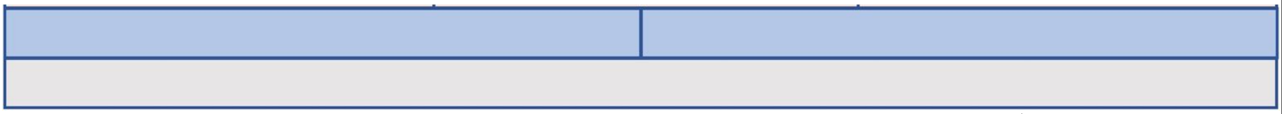
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

FRACTION WALL 30-MINUTE GLOBAL MATHS LESSON

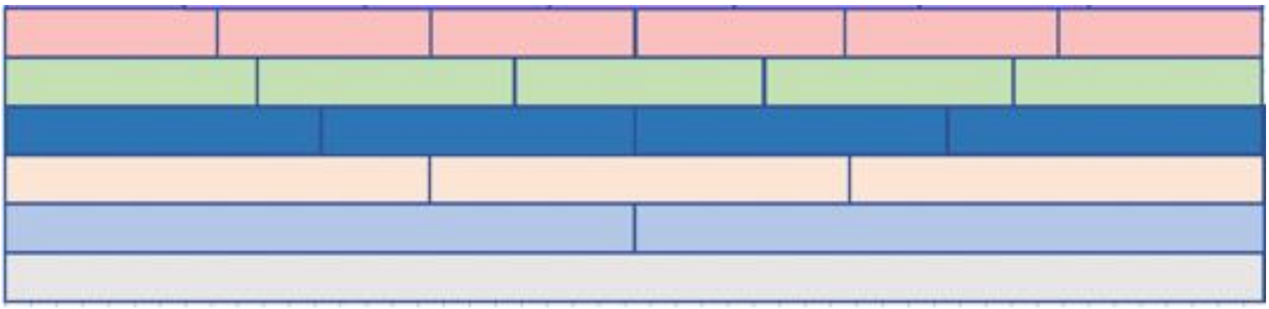
Build a Fraction Wall

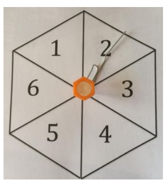


This wall is 1 unit wide. It shows one unit and two halves. Label the blocks 1 and $\frac{1}{2}$.

Cut out the strips from page 2.
Build a wall that has smaller and smaller blocks in each layer as it is built up.
Label the blocks 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$ and $\frac{1}{12}$.
Explain how the wall shows that one half, two quarters and three sixths are equivalent fractions $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$.
Explain how the wall shows that three quarters is greater than two thirds $\frac{3}{4} > \frac{2}{3}$.

Play the Fraction Wall Double Six Game





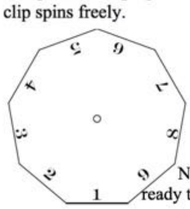
To play this starter game you need 2 dice, or a 1 – 6 spinner, and this fraction wall showing fractions 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$.
Play with 2 players or 2 teams. If you are a small group of 7 or fewer people, then everyone can play individually and, in turn, throw the dice for themselves.
Players throw 2 dice or use a spinner, make a fraction with the numbers on the dice putting the smaller number on top. For example, a 2 and a 3 make $\frac{2}{3}$.
Use the Fraction Wall to compare the fractions.
The largest fraction wins the round and scores a point. More than one player scores a point in the same round if they all get the largest fraction.

Variations of the Fraction Wall Game

If time, play with walls of different heights that include smaller fractions. You will need to make spinners that show the digits needed for your game. The instructions show how to make your own spinner with digits 1 to 9 for the Fraction Wall Game on a wall built up to the layer of ninths on top.

MAKE YOUR SPINNER
You will need a paper clip opened out as shown. Cut out the spinner. Mark sectors.

Hold the paper clip down at the centre of the spinner using a pencil so that the paper clip spins freely.



Now you are ready to play the games.

For secondary school students

Write down all the fractions you that you see from the wall are equivalent to (a) three quarters $\frac{3}{4}$ (b) five sixths $\frac{5}{6}$
(c) seven twelfths $\frac{7}{12}$ (d) six twenty-fourths $\frac{6}{24}$
Explain the rules for checking whether two fractions are equivalent.

HELP

You could print the strips, cut them out, build the wall and stick the strips onto a backing sheet, or into your notebook. Alternatively make your own wall on a larger scale from scrap cardboard or paper.

You could use a second set of separate strips so that you can match and compare the lengths of the parts of the strips (the fractions).

If you have cubes available (for example Multilink or Centicube) you can make your own fraction wall with the cubes or use Lego.

NEXT

How many red bits (twelfths) are equivalent to one blue bit (one half)?

How many red bits (twelfths) are equivalent to two cream bits (two thirds)?

Use the fraction wall to find how many units and twelfths give the answer to $\frac{1}{2}+\frac{2}{3}+\frac{5}{12}$

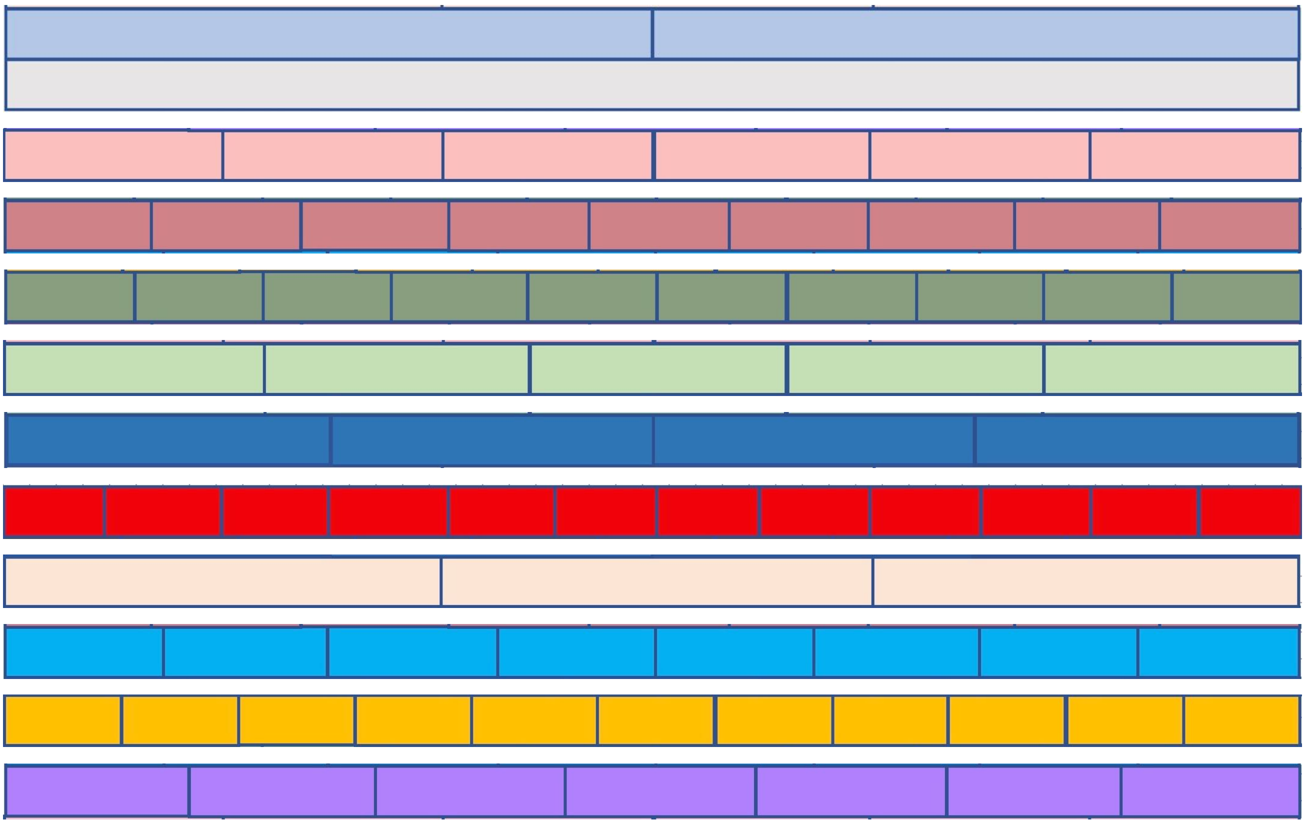
Make up some questions of your own using the fraction wall. Give answers to your questions.

Compare fractions and say which is bigger and which is smaller.

Build a Fraction Wall

Cut out the strips. Rearrange the order of the strips to build a wall with smaller and smaller blocks in each layer as it is built up.

Label the blocks 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$ and $\frac{1}{12}$.



NOTES FOR TEACHERS

Why do this activity?

This visual representation of fractions is very powerful. The main aim of the activity is for learners to develop confidence in their understanding of fractions through playing the game and connecting the visual image to the concept of equivalent fractions. Older students then deduce a "rule" (or more than one) for finding equivalent fractions without a picture and also use the image of the wall to underpin explanations of the processes of addition, subtraction, multiplication and division of fractions.

Learning objectives

In doing this activity students will have an opportunity to:

- describe and compare common fractions in diagram form;
- recognize and use equivalent forms of common fractions.

Generic competences

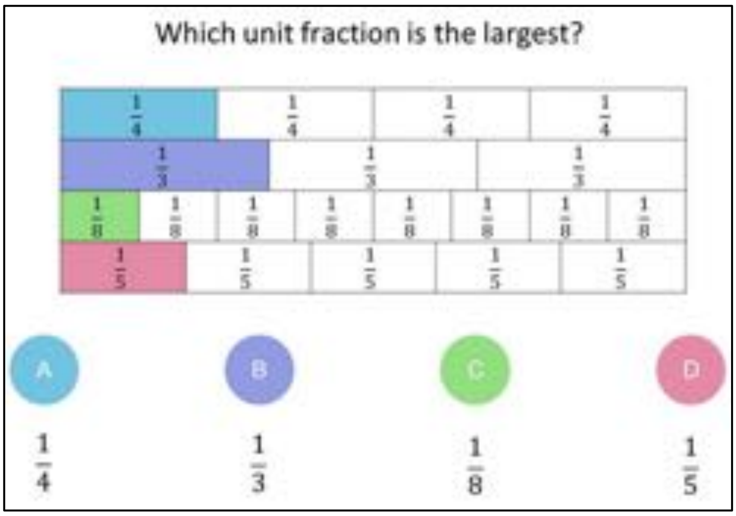
In doing this activity students will have an opportunity to **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.

Diagnostic Assessment

This should take about 5–10 minutes.

Use this diagnostic quiz at **the end of the session** to find out how much the learners have understood.
Show this question to the learners and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask all learners in your group why they gave their answers and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. Learners frequently guess at random. It is important to ask them to give reasons for their answers as it helps them to develop communication skills and it helps other learners who could not answer the question.
3. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class **again** to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

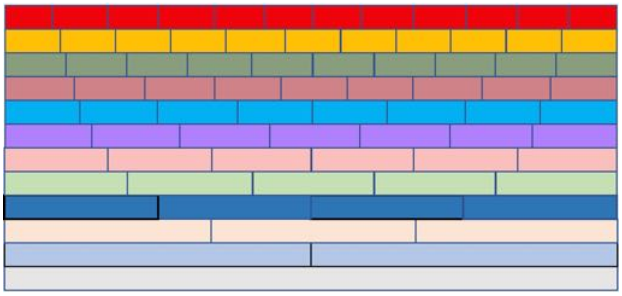
The fraction wall diagram helps learners to see that larger numbers in the denominator give smaller fractions.

B. is the correct answer.

Common Misconceptions

C. Learners may give the answer 1/8 because 8 is the largest number. https://diagnosticquestions.com

SOLUTION



In the diagram, two quarters (shown in dark blue) are the same width as one half (in light blue) showing $\frac{2}{4} = \frac{1}{2}$. The wall shows that three sixths, four eights, six twelfths and twelve twenty-fourths all belong to the set of fractions equivalent to one half.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12} = \frac{12}{24} = \cdots$$

There are more equivalent fractions in the set but, for now we are focussing on this fraction wall. Similarly:

three quarters $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{18}{24}$

five sixths $\frac{5}{6} = \frac{10}{12} = \frac{20}{24}$

seven twelfths $\frac{7}{12} = \frac{14}{24}$

six twenty-fourths $\frac{6}{24} = \frac{3}{12} = \frac{1}{4}$

To find an equivalent fraction multiply the top and bottom of the fraction by the same number, or divide the top and bottom of the fraction by the same number. This process is called **cancelling.** For example $\frac{9}{24} = \frac{6}{16} = \frac{3}{8}$ are all equivalent fractions.

Dividing the top and bottom of the fraction $\frac{9}{24}$ by the common factor 3 gives the fraction $\frac{3}{8}$.

Dividing the top and bottom of the fraction $\frac{6}{16}$ by the common factor 2 gives the fraction $\frac{3}{8}$.

A fraction like $\frac{3}{8}$ (called a fraction in its lowest terms) has no common factor between the numerator and denominator. It is equivalent to infinitely many other fractions that all have the same value. The set of all such fractions is called an equivalence class.

Looking at the fraction wall you see that the red bits are twelfths. To add fractions, all the fractions need to have the same denominator.

For example: $\mathbf{\frac{1}{2} + \frac{2}{3} + \frac{5}{12}}$ will be the same if you replace one half (one light blue bit) by 6 twelfths (6 red bits) and replace 2 thirds (2 cream bits) by 8 twelfths (8 red bits).

$\frac{1}{2} + \frac{2}{3} + \frac{5}{12} = \frac{6}{12} + \frac{8}{12} + \frac{5}{12} = \frac{21}{12}$.

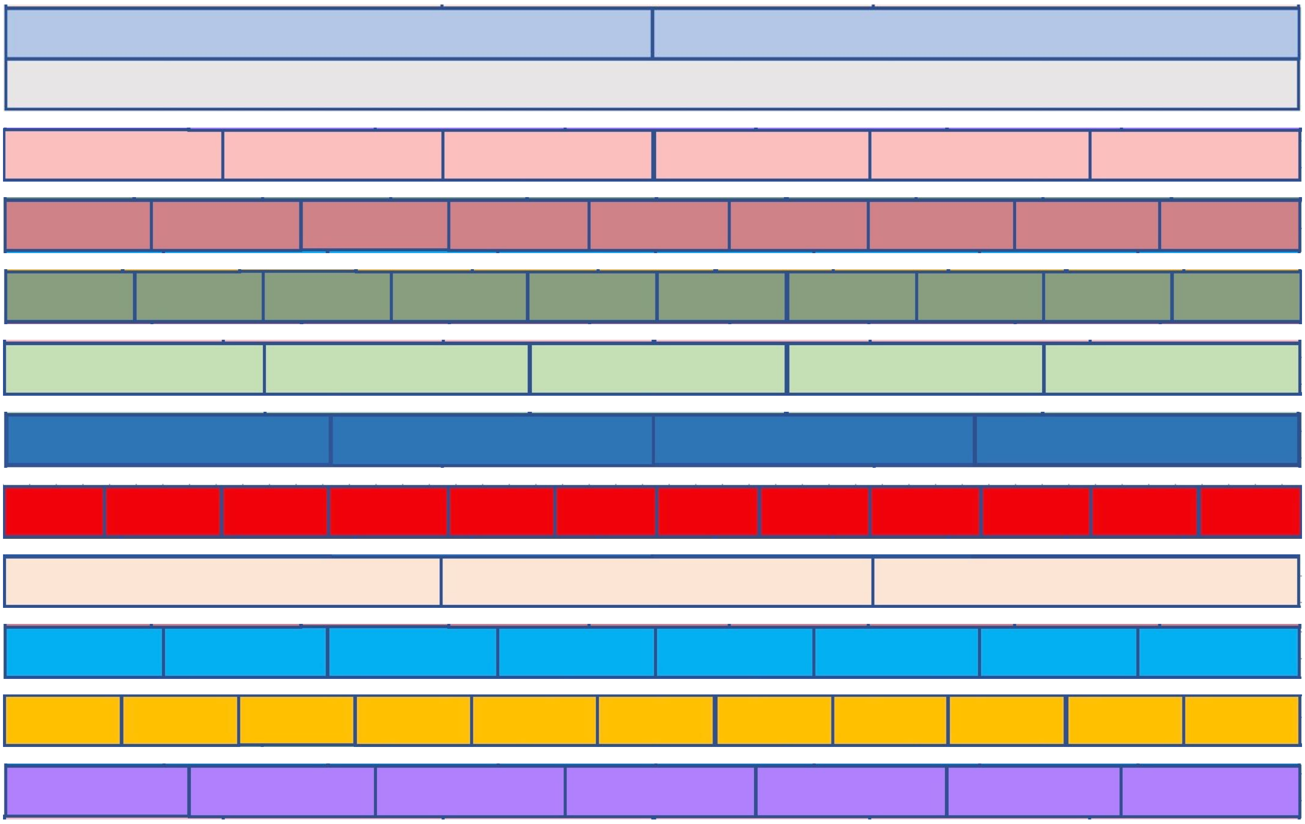
So $\mathbf{\frac{1}{2} + \frac{2}{3} + \frac{5}{12}}$ is equivalent to 21 twelfths (red bits) made up of (12+9 twelfths). In the Fraction Wall that is 1 grey unit bar and 9 red bits or 3 dark blue bits (9 twelfths or three quarters). So $\mathbf{\frac{1}{2} + \frac{2}{3} + \frac{5}{12} = 1\frac{3}{4}}$.

Use the Diagnostic Quiz for formative assessment.

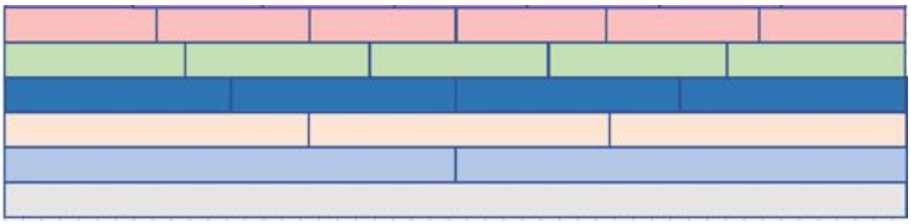
Use the HELP and NEXT sections where they are suitable for your students.

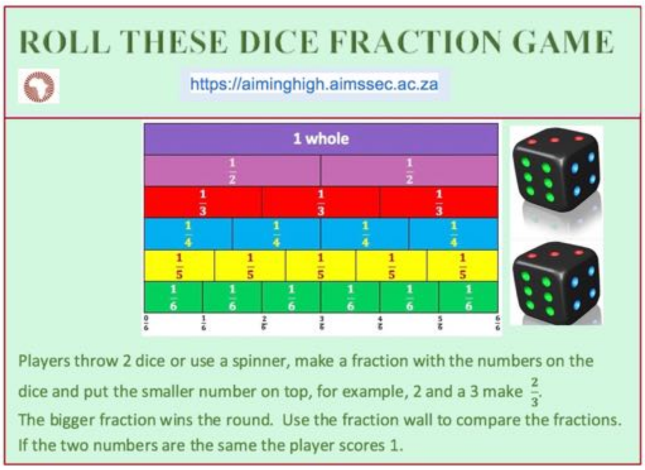
Suggestions for teaching

Introduce the Fraction Wall as a puzzle. You have to assemble the strips in order to build the wall with smaller and smaller fractions in each successive layer as described in the worksheet.



Choose a wall to suit your class. Younger learners should start playing the game with the wall showing sixths as the smallest fractions. They should fill in the fractions in each of the blocks.





Show the learners this picture and ask them to compare their answers. Having the wall this way up makes it natural to mark the scale of sixths along the bottom edge which is helpful when comparing the fractions.

Play the Fraction Wall Double Six Game often at the start of lessons for about 5 minutes.

To make the game more challenging, gradually build up more layers in the fraction wall. You will need to make spinners with the numbers required.

The rule for division of fractions is often used without understanding of the process.
Ask the learners to suggest ways of simplifying the expression two thirds divided by five eighths $\frac{2/3}{5/8}$.
Remind the learners that, if you ***multiply the top and bottom of a fraction by the same number,*** you get an equivalent fraction.
Suggest that learners simplify $\frac{2/3}{5/8}$ by multiplying the top and bottom by 24 to get rid of the fractions. This gives:
$\frac{2/3}{5/8} = \frac{2/3 \times 24}{5/8 \times 24} = \frac{2\times 8}{3\times 5} = \frac{2}{3} \times \frac{8}{5} = \frac{16}{15}$.
The same method applies to division of all fractions including fractions involving algebraic expressions.

Key Questions

- What do you notice about the lengths of the different bits in the picture?
- How many pink bits are there?
- How many of the orange bits match the unit (grey bit)?
- If the grey bit is one unit then what fraction is a purple bit?
- How many of the bits of this colour match how many bits of that colour?

Follow up

Primary and Lower Secondary

Chocolate fractions https://aiminghigh.aimssec.ac.za/chocolate-fractions/
Fractions by Halves https://aiminghigh.aimssec.ac.za/fractions-by-halves/
Fractions by Thirds https://aiminghigh.aimssec.ac.za/fractions-by-thirds/
Tangram Fractions https://aiminghigh.aimssec.ac.za/tangram-fractions/
Divide Divide https://aiminghigh.aimssec.ac.za/divide-divide/
Repetition https://aiminghigh.aimssec.ac.za/repetition/
Egyptian Fractions https://aiminghigh.aimssec.ac.za/egyptian-fractions/
The Greedy Algorithm https://aiminghigh.aimssec.ac.za/the-greedy-algorithm/

Upper Secondary

Peaches https://aiminghigh.aimssec.ac.za/peaches/
GP Algebraically https://aiminghigh.aimssec.ac.za/gp-algebraically/
GP Geometrically https://aiminghigh.aimssec.ac.za/gp-geometrically/

Years 11 to 13 FILLING AN INTERVAL WITH INFINITELY MANY BITS

This is an exercise about thinking mathematically and using your imagination and visualisation skills to work on an abstract concept.

Think of the fraction wall idea. Imagine starting with a bar precisely 1 unit long, then imagine laying a sequence of bars end to end each one precisely half the length of the previous bar. Imagine repeating this process on and on for ever. What would the lengths of all the bars add up to?

Draw a sketch of this process as follows:

1. Imagine a line segment AB exactly 2 units long. Start at A. Each time you will move towards the endpoint B, half-way along the gap between where you are and B. Imagine continuing this process indefinitely, on and on for ever.
2. Start at A. Imagine moving a distance 1 unit towards B and make a mark.
3. Next imagine moving a distance $\frac{1}{2}$ unit towards B and make a second mark. What is the distance of this mark from B?
4. Imagine moving a distance $\frac{1}{4}$ unit and make a mark. What is the distance of this mark from B?
5. Imagine the next step moving a distance $\frac{1}{8}$ unit and make a mark, then $\frac{1}{16}$ unit and make a mark.
6. Imagine continuing this process, moving distances $\frac{1}{32}=\frac{1}{2^5}$, then $\frac{1}{64}=\frac{1}{2^6}$, … $\frac{1}{2^n}$, and so on.

What happens in the limit as n tends to infinity?

This demonstrates the sum of the infinite geometric series with first term 1 and common ratio $\frac{1}{2}$.

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\frac{1}{2^n}+\frac{1}{2^{n+1}}+\cdots=2.$$

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13