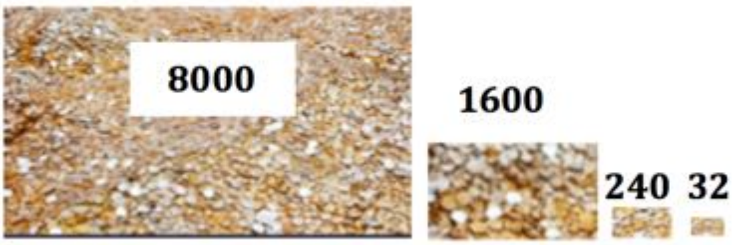
'SHARING' is the theme
for this INCLUSION AND HOME LEARNING GUIDE
This Guide suggests related learning activities for all ages from 4 to 18+
Just choose whatever seems suitable for your group of learners.

MAGIC NUMBERS

$9 - 1 =$
$98 - 2 =$
$987 - 3 =$
$9876 - 4 =$
$98765 - 5 =$
$987654 - 6 =$
$9876543 - 7 =$
$98765432 - 8 =$
$987654321 - 9 =$

Work out these subtractions.
Then divide each answer by 8.
What patterns do you see?

Suppose you share 9872 gold coins between 8 people.
You split them into 4 piles, as shown in the picture, chosen so that each pile is a multiple of 8.
Then you share each of the piles between the 8 people.
You give each person 1000 + 200 + 30 + 4 coins.



Use the coin sharing story to explain the division sum:

$$9872 \div 8 = 1000 + 200 + 30 + 4 = 1234$$

The brackets in these calculations tell you that you must do the calculation inside the brackets **first**.

Using the subtraction calculations that you have already done can you complete all of these calculations without using a calculator?

What patterns do you notice?

Why do you think that the ancient Egyptians called these numbers *magic numbers*?

$(9 - 1) \div 8 =$
$(98 - 2) \div 8 =$
$(987 - 3) \div 8 =$
$(9876 - 4) \div 8 =$
$(98765 - 5) \div 8 =$
$(987654 - 6) \div 8 =$
$(9876543 - 7) \div 8 =$
$(98765432 - 8) \div 8 =$
$(987654321 - 9) \div 8 =$

HELP Start with this simpler activity.

Fill in the boxes below:

136 ÷ 8 = 17 because 136 = 80 + ☐

280 ÷ 8 = 35 because 280 = ☐ + 40

456 ÷ 8 = 57 because 456 = ☐ + ☐

To do the division 2760 ÷ 8 by chunking.
Split up 2760 giving ☐ + 320 + 40.
Think how many 8's in the piles of hundreds, tens and units and fill in these boxes.

8 × ☐ + 8 × ☐ + 8 × ☐

Explain how you filled in the boxes.
Then try the challenge on page 1 again. You can do it!! Say to yourself YES I CAN.

NEXT MORE MAGIC NUMBERS

This challenge is an extension to MAGIC NUMBERS
https://aiminghigh.aimssec.ac.za/magic-numbers/

Find the numbers to put in the boxes to make the calculations in LIST 1 correct.

LIST .		LIST 2		
	☐ × 8 + 1 = 9		(9 - 1) ÷ 8 =	Complete the calculations in LIST 2.
	☐ × 8 + 2 = 98		(98 - 2) ÷ 8 =	
	☐ × 8 + 3 = 987		(987 - 3) ÷ 8 =	
	☐ × 8 + 4 = 9876		(9876 - 4) ÷ 8 =	
	☐ × 8 + 5 = 98765		(98765 - 5) ÷ 8 =	
	☐ × 8 + 6 = 987654		(987654 - 6) ÷ 8 =	
	☐ × 8 + 7 = 9876543		(9876543 - 7) ÷ 8 =	
	☐ × 8 + 8 = 98 765432		(98765432 - 8) ÷ 8 =	
	☐ × 8 + 9 = 987654321		(987654321 - 9) ÷ 8 =	

What do you notice about the connections between the equations:

☐ × 8 + 1 = 9 etc.

and the calculations:

(9 - 1) ÷ 8 = ? etc. ?

What are the connections between LIST 1 and LIST 2 and **inverse operations**?

You have been doing algebra!

The equations ☐ × 8 + 1 = 9 etc. can be written in the form $8x + 1 = 9$ etc.
where the letter x represents an unknown number and $8x$ means x multiplied by 8.

When you do algebra you are asked to solve equations, that is to find the number that the letter represents.

How would you use inverse operations to solve equations like:
$8x + 1 = 9$, $8x + 2 = 98$ etc. ?

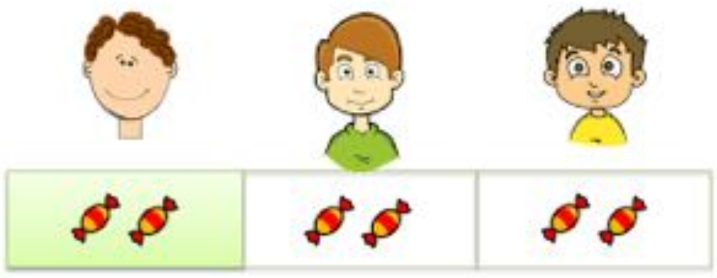
Inclusion and Home Learning Guide to a lesson for each age group on the theme of sharing.

Early Years - ALL ABOUT EQUAL SHARES

Talk about sharing and do some sharing, **not** dividing up to make fractions this time, but sharing collections of objects into **equal shares** and perhaps having some left over (**the remainder**).

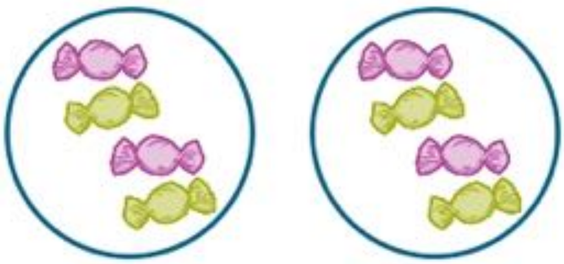
Don't make this into a lesson but rather let it be a part of everyday life.

Share a bunch of grapes between a group of children so everyone gets the same number of grapes, or share some sweets, or whatever there is to share.



How many different ways can you describe what you see in this picture?

Make up a story about this picture and then make up your own stories and draw your own pictures.



Make some play money. Then make up stories and play games that involve sharing out the money.

Collect some empty cartons and other items, set up a shop and play at shopping.

If you play games like Monopoly as a family let the children share out the money at the start.

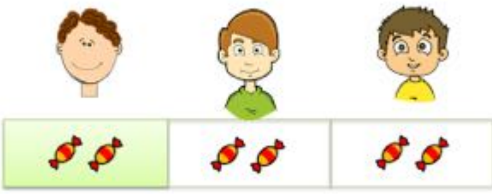
Lower Primary

Talk about sharing and do some sharing, **not** dividing up to make fractions this time, but sharing collections of objects into **equal shares** and perhaps having some left over (**the remainder**).

In school children are taught to count, to recognise numbers, to add numbers and to subtract numbers before they are taught about multiplication and division.

However, in many families and groups of young children, sharing is a very normal activity. Before starting school children understand the idea that, if there are 6 sweets to share between 3 children, each child will get 2 sweets.

If you ask questions about what you see in this picture children might say things like there are 3 lots of 2 making 6 and 6 shared 3 ways gives 2 each.

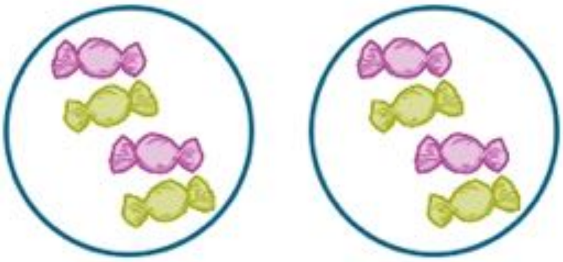


Already they can recognise that there is a connection between the concept $3 \times 2 = 6$ and the concept $6 \div 3 = 2$.

Do not introduce the formal way to write down division calculations, that will come later.

Share a bunch of grapes between a group of children so everyone gets the same number of grapes, or share some sweets, or whatever there is to share.

Ask questions like: How many different ways can you describe what you see in this picture?



In order to lay a good foundation for later work, children should make up number stories that involve sharing situations.

Your group could tell stories based on these pictures and make up their own stories and draw their own pictures to illustrate them.

Sadly, the way that children are taught about multiplication and division often makes it seem difficult and unconnected with everyday life.

You could make some play money. Then make up stories and play games that involve sharing out the money.

If you play games like Monopoly as a family let the children share out the money at the start and talk about the sharing.

Upper Primary

Tell the children that you are going to read a story and they must listen carefully because you are going to ask some questions at the end.

You and your friends and family, 8 of you altogether, were playing on the beach and you noticed that the sand had shifted to reveal something made of wood.

So, you all started digging and found a treasure chest full of gold coins. The grown-ups said that you could not keep the money, although perhaps it had been buried for a long time, but there would be no harm in counting it.

To count the gold coins, you counted out piles of a hundred gold coins.

There were 98 hundreds.

You separated them into piles of a thousand and smaller piles of a hundred.

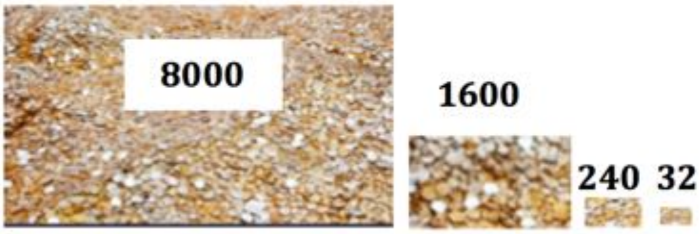
Then you counted how many were left and found there were 72.

You sorted them into piles of ten with 2 left over.

Now ask the questions. You may need to read the story again before they can answer.

How many thousands? How many hundreds? How many tens? How many units?

How many altogether?



Suppose you share the 9872 gold coins from the treasure chest between 8 people.
You split them into 4 piles, as shown in the picture, chosen so that each pile is a multiple of 8.
Then you share each of these piles between the 8 people.
How much would each person get?

Ask the children to answer the question: ‘how much would each person get?’
and then ask them to explain this method of doing the division sum 9872 ÷ 8.

The answer is: The piles of thousands, hundreds, tens and units must have a multiple of 8 thousands, hundreds, tens and units respectively.
You give each person 1000 + 200 + 30 + 4 coins
so the answer to the division sum is 1234.

Investigate: 1×1
11×11
111×111
…
111111111×111111111

As an extra activity explore this ‘Ones Galore’ pattern and explain it.

Lower Secondary

Start with the Diagnostic Quiz as a warm up.

$$9 - 1 =$$
$$98 - 2 =$$
$$987 - 3 =$$
$$9876 - 4 =$$
$$98765 - 5 =$$
$$987654 - 6 =$$
$$9876543 - 7 =$$
$$98765432 - 8 =$$
$$987654321 - 9 =$$

Lesson starter (10 minutes) Write down this pattern of subtractions and ask the learners to copy them into their notebooks, to work them out and, when they have finished, to make a note of what they have noticed.

Main lesson: The learners should do the question on page 1.

This lesson focusses on division. Talk about the gold coins sharing story. Ask the learners to talk with their partners, or in small groups, to find the best way to use the coin sharing story to explain the division sum: 9872 ÷ 8 = 1000 + 200 + 30 + 4 = 1234. After about 5 minutes have a group discussion about this.

$$(9 - 1) \div 8 =$$
$$(98 - 2) \div 8 =$$
$$(987 - 3) \div 8 =$$
$$(9876 - 4) \div 8 =$$
$$(98765 - 5) \div 8 =$$
$$(987654 - 6) \div 8 =$$
$$(9876543 - 7) \div 8 =$$
$$(98765432 - 8) \div 8 =$$
$$(987654321 - 9) \div 8 =$$

Then copy out this list of calculations. Tell the learners that the brackets tell you that you must do the calculation in the brackets first and ask them to complete the calculations.

Depending on the stage that your learners are at you could allow the use of calculators or not.

Finally, to end the lesson:

$$(9 - 1) \div 8 = 1$$
$$(98 - 2) \div 8 = 12$$
$$(987 - 3) \div 8 = 123$$
$$(9876 - 4) \div 8 = 1234$$
$$(98765 - 5) \div 8 = 12345$$
$$(987654 - 6) \div 8 = 123456$$
$$(9876543 - 7) \div 8 = 1234567$$
$$(98765432 - 8) \div 8 = 12345678$$
$$(987654321 - 9) \div 8 = 123456789$$

Write out this second list in this patterned layout, asking the learners to tell you what to fill in line by line and writing down what they tell you.

Ask the learners what they have noticed.

To help the learners to understand, and to remember, what they have learned about division and about the use of brackets, finish the lesson with a summary what they have done.

Diagnostic Assessment This should take about 5–10 minutes.

Write out the question and say to the learners:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

$$60 \div (5 \times 2) + 3$$

A	B	C	D
10	8	9	4

1. Notice how the learners respond. Ask them to explain why they gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer to practice their communication skills and sort out their thinking by putting their ideas into words.
3. Try to make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. As the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is C.

A. and **B.** Students giving these answers seem to be very poor at simple arithmetic.

D. One student explained his answer: Because 5x2 = 10 + 3 = 13 and 60 / 13 = 4

https://diagnosticquestions.com

Why do this activity?

Discovering the pleasing patterns should give this activity an appeal for learners. They will get practice in subtraction and division in an interesting way. Moreover, the activity is designed to help learners to understand three fundamental mathematical ideas: inverse operations; the process of division and how it is equivalent to 'sharing'; and how solving linear equations involves the use of inverse operations.

Learning objectives

In doing this activity students will have an opportunity to gain a better understanding of: (a) the processes of subtracting and dividing;

(b) what inverse operations are and how to use them;

(c) how to solve linear equations.

Generic competences

In doing this activity students will have an opportunity to develop the skill of interpreting, creating and visualising images to represent concepts and situations.

Key questions

- What do you notice?
- How does splitting the dividend into separate parts help you to share it between a number of people?
- How does splitting a collection of objects into separate parts naturally start the division process involved in sharing?
- Why do you think we have put brackets in those calculations?

Upper Secondary Years 10 and 11

This challenge is an extension to MAGIC NUMBERS
https://aiminghigh.aimssec.ac.za/years-7-11-more-magic-numbers/

Find the numbers to put in the boxes to make the calculations in LIST 1 correct.

LIST

$$\square \times 8 + 1 = 9$$
$$\square \times 8 + 2 = 98$$
$$\square \times 8 + 3 = 987$$
$$\square \times 8 + 4 = 9876$$
$$\square \times 8 + 5 = 98765$$
$$\square \times 8 + 6 = 987654$$
$$\square \times 8 + 7 = 9876543$$
$$\square \times 8 + 8 = 98\ 765432$$
$$\square \times 8 + 9 = 987654321$$

LIST 2

$$(9 - 1) \div 8 =$$
$$(98 - 2) \div 8 =$$
$$(987 - 3) \div 8 =$$
$$(9876 - 4) \div 8 =$$
$$(98765 - 5) \div 8 =$$
$$(987654 - 6) \div 8 =$$
$$(9876543 - 7) \div 8 =$$
$$(98765432 - 8) \div 8 =$$
$$(987654321 - 9) \div 8 =$$

Complete the calculations in LIST 2.

What do you notice about the connections between the equations:

$$\square \times 8 + 1 = 9 \text{ etc.}$$

and the calculations:

$$(9 - 1) \div 8 = ? \text{ etc. } ?$$

What do **inverse operations** have to do with the connections between LIST 1 and LIST 2?

You have been doing algebra!

The equations $\square \times 8 + 1 = 9$ **etc.** can be written in the form

$$8x + 1 = 9 \text{ etc.}$$

where the letter x represents an unknown number and $8x$ means x multiplied by 8.

When you do algebra you are asked to solve equations. That means to find the number that the letter represents.

How would you use inverse operations to solve equations like $8x + 1 = 9$
$8x + 2 = 98$ **etc.** ?

Upper Secondary Years 12 and 13

Why do some fractions have recurring decimal expansions with patterns of nines?

Observe $\frac{1}{7} = 0.\ 142857$ ... recurring. Notice that 142 + 857 = 999.

Look at the pattern in the decimal expansions for sevenths with other numerators and for thirteenths and elevenths. Do you notice any similar patterns there?

Similar patterns arise for denominators 101, 1001 etc.

For example, notice that $\frac{\mathbf{32}}{\mathbf{101}}$ ...= 0.31683168... and 31 + 68 = 99 and investigate other fractions with these denominators.

A fraction n with a decimal expansion of the special periodic form $0.a_1a_2a_3...a_kb_1b_2b_3...b_k$ recurring with $(a_1a_2a_3...a_k) + (b_1b_2b_3...b_k) = 9 ... 9 = 10^k - 1$ is given by the formula: $n = \frac{a_1a_2a_3...a_k + 1}{10^k + 1}$.

For example, for $k = 4$, suppose $a_1a_2a_3a_4 = 3251$

then $\frac{3252}{10^4 + 1} = \frac{3252}{10001} = 0.32516748...$

and 3251 + 6748 = 9999

This result can be proved in general using the sum of geometric series. See page 10.

PROOF SORTER ACTIVITY

Why do some fractions have recurring decimal expansions with patterns of nines?

Cut out the strips and put them in the correct order to give a proof of the theorem.

Theorem 1. *Suppose n has a decimal expansion of the special periodic form*

$$n = 0{\cdot}\overline{a_1 \cdots a_k b_1 \cdots b_k}$$

where $a_1 \cdots a_k + b_1 \cdots b_k = 9 \cdots 9 = 10^k - 1$. Then

$$n = \frac{a_1 \cdots a_k + 1}{10^k + 1}$$

$$= \frac{10^k a_1 \cdots a_k + [(10^k - 1) - a_1 \cdots a_k]}{10^{2k} - 1}$$

Proof. We are given that $n = 0{\cdot}\overline{a_1 \cdots a_k b_1 \cdots b_k}$ so, writing this recurring decimal as an infinite series, we have

$$n = \frac{a_1 \cdots a_k}{10^k}\left(1 + \frac{1}{10^{2k}} + \frac{1}{10^{4k}} + \cdots\right) + \frac{b_1 \cdots b_k}{10^k}\left(1 + \frac{1}{10^{2k}} + \frac{1}{10^{4k}} + \cdots\right)$$

$$= \frac{a_1 \cdots a_k + 1}{10^k + 1}.$$

$$= \frac{(10^k - 1)(1 + a_1 \cdots a_k)}{(10^k - 1)(10^k + 1)}$$

$$= \left(\frac{10^k a_1 \cdots a_k + b_1 \cdots b_k}{10^{2k}}\right)\left(\frac{1}{1 - 1/10^{2k}}\right)$$

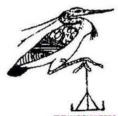
MAGIC NUMBERS SOLUTION

This is a 'pre-algebra' activity. The gold coins story is designed to help learners to understand the process of division. Completing the number patterns leads naturally to solving linear equations.

9 – 1 = 8
98 – 2 = 96
987 – 3 = 984
9876 – 4 = 9872
98765 – 5 = 98760
987654 – 6 = 987648
9876543 – 7 = 9876534
98765432 – 8 = 98765424
987654321 – 9 = 987654312

Many patterns can be observed in the solutions to these subtraction calculations. Learners need to understand the use of brackets in the calculations.

The subtractions are the first step to calculating the answers to the calculations (9 – 1) ÷ 8 = ? etc. that are given in the Extension Question 'More Magic Numbers'.



THE MAGIC NUMBERS

1 x 8 + 1 = 9
12 x 8 + 2 = 98
123 x 8 + 3 = 987
1234 x 8 + 4 = 9876
12345 x 8 + 5 = 98765
123456 x 8 + 6 = 987654
1234567 x 8 + 7 = 9876543
12345678 x 8 + 8 = 98765432
123456789 x 8 + 9 = 987654321

This illustration gives the solution to the MORE MAGIC NUMBERS CHALLENGE.

Solution to PROOF SORTER for:
Why do some fractions have recurring decimal expansions with patterns of nines?

Theorem 1. *Suppose n has a decimal expansion of the special periodic form*

$$n = 0{\cdot}\overline{a_1 \cdots a_k b_1 \cdots b_k}$$

where $a_1 \cdots a_k + b_1 \cdots b_k = 9 \cdots 9 = 10^k - 1$. Then

$$n = \frac{a_1 \cdots a_k + 1}{10^k + 1}$$

Proof. We are given that $n = 0{\cdot}\overline{a_1 \cdots a_k b_1 \cdots b_k}$ so, writing this recurring decimal as an infinite series, we have

$$\begin{aligned} n &= \frac{a_1 \cdots a_k}{10^k}\left(1 + \frac{1}{10^{2k}} + \frac{1}{10^{4k}} + \cdots\right) + \frac{b_1 \cdots b_k}{10^k}\left(1 + \frac{1}{10^{2k}} + \frac{1}{10^{4k}} + \cdots\right) \\ &= \left(\frac{10^k a_1 \cdots a_k + b_1 \cdots b_k}{10^{2k}}\right)\left(\frac{1}{1 - 1/10^{2k}}\right) \\ &= \frac{10^k a_1 \cdots a_k + [(10^k - 1) - a_1 \cdots a_k]}{10^{2k} - 1} \\ &= \frac{(10^k - 1)(1 + a_1 \cdots a_k)}{(10^k - 1)(10^k + 1)} \\ &= \frac{a_1 \cdots a_k + 1}{10^k + 1}. \end{aligned}$$

Investigate: 1×1

11×11

111×111

...

111111111×111111111

1
121
12321
1234321
123454321
12345654321
1234567654321
123456787654321
12345678987654321

This is the pattern of the products of these numbers.

Consider the product 111111 × 111111 represented in the array shown below as a sum of products of powers of 10. Each power of 10 in the final product appears on a diagonal highlighted by a different colour.

111 111 × 111 111

$= (10^{10}\times1) + (10^9\times2) + (10^8\times3) + (10^7\times4) + (10^6\times5) + (10^5\times6) + (10^4\times5) + (10^3\times4) + (10^2\times3) + (10\times2) + 1$

= 12 345 654 321

×	10^5	10^4	10^3	10^2	10	1
10^5	10^{10}	10^9	10^8	10^7	10^6	10^5
10^4	10^9	10^8	10^7	10^6	10^5	10^4
10^3	10^8	10^7	10^6	10^5	10^4	10^3
10^2	10^7	10^6	10^5	10^4	10^3	10^2
10	10^6	10^5	10^4	10^3	10^2	10
1	10^5	10^4	10^3	10^2	10	1

The pattern works in the same way when the highest power of 10 is 10^1, 10^2, 10^3, … 10^9.

The pattern breaks down when the highest power of 10 gives 10^{10}, 10^{11}… and so on when there will be more than 10 terms on the diagonals in arrays like the one shown above that represent the product, for example: $(10^{10}+10^9+10^8+\ldots10+1)\,(10^{10}+10^9+10^8+\ldots10+1)$.

Follow up

The MORE MAGIC NUMBERS challenge is an extension of the MAGIC NUMBERS activity leading to methods of solving linear equations.
https://aiminghigh.aimssec.ac.za/years-7-11-more-magic-numbers/

Also see Beautiful Numbers:
https://aiminghigh.aimssec.ac.za/years-6-8-beautiful-numbers/

MATHS TOYS

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13