to do a different fold and cut along their fold-line to show that the two pieces are the same and so they must be halves. Then ask them to colour or decorate their two pieces and write their names on the pieces. Make a poster with the learner's work but only use one example for each method of splitting the square. Ask the learners whose work has not been included in the poster to stick their pieces into one of their own workbooks.

PART 2 – drawing a line to split a square in half

Just sow the seeds of this idea, you should not be trying to teach them that the two pieces are halves. Don't dwell on it if some of the learners are not convinced. If they do more activities like this, they'll come to understand the concept in their own good time. Show these pictures and ask the learners if the squares are split exactly in half. They are, but the learners may have difficulty in recognising it. The first two may already have been found by folding and the second one using many different angles.

The last two can't be done by paper folding.

Cut out number 3. Show that the black and white shapes are the same so they must both be halves of the square. Encourage the class to talk about other reasons why the two pieces are both halves. Ask them to describe the shapes. They might say that the shapes in number 3 are rectangles with a piece sticking out and a bite taken out of the rectangle that's exactly the same as the piece that sticks out.



Give out squared paper and ask the learners to draw a square and to draw a line that splits the square in half. Some learners may do more than one square.

Finish the lesson by discussion of the different ways that learners have done this. Draw their designs on the chalk board. Make a poster of the learners' work.

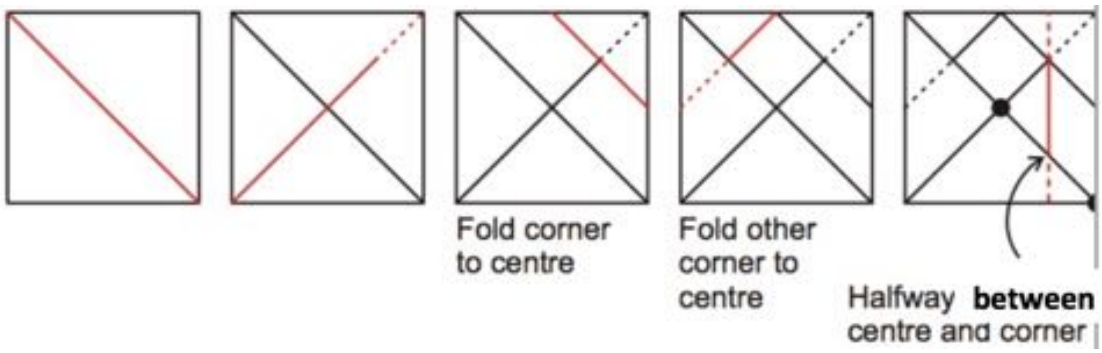
Upper Primary and Lower Secondary

Start with the diagnostic quiz (see page 7). Make sure if possible that all the learners understand that there are 16 small squares making up an area of 1 square unit so the area of each small square is $\frac{1}{16}$. The number in the denominator (16) names the number of equal parts into which the whole unit has been split. Different fractions are made up by combining some of the sixteenths ($\frac{1}{16}$ ths)

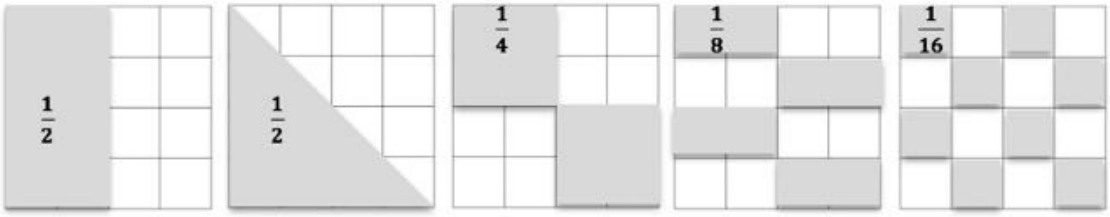
For example, if you have 3 of the small squares you have three sixteenths ($\frac{3}{16}$ ths)

Give each learner a square of paper. Explain and demonstrate step by step how to fold the paper to make the 7 tangram pieces without any measuring. Each fold-line gives an edge for one of the pieces. When you make each fold ask the learners "What is being halved by this fold?"

Follow the instructions in the video or the instructions below. In each diagram the red line shows the fold to make. Help the learners to make their own Tangram Puzzle pieces and cut out the 7 pieces.



The area of the whole Tangram square is taken to be 1 square unit. **Before you ask** the learners to find the areas of each of the Tangram pieces and fractions of the whole, first discuss the areas and the fractions in the diagram below

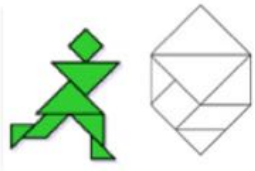


Then give the learners time to work in pairs and decide on the fractions for each of the 7 tangram pieces.

In a whole class discussion ask the learners to tell you the fractions for the 7 pieces and draw the diagram as given in the solution section writing in the fractional parts or areas. Ask the learners to name the shapes. Ask about the angles. Perhaps ask which shapes are similar.

Play the Tangram Fractions Game (see page 1). Split the class into two teams to play this game. As a lesson starter it will take less than 5 minutes and it can be repeated in many lessons to give practice in adding fractions.

When each learner has cut out their 7 pieces ask them to put the pieces together again to make the original square. Then ask them to make a smaller square from pieces A, G, C and E.



Draw this little green tangram boy on the chalk board and ask the learners to arrange their pieces to give the same picture of the boy running along. Draw the tangram hexagon on the board and ask the learners to arrange their pieces to give the same picture.

The man with an umbrella is another puzzle for learners to try.

Also encourage learners to be creative and make their own tangram people and other pictures.

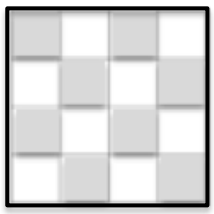
Key questions

- What are the angles?
- What angle do you need to fit in there?
- Does the diagonal (fold line) split the square (triangle) into two equal parts?
- What is the same and what is different about these two shapes?

DIAGNOSTIC QUIZ This should take about 5–10 minutes.

Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The outer square has area 1 square unit.
What is the area of **one** of the grey squares?

A. $\frac{1}{8}$

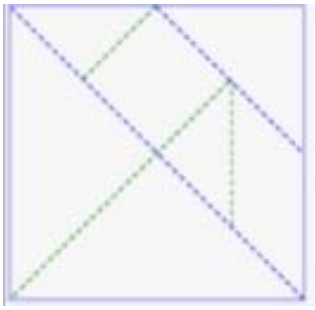
B. $\frac{1}{16}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

The correct answer is: B

Upper Secondary

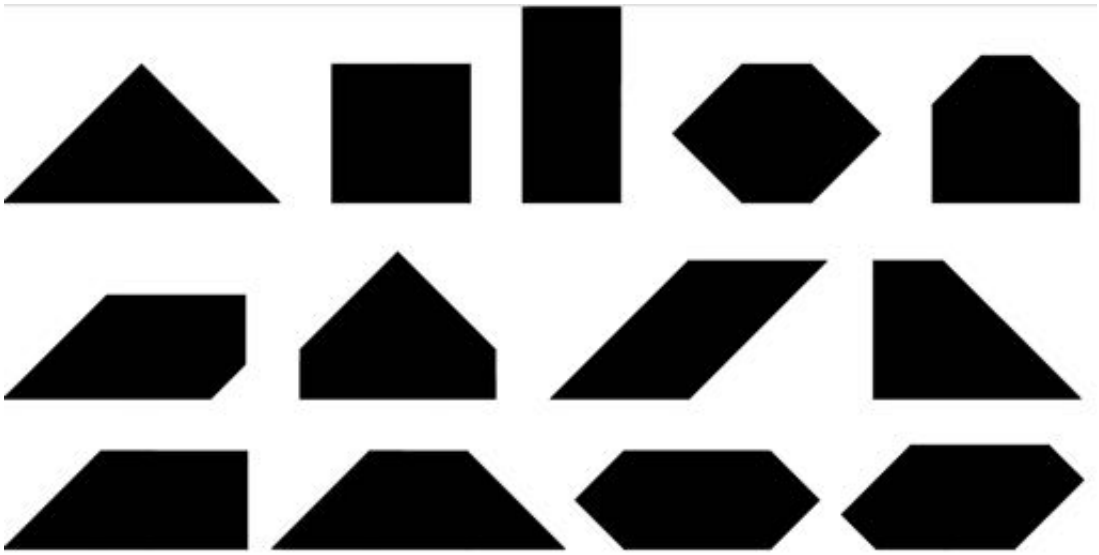


Fold a square of scrap paper or card and cut out the 7 pieces of the Tangram Puzzle accurately without any measuring. See the video https://youtu.be/1l_U31PIfLQ

Can you make this little man with the umbrella?

Tangram 2D Shapes https://aiminghigh.aimssec.ac.za/tangram-2d-shapes/

A Bigger Challenge: In a **convex polygon** all the interior angles are less than 180°. Only 13 convex polygons can be made using all 7 tangram pieces. Can you find them? As a challenge for the teacher, can you prove that it is only possible to make 13 convex polygons and no more, or find the proof on the internet and understand it?

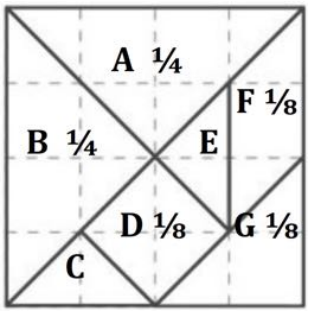


Why do this activity?

This activity can be described a 'low entry point – high ceiling' problem making it suitable for a wide age range and for low and high attaining learners. The area of a triangle is introduced in an informal way as half the area of the square when the diagonal of the square is drawn.

This activity gives good practice in working with areas and fractions and adding fractions to check that the total area is 1 square unit. It also gives practice with angles. Young learners in primary school can find the arrangements of the Tangram Puzzle pieces to make the little people, and they can work out the areas of the 7 pieces. Finding the area of the square made from 4 pieces takes lower secondary learners into working with surds. Perhaps every learner can find some polygons and maybe between them find all 13 convex polygons. Proving that exactly 13 convex polygons can be made and no more is beyond the scope of school mathematics.

SOLUTION



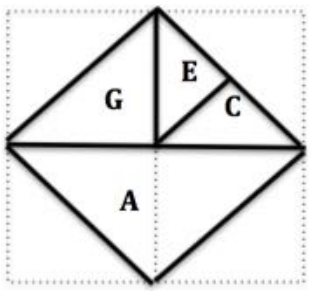
Draw the tangram on squared paper to help in finding the areas. The 16 small squares each have an area of 1/16.

The edge of the square is 1 unit. The area of the big square is 1 square unit.

Pieces C and E are made up of 2 half squares so they have an area of 1/16.

Pieces D, G and F are each made up of 2 small squares and have an area of 2/16 or 1/8.

Area C + Area E + Area D + Area G + Area F + Area A + Area B
= 1/16 + 1/16 + 1/8 + 1/8 + 1/8 + 1/4 + 1/4 = 1 square unit

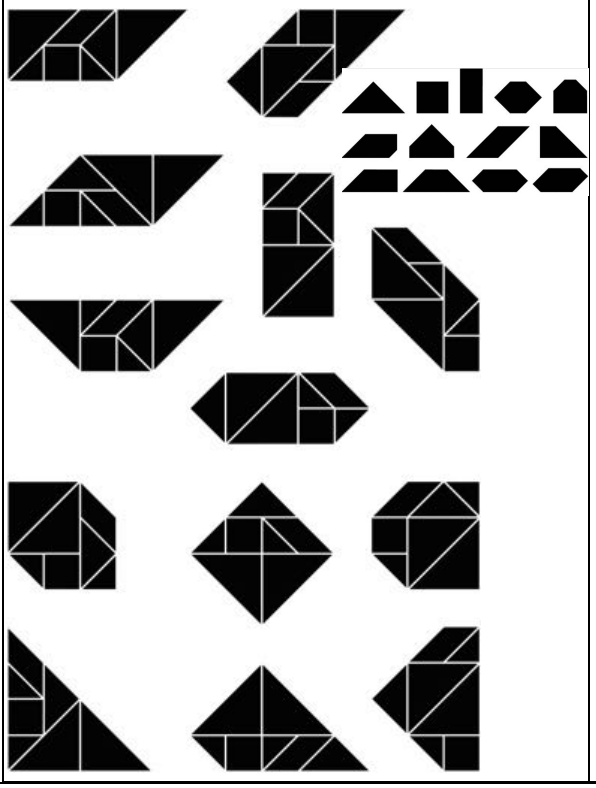


The diagram shows a square made up from pieces A, G, C and E. The area of the square is 1/2 and the edge length is $1/\sqrt{2}$

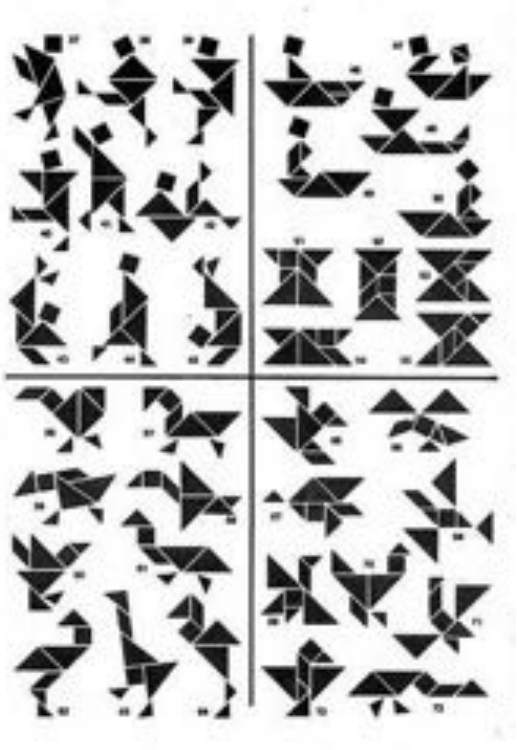
The diagram below on the left shows the 13 convex polygons that can be constructed using the 7 tangram pieces.

The diagram on the right gives solutions to some tangram puzzles. Many more of these puzzles and their solutions can be found by doing a Google search.

Solutions for the 13 convex polygons.



Solutions for a miscellaneous set of tangram pictures. Try them.



Learning objectives

In doing this activity students will have an opportunity to:

- practise finding areas from thinking about splitting squares rather than using formulae;
- make sense of fractions by thinking about dividing up 1 square unit.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically, reason logically and give explanations;
- think flexibly, be creative and innovative and apply knowledge and skills;
- visualize and develop the skill of interpreting and creating visual images to
- represent concepts and situations;
- persevere and work systematically to investigate all possible cases.

Follow up

Making the complete set of 13 polygons is a good follow up activity. Challenge the learners to make as many different convex polygons as they can using all of the 7 pieces.

You could make a poster for the classroom wall and paste up each polygon as one of the learners finds a new one. With encouragement the learners could go on searching for new polygons for a long period. When most have been found, the teacher can give hints about what shapes to look for to find them all.

See also: Tangram 2D Shapes
https://aiminghigh.aimssec.ac.za/tangram-2d-shapes/
Tangram Pattern
https://aiminghigh.aimssec.ac.za/tangram-pattern/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13