AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

Sweets

SAME SWEETS

Six bags of sweets each contain green, white, yellow, orange and red sweets with equal numbers of each colour (5 flavours). You pick sweets from different bags without looking.

If you pick 2 sweets what different combinations of colours can you get?

If you pick 2 sweets how likely are you to pick two of the same colour?

If you pick 6 sweets what is the probability that two are the same colour?

HELP

One of the 'golden rules' of problem solving is to work on simple cases when a problem seems difficult.

If you find this problem difficult first solve the simpler problem for a bag of sweets with only 2 colours. Ask the same questions.

If you pick 2 sweets how likely are you to pick two of the same colour?

If you pick 6 sweets what is the probability that two are the same colour?

After that, progress to a bag of sweets with three colours and solve the same problem. Then solve the problem for a bag of sweets with 4 colours.

You should then easily be able to solve the problem for 5 colours.

NEXT

What is the probability that, in a group of 6 people, two people have birthdays in the same month?

What if the group has 7 people?

What if the group has more than twelve people?

https://aiminghigh.aimssec.ac.za/same-birth-month/

NOTES FOR TEACHERS

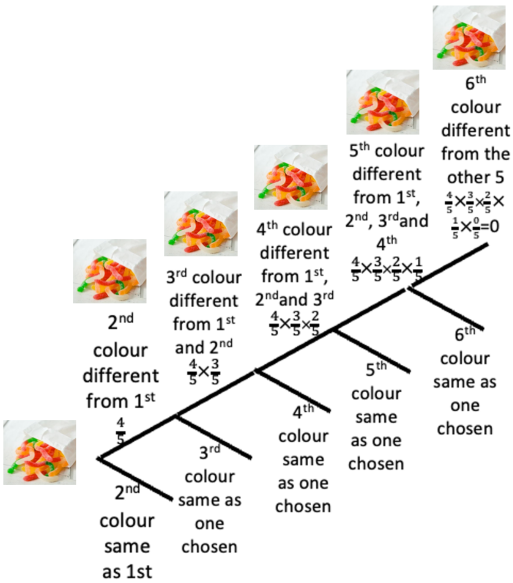
SOLUTION

There are 25 different possible combinations of colours with 5 pairs the same colour. (Key. G denotes green, W for white, O for orange, y for yellow, R for red)

WW	OW	YW	GW	RW
W0	OO	YO	GO	RO
WY	OY	YY	GY	RY
WG	OG	YG	GG	RG
WR	OR	YR	GR	RR

If you pick 2 sweets the probability that two sweets are the same colour is

$\frac{5}{25} = \frac{1}{5} = 0.2$



At each fork in the tree diagram, **either** all the sweets chosen are different colours **or** there are two of the same colour.

If you pick 6 sweets then there must be two of the same colour, the probability of two of the same colour is 1.

This is called the Pigeon Hole Principle. If the first five sweets picked are all different colours, then the sixth sweet must be one of those colours. because they are the only 5 possible colours.

<table>
<tr><td rowspan="6">The probability that an event does not happen
= 1 – probability it does happen.

In the last two columns one probability is 1 minus the other.</td><th>Number of sweets picked</th><th>Probability that all sweets picked are different colours.</th><th>Probability that two sweets picked are the same colour</th></tr>
<tr><td>2</td><td>$\frac{4}{5}=0.8$</td><td>0.2</td></tr>
<tr><td>3</td><td>$\frac{4}{5}\times\frac{3}{5}=0.48$</td><td>0.52</td></tr>
<tr><td>4</td><td>$\frac{4}{5}\times\frac{3}{5}\times\frac{2}{5}=0.192$</td><td>0.818</td></tr>
<tr><td>5</td><td>$\frac{4}{5}\times\frac{3}{5}\times\frac{2}{5}\times\frac{1}{5}=0.0384$</td><td>0.9616</td></tr>
<tr><td>6</td><td>0</td><td>1</td></tr>
</table>

Why do this activity?

This learning activity provides a context to which learners can easily relate where they can discover answers for themselves and not need to use a formula. Learners have to work systematically to find out that there are 25 different colour combinations. This could be where you stop with Years 4 or 5.

Each colour combination for 2 sweets is equally likely. With less information than given on this sheet you would need to discuss with the class whether it is reasonable to assume this.

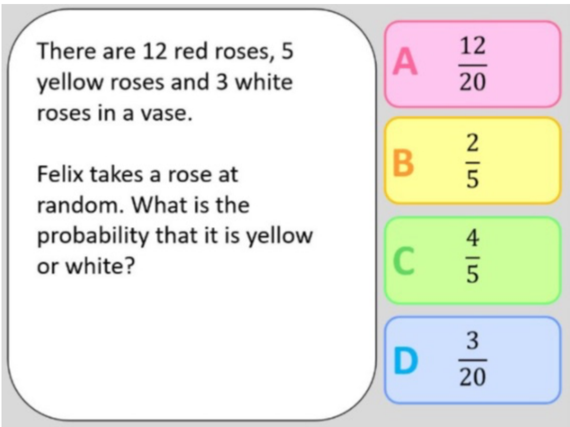
With a little guidance from the teacher learners can then experience the thinking about probability that attaches a value of 1/25 = 0.04 or 4% to the probability for each colour combination. Given enough time to explore and discuss the idea, most learners in Years 6 or 7 will be able to make the next step to realising that the probability of picking 2 sweets the same colour is 5/25 = 0.2 = 20%.

The reasoning in this problem is exactly like the reasoning for solving the famous Birthday Problem and high fliers can be introduced to the Birth---month Problem (see NEXT) and possibly to the Birthday Problem.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes at the end of the lesson

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is: **B** is the correct answer. Eight of the roses are yellow or white so the probability is 8/20 = 2/5

Common Misconceptions

A. Learners have chosen the probability of picking a red rose. They have not understood the question.

C. They have not understood the question.

D. Learners have chosen the probability of picking a white rose.

https://diagnosticquestions.com

Learning objectives

In doing this activity students will have an opportunity to:

- learn to plan and work systematically to find answers and to record results;
- develop an understanding of probability;
- improve their problem solving skills.

Generic competences

In doing this activity students will have an opportunity to:

- learn to plan and work systematically to find answers and to record results;
- improve their problem solving skills;
- Develop team working and communication skills.
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Suggestions for teaching

Much the most important learning experience for learners in doing this activity is to discover, without being told, that they need to record their results systematically in order to be sure that they have found all possible cases. They should be given time to do this.

It is important to let learners find out for themselves how many colour combinations there are and find their own way to represent and explain it. Ideally they will discover a neat way to organise the representation in something like the 5 by 5 layout above but the teacher should let the learners discover this for themselves. At first it is likely that they will just record all the different colour combinations they can think of in no special order. Give the class plenty of time to discuss how to record the results to be sure they have found all possible combinations. This works well for a '**One – two – four – more'** lesson where learners work individually, then in pairs, then in fours, then the whole class discusses the problem with learners presenting their ideas on the chalkboard.

Key question 7 requires an application of the Pigeon-hole principle but again the teacher should not give the rules at the start but only at the end of the lesson when the learners have discovered the logic behind the concepts for themselves.

In summary, at the end of the lesson:

RULES OF PROBABILITY

1. All events have a probability between 0 and 1.
2. The probability of an event NOT happening is:

   1 – the probability that the event happens.
3. The probability of an event is the fraction:

$$\frac{\text{the number of ways the event can happen}}{\text{the total number of different possible events}}$$

4. **PIGEON HOLE PRINCIPLE, explained by an example**: If there are 30 post boxes, and more than 30 letters are put in the boxes, then at least one box must have 2 or more letters in it.

Key questions

1. Can you find a neat way to record all the colour combinations?
2. Are you sure that you have found all the possibilities? How do you know?
3. Are all the different combinations equally likely? How do you know?
4. What is the probability of each colour combination?
5. How many combinations are there with 2 sweets the same colour?
6. How likely is it that you pick 2 sweets the same colour?
7. If you picked 6 sweets what is the probability that there would be 2 the same colour?

Follow up

Same Birth Month https://aiminghigh.aimssec.ac.za/same-birth-month/
Same Birthday https://aiminghigh.aimssec.ac.za/same-birthday/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13