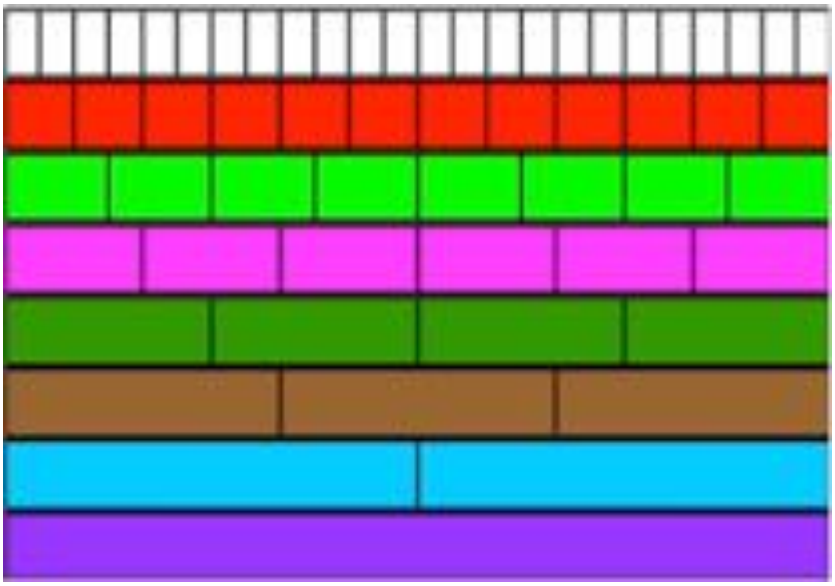
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

THE FRACTION WALL and FRACTION WALL GAME Inclusion and Home Learning Guide provides related activities for lessons in school and for home learning, for all ages and learning stages from pre-school to school-leaving. Common Theme FRACTIONS.
Choose what seems suitable for the age or attainment level of your learners.
https://aiminghigh.aimssec.ac.za/fraction-wall-game/

FRACTION WALL



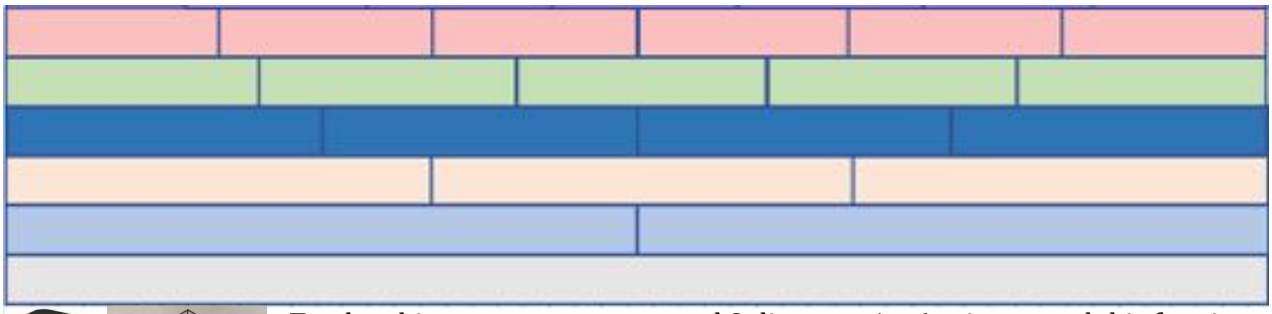
This wall is 1 unit wide.
What fractions does it show?

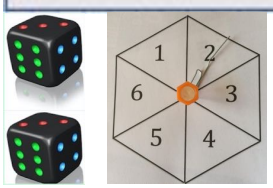
Explain how the wall shows that two thirds and four sixths and sixteen twenty-fourths are equivalent fractions $\frac{2}{3} = \frac{4}{6} = \frac{16}{24}$.
Which other fractions belong to the set of fractions equivalent to two thirds?

Write down all the fractions you that you see from the wall are equivalent to:
(a) one half $\frac{1}{2}$ (b) ten twelfths $\frac{10}{12}$ (c) six eighths $\frac{6}{8}$ (d) six twenty-fourths $\frac{6}{24}$
Explain the rules for checking whether two fractions are equivalent?
How many white bits (twenty fourths) are equivalent to one blue bit (one half)?
How many white bits (twenty fourths) are equivalent to ten two brown bits (two thirds)?
Use the fraction wall to find how many purple and dark green bits give the answer to

$$\frac{1}{2} + \frac{10}{12} + \frac{3}{8}$$

PLAY THE FRACTION WALL DOUBLE SIX GAME





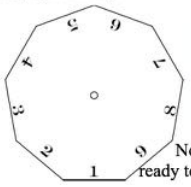
To play this starter game you need 2 dice, or a 1 – 6 spinner, and this fraction wall showing fractions 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$. Play with 2 players or 2 teams. If you are a small group of 7 or fewer people, then everyone can play individually and, in turn, throw the dice for themselves. Players throw 2 dice or use a spinner, make a fraction with the numbers on the dice putting the smaller number on top.

For example, a 2 and a 3 make $\frac{2}{3}$. Use the Fraction Wall to compare the fractions. The player (or players) with the largest fraction win that round and score a point.

Variations of the Fraction Wall Game

MAKE YOUR SPINNER
You will need a paper clip opened out as shown. Cut out the spinner. Mark sectors.

Hold the paper clip down at the centre of the spinner using a pencil so that the paper clip spins freely.



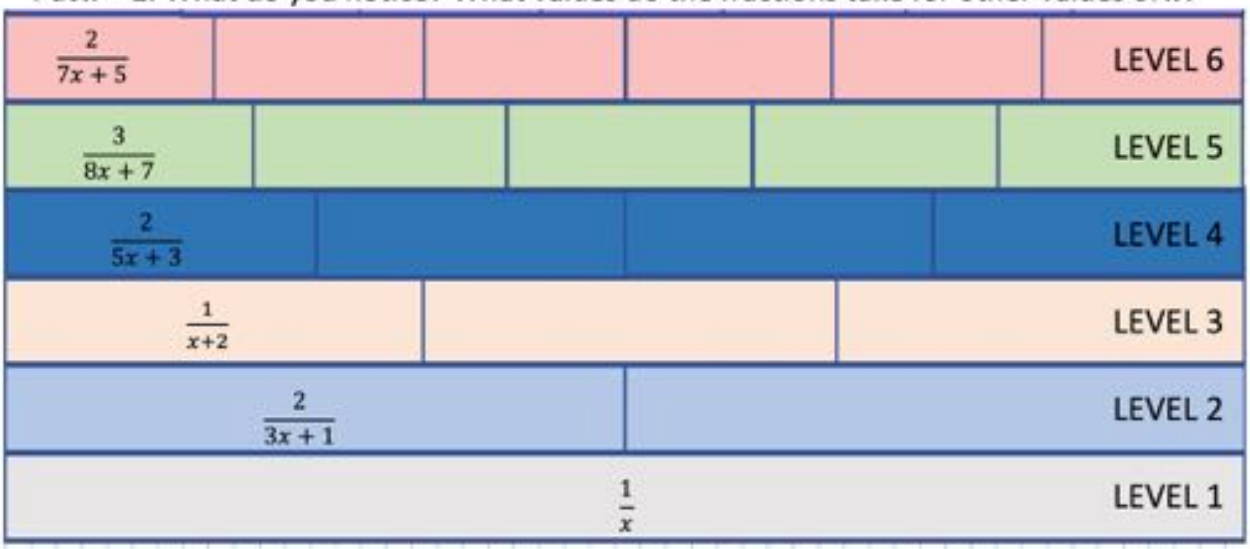
Now you are ready to play the games.

Play with walls of different heights that include smaller fractions or play the game with the Algebraic Fraction Wall. You will need to make spinners that show the digits needed for your game. The instructions show how to make your own spinner with digits 1 to 9 for the Fraction Wall Game on a wall built up to the layer of ninths on top.

The rules for the **Algebraic Fraction Wall Game** are similar. Each player throws 2 dice or spins a 1 to 6 spinner twice. The value of x is given by one die or the first spin. The level is given by the other die or the second spin. For example, if Player A throws a 5 followed by 2 (x = 5 and LEVEL 2) and Player B throws 4 followed by 5 (x = 4 and LEVEL 5) then Player A has the larger fraction and wins that round because $\frac{2}{16} = \frac{1}{8} > \frac{3}{39} = \frac{1}{13}$.

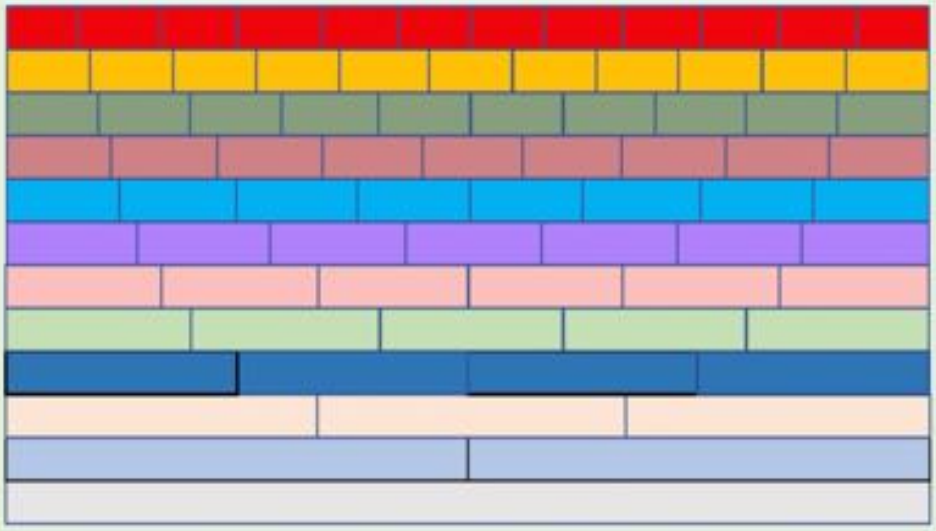
The Algebraic Fraction Wall

Put x = 1. What do you notice? What values do the fractions take for other values of x?



FRACTION WALL GAME – ADD 3 FRACTIONS VERSION

https://aiminghigh.aimssec.ac.za/fraction-wall-game/



The grey bar is one unit. The red bars are twelfths.
Label all the bars on the wall with the fractions represented.
Write 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$ on 7 identical cards.
Put the 7 number cards in a bag. Play the game.

Each player picks 3 cards at random from the bag and adds the fractions using the wall to check answers. The highest total wins the round.

If you want to make this version of the game easier, then play with 6 cards leaving out the one eighth. Then, when everyone finds the addition calculations easy because you only need to work with denominator 12, bring in the one eighth card which introduces calculations with the denominator 24.

HELP

You could make your own walls from scrap cardboard or paper.

Cut a copy of the wall into strips so re-building the wall becomes a puzzle. You can also match the lengths of parts of the strips (the fractions) to compare the sizes of factions.

If you have cubes available (for example Multilink or Centicube) you can make your own fraction wall with the cubes.

NEXT

Make up some questions of your own that use the fraction wall. Give answers to your questions.

Compare fractions and say which is bigger and which is smaller.

LESSON STARTER FOR ALL AGES

This is a starter wall.

Write the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ on the building blocks.

Play the FRACTION WALL DOUBLE SIX GAME

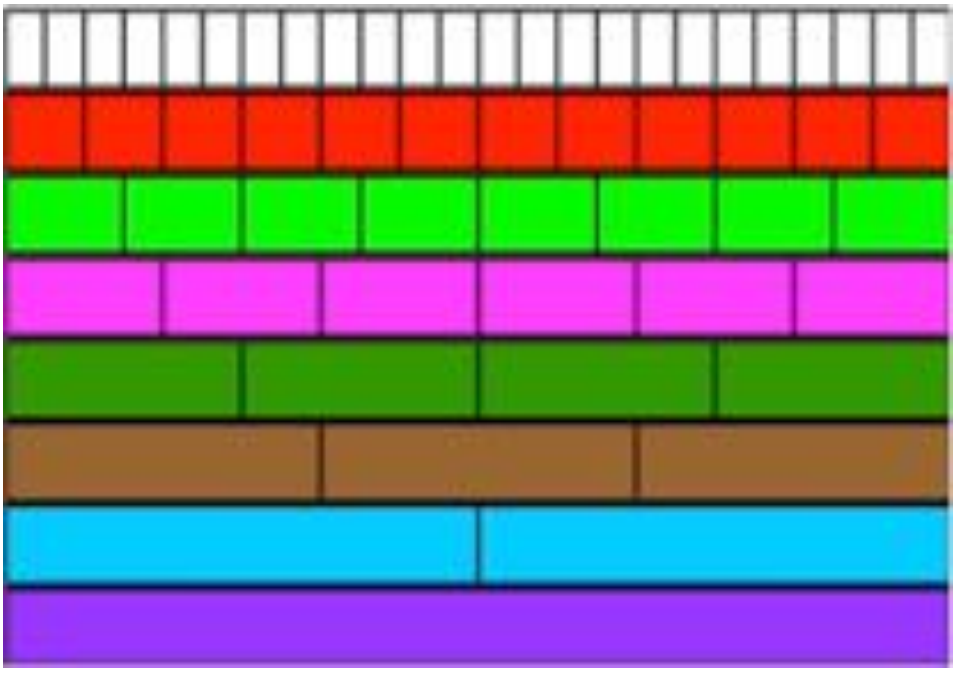
FRACTION WALL GAME DOUBLE 6

https://aiminghigh.aimssec.ac.za/fraction-wall-game/

Play with 2 players or 2 teams or alternatively with up to 7 players around a table and everyone throwing the dice in turn.

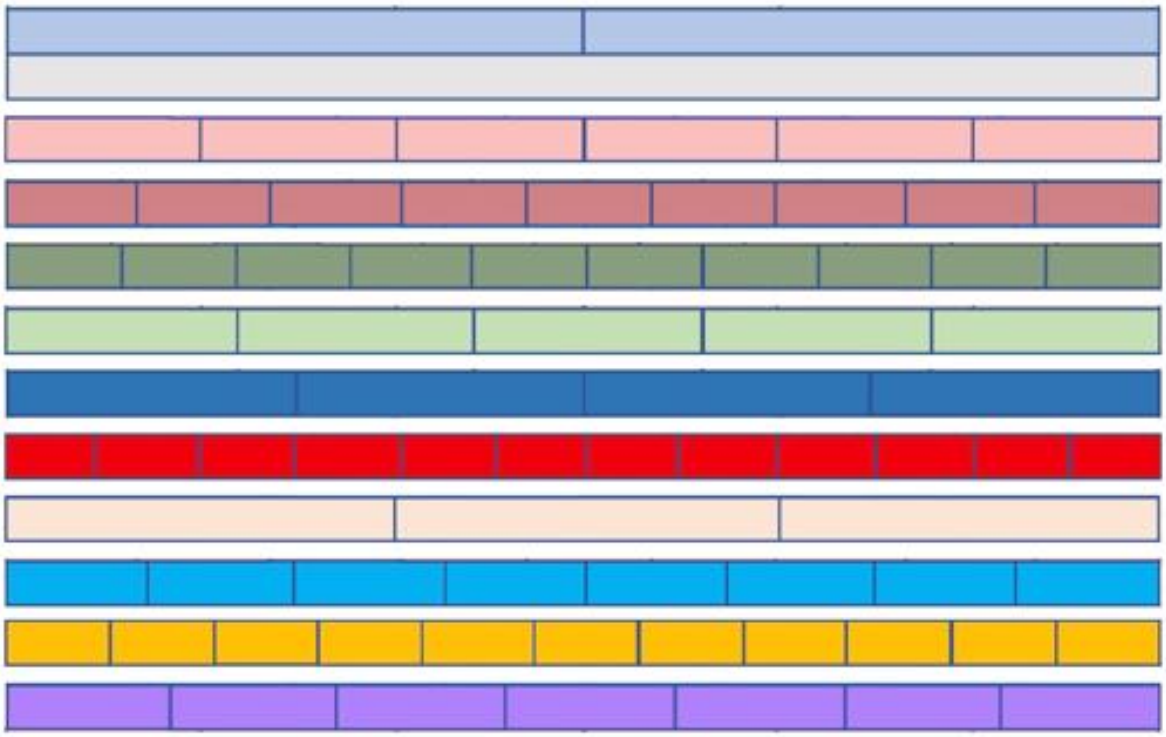
Players throw 2 dice or use a spinner, make a fraction with the numbers on the dice and put the smaller number on top. For example, a 2 and a 3 make $\frac{2}{3}$. The biggest fraction wins the round and scores a point.

Use the fraction wall to compare the fractions

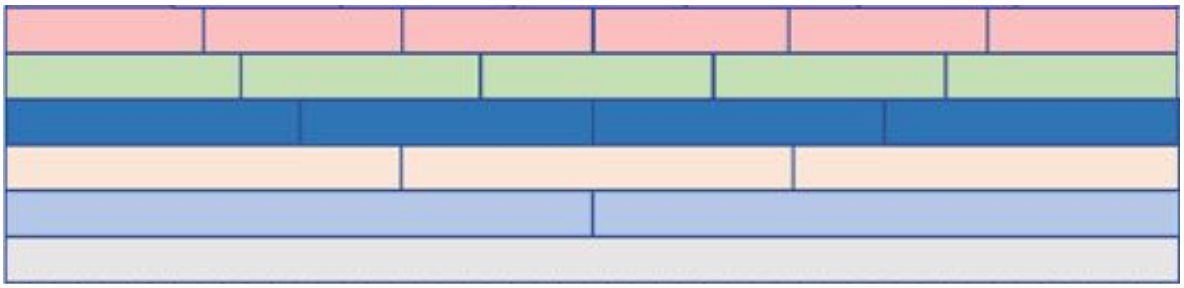


More fraction walls

FRACTION WALL PUZZLE: Cut out the strips and build the wall with the fractions getting smaller as the wall goes up.

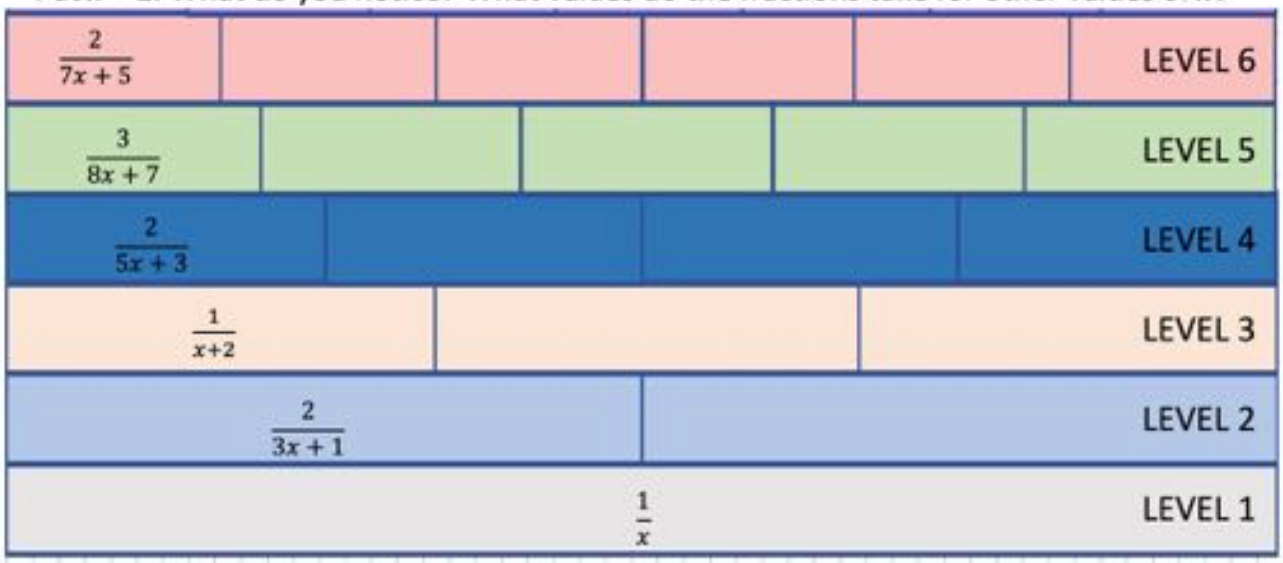


FRACTION WALL FOR DOUBLE SIX GAME



The Algebraic Fraction Wall

Put $x = 1$. What do you notice? What values do the fractions take for other values of x?



HOME LEARNING AND INCLUSION GUIDE

THEME: FRACTIONS

Early Years – Fraction Wall Puzzle

Divide a cake or a loaf of bread or a bar of chocolate between a group of people so that each person gets (as nearly as possible) the same amount.

Talk about everyone getting the same fraction giving them a fair share.

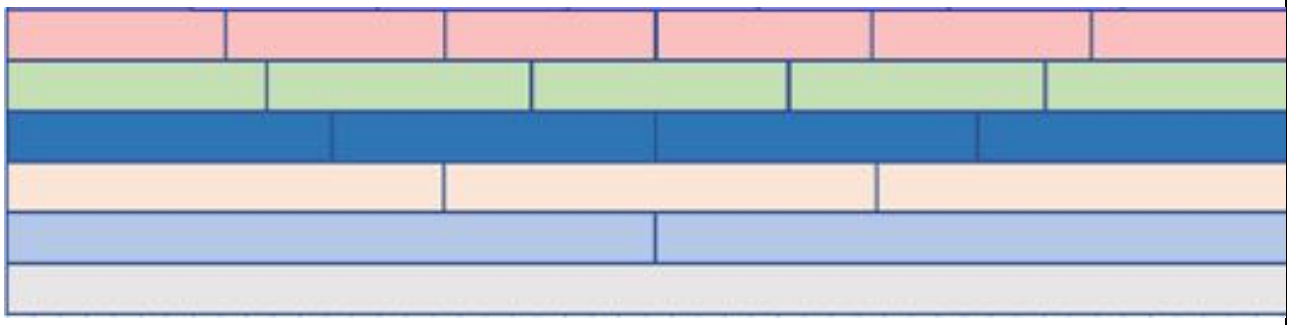
With very young children introduce the fraction names gradually. Talk about two equal shares giving half each. Build up the concept of equal shares and as the children get older you can introduce the words third, quarter etc. into the everyday vocabulary.

Make a large fraction wall from cardboard in different colours as shown below and cut it carefully into pieces.

Let the children play with the pieces.

You might suggest matching lengths, for example two dark blue make one light blue, the six pink make ONE UNIT etc.

Talk about breaking the ONE UNIT into smaller pieces. You might compare this with sharing a bar of chocolate or a cake into equal shares.



Show the children how to build the Fraction Wall Puzzle with the largest piece for ONE UNIT at the bottom and smaller and smaller pieces in each layer as the wall is built up. Then mix the pieces up and ask them to re-build the wall.

If you have cubes that link together such as Multilink, let the learners play with the cubes. Then make a fraction wall similar to the picture. Start by sorting out 12 cubes of one colour and joining them to make a bar. Then make:

2 bars with 6 cubes in each (brown and pink in the diagram),

3 bars of 4 cubes each (orange and yellow in the diagram),

4 bars of 3 units each (light green and grey in the diagram),

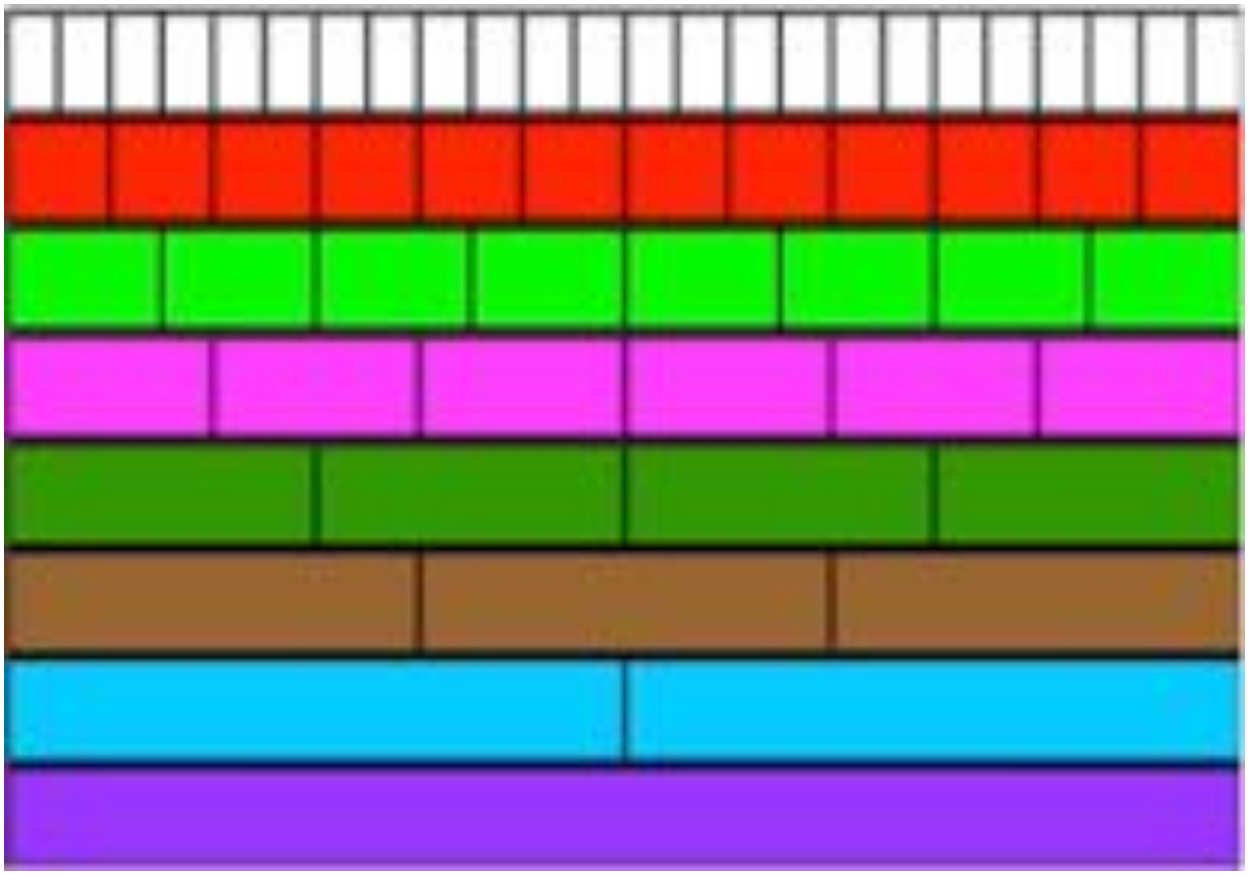
6 bars of 2 units each (dark green and blue in the diagram, and

12 single cubes (black and white in the diagram)

Years 1 to 3

In Years 1 – 3 engage with the fraction puzzle in a similar way to that described for the Early Years and talk about sharing the unit into smaller equal parts that are called fractions. Gradually begin to introduce the names half, quarter, fifth and sixth.

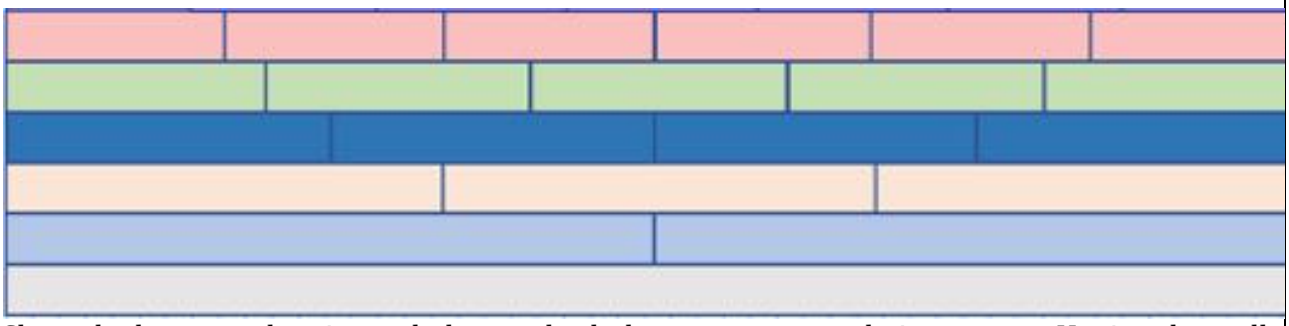
When the learners can easily do the easier Fraction Wall jig-saw puzzle introduce the one pictured below. Print the puzzle from page 3 and cut it up into bits. Show the class how the bits can be put together to make the fraction wall. Then mix up the bits and ask the children to solve the jig-saw puzzle for themselves working in small groups.



Years 4 to 6

Introduce the simple Fraction Wall as a jig-saw puzzle.

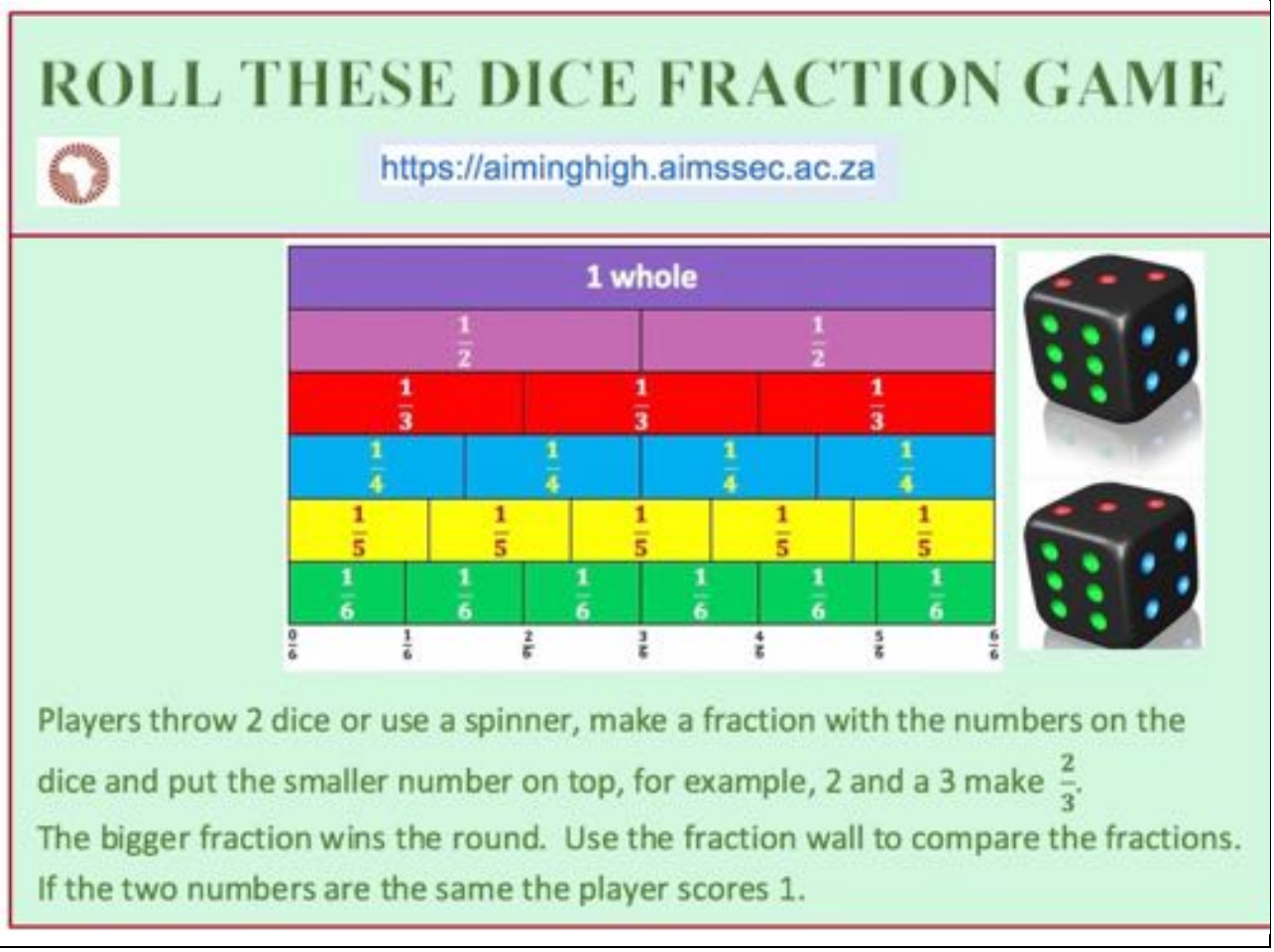
Then fill in the fractions in each of the blocks.



Show the learners the picture below and ask them to compare their answers. Having the wall with the smallest fractions on the bottom layer makes it natural to mark the scale of sixths along the bottom edge. An understanding of this way of labelling is helpful when comparing the fractions.

Often play the Fraction Wall Double Six Game for 5 minutes at the start of lessons. It is the same game as the Lesson Starter on page 2.

To make the game more challenging, gradually build up more layers in the fraction wall. You will need to make spinners with all the numbers required (see page 8).



Why do this activity?

This visual representation of fractions is very powerful. The main aim of the activity is for learners to find equivalent fractions using the image, but then to deduce a "rule" (or more than one) for finding equivalent fractions without a picture.

Learning objectives

In doing this activity students will have an opportunity to:

- describe and compare common fractions in diagram form;
- recognize and use equivalent forms of common fractions.

Generic competences

In doing this activity students will have an opportunity to **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.

Suggestions for teaching

Ask the learners if they can find rules for finding equivalent fractions and for checking whether two fractions are equivalent. Give the learners time to talk to each other about this and then orchestrate a discussion asking the learners to explain their rules.

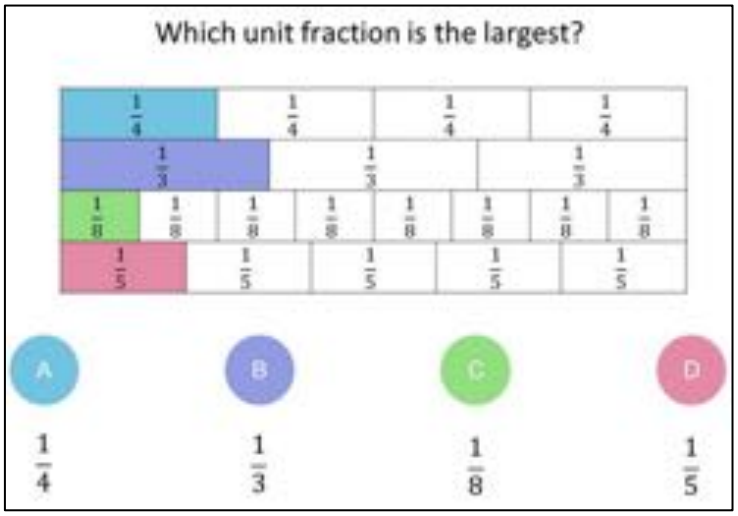
Finally summarize what they have learnt giving the explanations that you think are needed. Understanding is all important so it's better to cover less content and understand it all than to cover more and get confused. Don't rush this. It's sometimes best to leave the topic for another day especially with children age 10 or younger.

Diagnostic Assessment

This should take about 5–10 minutes.

Use this diagnostic quiz at **the end of the session** to find out how much the learners have understood. Show this question to the learners and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask all learners in your group why they gave their answers and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. Learners frequently guess at random. It is important to ask them to give reasons for their answers as it helps them to develop communication skills and it helps other learners who could not answer the question.
3. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class **again** to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The fraction wall diagram helps learners to see that larger numbers in the denominator give smaller fractions.

B. is the correct answer.

Common Misconceptions

C. Learners may give the answer 1/8 because 8 is the largest number. https://diagnosticquestions.com

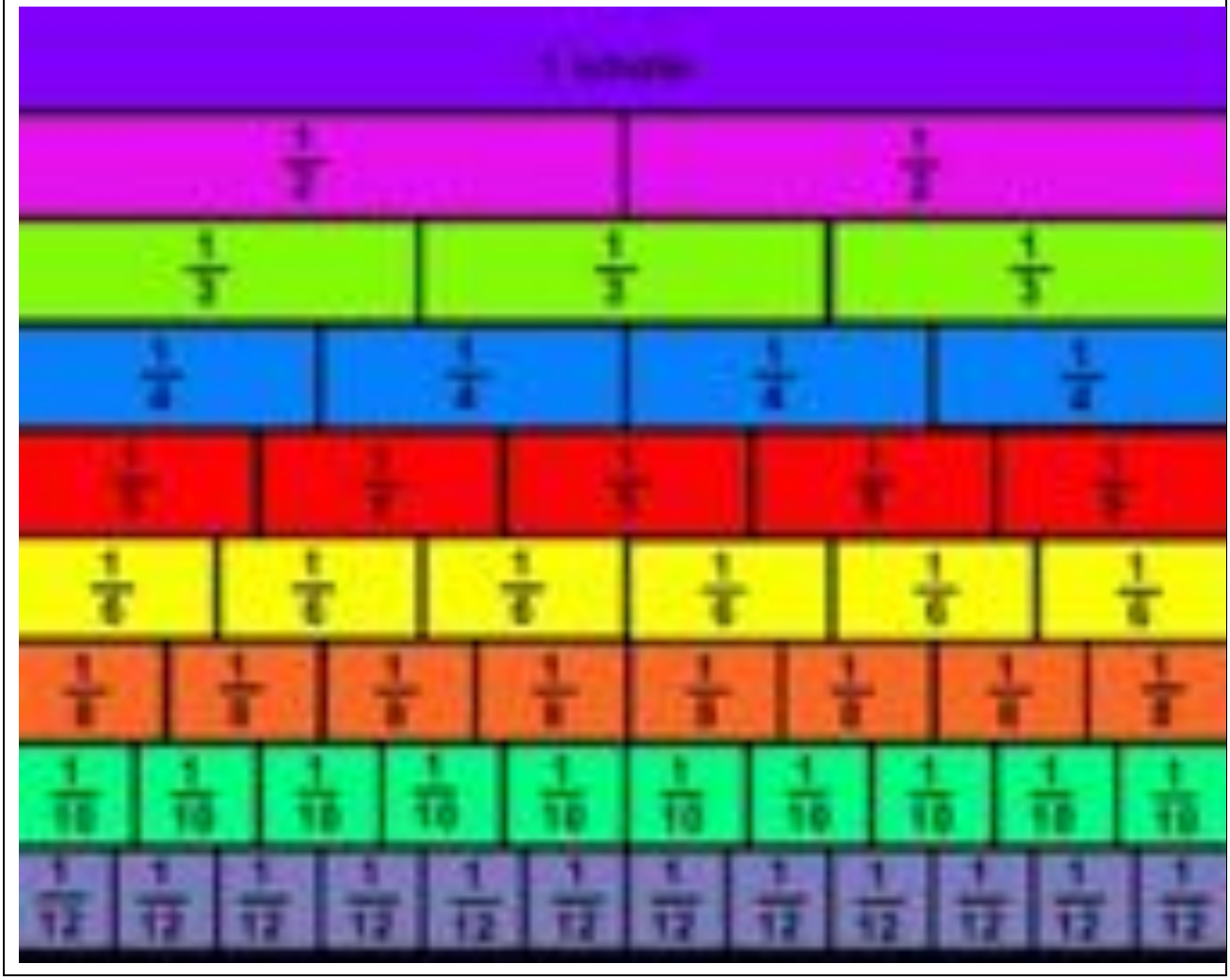
Years 7 to 9

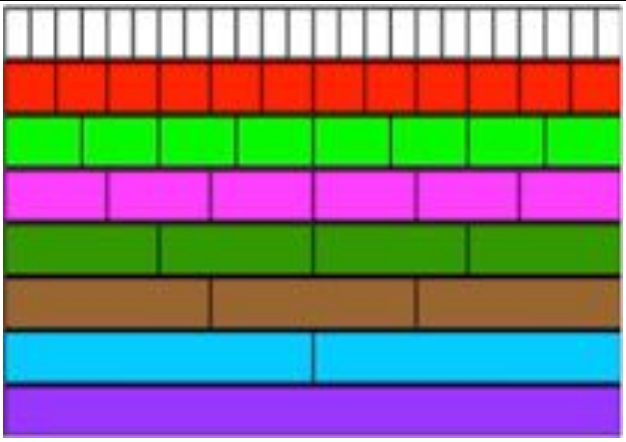
Teach the lesson exactly as on page 1. Use the HELP slip for learners who have difficulties and the NEXT strip for those who finish the work quickly.

If you have a group of learners working together, especially if it is a mixed age group, then mini whiteboards or showboards will be very useful. Ask everyone to write an answer on their showboard. Then, at a signal from you, everyone must raise their showboard at the same time so that you can see what they have written. In this way you can quickly assess what the children do and do not understand and who are succeeding and who need more support and help.

Showboards can be made by laminating a sheet of A4 paper that is plain on one side and has squares on the other. Each learner needs a showboard, a marker pen (dry marker) and piece of cloth to clean the board.

You might like to make large photocopies of this fraction wall, A4 or A3 size, to demonstrate and then display on the classroom wall so that learners can refer to it.





This wall without the fractions written in (as on page 1 and page 3) is also useful because it is good for the learners to work out for themselves what the fractions are. The wall with the fractions written in is good for a wall display because it provides a helpful reminder. Each learner might draw and colour a fraction wall for themselves on squared paper, making the wall 24 squares wide, and paste it in their notebook.

If the learners have not met Fraction Walls before show them the picture and ask the learners what they see in it without telling them at the start that it will be a lesson on fractions. They may spontaneously talk about fractions. If not ask some of the key questions such as "What do you notice about the lengths of the different bits in the picture?" How many pink bits are there?", "How many of the brown bits match the purple bit?" and "If the purple bit is one unit then what fraction is a brown bit?".

Tell the children "This wall is 1 unit wide" and ask "What fractions does it show?"

Then ask the learners to explain how the picture shows that two quarters are equivalent to one half $\frac{2}{4} = \frac{1}{2}$. (Two dark green bits are the same length as one turquoise blue bit).

Ask several learners to explain this and then repeat the explanation to reinforce the idea.

Then ask them to write on their showboards and hold up fractions **equivalent to** $\frac{3}{4}$.

If this list does not include all the equivalent fractions that can be seen on the fraction wall, ask the learners if they can find another one and write it on their showboard. Then ask them to hold their showboards up when you say 'NOW'.

Write on your board all the answers that the class give as equivalent whether correct or not, then ask if all the answers on your board are correct and if not why not. Lead a discussion where the learners try to explain which are correct answers and which are incorrect and to give reasons why. Do NOT name the learners who supplied the wrong answers.

When you are satisfied that the learners know and understand which fractions are equivalent to $\frac{3}{4}$ move on to the other fractions $\frac{5}{6}$, $\frac{7}{12}$ and $\frac{6}{24}$ etc.

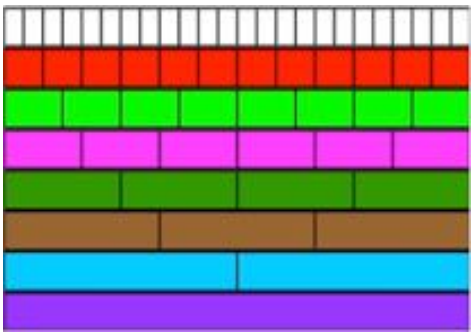
Then use the wall to add the fractions $\frac{1}{2} + \frac{2}{3} + \frac{5}{12}$. Learners should practise adding other fractions using the fraction wall. If they first meet addition of fractions using a fraction wall learners will easily understand the concept of a common denominator when it is introduced.

Key Questions

- What do you notice about the lengths of the different bits in the picture?
- How many pink bits are there?
- How many of the brown bits match the purple bit?
- If the purple bit is one unit then what fraction is a brown bit?
- How many of the bits of this colour match how many bits of that colour?

SOLUTION

In the diagram, two quarters (shown in dark green) are the same width as one half (in blue) showing $\frac{2}{4} = \frac{1}{2}$. The wall shows that three sixths, four eights, six twelfths and twelve twenty-fourths all belong to the set of fractions equivalent to one half.



$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12} = \frac{12}{24} = \cdots$$

There are more equivalent fractions in the set but, for now we are focussing on this fraction wall. Similarly:

three quarters $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{18}{24}$

five sixths $\frac{5}{6} = \frac{10}{12} = \frac{20}{24}$

seven twelfths $\frac{7}{12} = \frac{14}{24}$

six twenty-fourths $\frac{6}{24} = \frac{3}{12} = \frac{1}{4}$.

To find an equivalent fraction multiply the top and bottom of the fraction by the same number or divide the top and bottom of the fraction by the same number. This process is called **cancelling.** For example $\frac{9}{24} = \frac{6}{16} = \frac{3}{8}$ are all equivalent fractions.

Dividing the top and bottom of the fraction $\frac{9}{24}$ by the common factor 3 gives the fraction $\frac{3}{8}$.

Dividing the top and bottom of the fraction $\frac{6}{16}$ by the common factor 2 gives the fraction $\frac{3}{8}$.

A fraction like $\frac{3}{8}$ (called a fraction in its lowest terms) has no common factor between the numerator and denominator. It is equivalent to infinitely many other fractions that all have the same value. The set of all such fractions is called an equivalence class.

Looking at the fraction wall you see that the white bits are twenty-fourths and the light green bits are eighths. You can also see that 3 twenty-fourths is the same as one eighth and 9 twenty-fourths is the same as 3 eighths.

To add fractions, all the fractions need to have the same denominator.

For example: $\mathbf{\frac{1}{2} + \frac{2}{3} + \frac{5}{12}}$ will be the same if you replace one half (one blue bit) by 6 twelfths (6 red bits) and replace 2 thirds (2 brown bits) by 8 twelfths (8 red bits).

$$\frac{1}{2} + \frac{2}{3} + \frac{5}{12}$$
$$= \frac{6}{12} + \frac{8}{12} + \frac{5}{12}$$
$$= \frac{21}{12}$$

So $\mathbf{\frac{1}{2} + \frac{2}{3} + \frac{5}{12}}$ is equivalent to 21 twelfths (red bits) made up of (12+9 twelfths). In the Fraction Wall that is 1 purple unit bar and 9 red bits or 3 dark green bits (9 twelfths or three quarters). So $\frac{1}{2} + \frac{2}{3} + \frac{5}{12} = 1\frac{3}{4}$.

Years 9 and 10 Play the ALGEBRAIC FRACTION WALL GAME

as described on page 2.

Years 10 to 12 FILLING AN INTERVAL WITH INFINITELY MANY BITS

This is an exercise about thinking mathematically and using your imagination and visualisation skills to work on an abstract concept.

Think of the fraction wall idea. Imagine starting with a bar precisely 1 unit long, then imagine laying a sequence of bars end to end each one precisely half the length of the previous bar. Imagine repeating this process on and on for ever. What would the lengths of all the bars add up to?

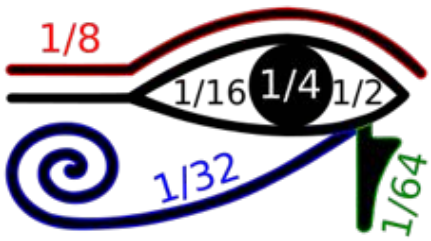
Draw a sketch of this process as follows:

1. Imagine a line segment AB exactly 2 units long. Start at A. Each time you will move towards the endpoint B, half-way along the gap between where you are and B. Imagine continuing this process indefinitely, on and on for ever.
2. Start at A. Imagine moving a distance 1 unit towards B and make a mark.
3. Next imagine moving a distance $\frac{1}{2}$ unit towards B and make a second mark. What is the distance of this mark from B?
4. Imagine moving a distance $\frac{1}{4}$ unit and make a mark. What is the distance of this mark from B?
5. Imagine the next step moving a distance $\frac{1}{8}$ unit and make a mark, then $\frac{1}{16}$ unit and make a mark.
6. Imagine continuing this process, moving distances $\frac{1}{32}=\frac{1}{2^5}$, then $\frac{1}{64}=\frac{1}{2^6}$, … $\frac{1}{2^n}$, and so on.

What happens in the limit as n tends to infinity?

This demonstrates the sum of the infinite geometric series with first term 1 and common ratio $\frac{1}{2}$.

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\frac{1}{2^n}+\frac{1}{2^{n+1}}+\cdots=2.$$



The Eye of Horus was believed to have protective magical power and it appeared frequently in ancient Egyptian art. Egyptians wrote all their fractions with 1 on the top, or as the sum of other fractions. For example, the fraction $\frac{7}{8}$ would be written $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$.

Do your own investigation into the history of fractions, especially Egyptian fractions.

Follow up

Primary and Lower Secondary

Chocolate fractions https://aiminghigh.aimssec.ac.za/chocolate-fractions/
Fractions by Halves https://aiminghigh.aimssec.ac.za/fractions-by-halves/
Fractions by Thirds https://aiminghigh.aimssec.ac.za/fractions-by-thirds/
Tangram Fractions https://aiminghigh.aimssec.ac.za/tangram-fractions/
Divide Divide https://aiminghigh.aimssec.ac.za/divide-divide/
Repetition https://aiminghigh.aimssec.ac.za/repetition/
Egyptian Fractions https://aiminghigh.aimssec.ac.za/egyptian-fractions/
The Greedy Algorithm https://aiminghigh.aimssec.ac.za/the-greedy-algorithm/

Upper Secondary

Peaches https://aiminghigh.aimssec.ac.za/peaches/
GP Algebraically https://aiminghigh.aimssec.ac.za/gp-algebraically/
GP Geometrically https://aiminghigh.aimssec.ac.za/gp-geometrically/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13